Humboldt-Universität zu Berlin
Institut für Physik
AG Theoretische Optik & Photonik
https://top.physik.hu-berlin.de

Computational Photonics/ Computerorientierte Photonik (SoSe 2019)

Exercise 2 (released on April 18th, 2019) *Due on May 3rd, 2019 at 8 am!*

Lecturer: Prof. Dr. Kurt Busch
Tutor: Dr. Dan-Nha Huynh

1 Numerical Dispersion Relation

(a) Derivation of the Dispersion Relation

Explicitly derive the numerical dispersion relation for the wave equation as sketched in the lecture and verify the result presented there. To that end, consider a wave of the form $E(x,t) = E_0 e^{\mathrm{i}(\tilde{\omega} n \Delta t - \tilde{k} j \Delta x)}$ and derive the relation $\tilde{\omega} = \tilde{\omega}(\tilde{k})$ by plugging this into the wave equation. Assume that $\Delta t \to 0$. State for which values of $\Delta x, \Delta t$ the angular frequency $\tilde{\omega}$ remains real. What happens for complex $\tilde{\omega}$?

(b) Relation for $\tilde{k}$

Derive the dispersion relation $\tilde{k} = \tilde{k}(\tilde{\omega})$ and examine for what values of $\Delta t, \Delta x$ the wave number $\tilde{k}$ remains real.

2 1D FDTD

(a) Reduction of Maxwell's equations to one dimension

Consider a system that is isotropic and homogeneous in y- and z-direction. Then, the six coupled differential equations can be reduced to two distinct systems, each with two coupled differential equations. Perform this reduction. Why is it sufficient to consider only one set of the coupled equations?

(b) Discretization of the equations by finite differences

In order to solve the reduced equations numerically, we have to discretize them in time and space. FDTD uses a special discretization where E- and H-field are not discretized at the same grid. Instead, it employs two staggered grids. The grid of the H-field is shifted by half of an increment in space and time with respect to the grid of the E-field. The step width in space is Δx and in time Δt. The grid for the E-field is $x_\mathrm{E} = i\Delta x$, $t_\mathrm{E} = n\Delta t$ and thus for the H-field $x_\mathrm{H} = (i+0.5)\Delta x$, $t_\mathrm{H} = (n+0.5)\Delta t$, where $i, n \in \mathbb{N}$. All differential operators are approximated by finite differences of the form $\frac{\partial f(r)}{\partial r} \approx \frac{f(r+\Delta r/2) - f(r-\Delta r/2)}{\Delta r}$. Write down the discretized equations and solve them for the latest value in time to obtain an update algorithm for both of the field components.

(c) Implementation of a 1D FDTD code

Implement a basic FDTD code that simulates the time evolution of an electromagnetic pulse in a one-dimensional system. Consider a system that does not contain any dielectric or magnetic materials. Make sure that the first and last point of the system are points of the electric field. The last point should always be zero during the simulation (perfect electric conductor). The first point is used as a source, so its value should be set, e.g. to a rectangular function, a sine-function or a Gaussian pulse in time. Play with the space and time steps. Try $\Delta t = 0.99\Delta x$, $\Delta t = 1.01\Delta x$ and $\Delta t = \Delta x$. Also investigate the influence of the number of grid points. What can you observe?

Hint: Use the MatLab function `diff` to efficiently calculate the finite differences.

(d) Poynting vector

Implement the calculation of the Poynting vector at a given discretization point. Remember that the electric and magnetic field are defined on shifted grids. The time-averaged Poynting vector at a fixed position x and for a given frequency is given by

$$\langle \mathbf{S}(x, \omega) \rangle = \frac{1}{2} \mathrm{Re} \left(\mathbf{E}(x, \omega) \times \mathbf{H}^*(x, \omega) \right)$$

Here, $\mathbf{E}(\omega)$ and $\mathbf{H}(\omega)$ are the Fourier components of the fields at fixed position x and given angular frequency ω. Note that both field have to be Fourier transformed *before* taking the cross product.

Examine the spectral properties of different Gaussian pulses.

(e) Spatial resolution

If you succeeded in problem set $\boxed{\mathbf{1}}$ task **(a)**, try to violate the stability criterion you have derived before and examine the effect in the solution of the numerical problem.

(f) Introduction of dielectric materials

Modify your program to include dielectric materials. Fill the right half of the system with a dielectric ($n = 2$). What happens with the pulse at the boundary? Again, check the influence of the space and time step.

Discussion in the exercise on May 3rd, 2019

Problems marked with * are voluntary. All others are **due** on **May 3rd, 2019** at **8am** via e-mail to *dhuynh@physik.hu-berlin.de* or has to be brought to the exercise class on a flash drive. Analytical solutions are accepted handwritten or in the .pdf format, numerical solution in a digital version.