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Part I

HPC and Exascale

Introduction

This part of my PhD thesis presents a state of the art of High Performance Computing. It describes the tools we need for our study. High Performance Computing, HPC, does not have a strict definition. The history starts with domain scientists in need of more complex and longer computation for models checking or simulations. They developed their own tools beginning with vacuum tubes computers which can be consider as a milestone in HPC history. Since this first machine the technology became more and more complex at every layer: the hardware conception, the software to handle it and even the models and topologies. HPC is now a scientific field on its own but always dedicated to the end purpose, domain scientist computations. HPC experts are interested in supercomputer construction, architecture and code optimization, interconnection behaviors and creating more software, framework or tools to facilitate access to these very complex machines.

In this part we give a non-exhaustive definition of HPC focusing models, hardware and tools required for our study in three chapters.

We first focus on what are the theoretic models for the machines and the memory we base our work on. This first chapter also presents what is defined as performance for HPC and the main laws that concern it.

The second chapter details the architecture base on these models. We present nowadays platforms with dominant constructors and architectures from multi-cores to specific many-cores machines. Representative members of today's supercomputers are described. We show that hybrid architectures seem to be the only plausible way to reach the exascale: they offer the best performance per watt ratio and nowadays API/tools allows to target them more easily and efficiently.

In the third chapter we detail the main software to target those complex architectures. We present tools, frameworks and API for shared and distributed memory. We also introduce the main benchmarks used in the HPC world in order to rank the most powerful supercomputers. This chapter also shows that those benchmarks are not the bests to give an accurate score or scale for "realistic" domain scientists applications.

Chapter 1

Models of HPC

1.1 Introduction

High Performance Computing (HPC) takes his roots from the beginning of computer odyssey in the middle of 20th century. A lot of rules, observations, theories emerged from it and even Computer Sciences fields. In order to understand and characterize HPC and supercomputers, some knowledge on theory is required. This part describes the Von Neumann model, the generic model of sequential computer on which every nowadays machine is built. It is presented along with the Flynn taxonomy that is a classification of the different execution models. We also present the different memory models based on those elements.

Then we give more details on what is parallelism and how to reach performances though it. And thus we define what performance implies in HPC. The Amdahl's and Gustafson's laws are presented and detailed along with the strong and weak scaling used in our study.

1.2 Von Neumann Model

First computers, in early 20th, were built using vacuum tubes making them high power consuming, hard to maintain and expansive to build. The most famous of first vacuum tubes supercomputers, the ENIAC, was based on decimal system. It might be the most known of first supercomputers but the real revolution came from its successor. In 1944 the first binary system based computer, called the Electric Discrete Variable Automatic Computer (EDVAC), was created. In the EDVAC team, a physicists described the logical model of this computer and provides a model on which every nowadays computing device is based.

John Von Neumann published its *First Draft of a Report on the EDVAC* [VN93] in 1945. Extracted from this work, the model know as the Von Neumann model or more generally Von Neumann Machine appears. The model is presented on figure 1.1.

On that figure we identify three parts, the input and output devices and in the middle the computational device itself.

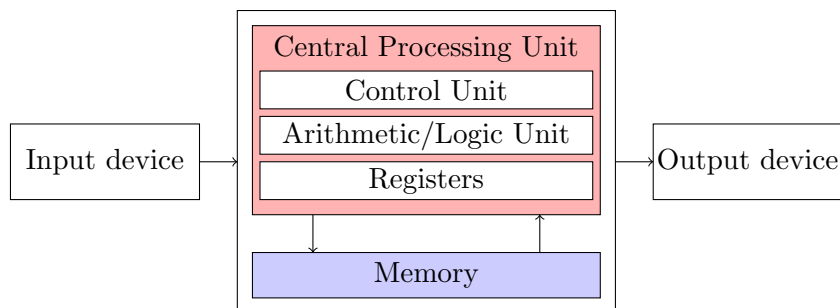


Figure 1.1: Von Neumann model

Input/Output devices The input and output devices are used to store in a read/write way data. They can be represented as hard drives, solid state drives, monitors, printers or even mouse and keyboard. The input and output devices can also be the same, reading and writing in the same area.

Inside the computational device we find the memory, for the most common nowadays architectures it can be considered as a Random Access Memory (RAM). Several kind of memory exists and will be discussed later.

Central Processing Unit The Central Processing Unit, CPU, is composed of several elements in this model. On one hand, the *Arithmetic and Logic Unit*, ALU, which takes as input one or two values and apply an operation on those data. They can be either logics with operations such as AND, OR, XOR, etc. or arithmetics with operations such as ADD, MUL, SUB, etc. Of course those operations are way more complex on modern CPUs. On the other hand, we find the *Control Unit*, CU, which control the data carriage to the ALU from the memory and the operation to be perform on data. It is also the part that takes care of the Program Counter (PC), the address of the next instruction in the program. We can also identify the Register section which represent data location used for both ALU and CU to store temporary results, the current instruction address, etc. Some representation may vary, the Registers can be represented directly inside the ALU or the CU.

Buses The links between those elements are called Buses and can be separated between data buses, control buses and addresses buses. They will have a huge importance for the first machine optimization, growing the size of the buses from 2, 8, 16, 32, 64 and even more for vector machine with 128 and 256 bits.

The usual processing flow on such an architecture can be summarized as a loop:

- Fetch instruction at current PC from memory;
- Decode instruction using the Instruction Set Architecture (ISA). Known ISA are Reduce Instruction Set Computer architecture (RISC) and Complex Instruction Set Computer architecture (CISC);
- Evaluate operand(s) address(es);
- Fetch operand(s) from memory;
- Execute operation(s), with some instructions sets and new architectures several similar operations can be processed in the same clock time;
- Store results, increase PC.

Every devices or machines we describe in the next chapter have this architecture as a basis. One will consider execution models and architecture models to characterize HPC architectures.

1.3 Flynn taxonomy and execution models

The Von Neumann model gives us a generic idea of how a computational unit is fashioned. The constant demand in more powerful computers required the scientists to find more way to provide this computational power. In 2001, IBM proposed the first multi-core processor on the same die, the Power4 with its 2 cores. This evolution required new paradigms. A right characterization is then essential to be able to target the right architecture for the right purpose. The Flynn taxonomy presents a hierarchical organization of computation machines and executions models.

In this classification [Fly72] from 1972, Michael J. Flynn presents the SISD, MISD, MIMD, and SIMD models represented on in table 1.1 and figure 1.2. Every of those execution model correspond to a specific machine and function.

		Data Stream(s) →	
Instruction Stream(s) ↓		Single Data (SD)	Multiple Data (MD)
	Single Instruction (SI)	SISD	SIMD <i>SIMT</i>
	Multiple Instruction (MI)	MISD	MIMD <i>SPMD/MPMD</i>

Table 1.1: Flynn taxonomy for execution models completed with SPMD and SIMT models

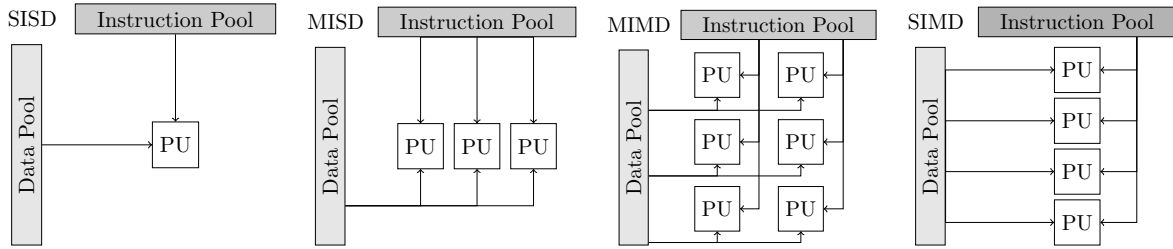


Figure 1.2: Flynn taxonomy schematic representation of execution models

1.3.1 Single Instruction, Single Data: SISD

This is the model corresponding to a single core CPU like in the Von Neumann model. This sequential model takes one instruction, operates on one data and the result is then store and the process continues over. SISD is important to consider as a reference computational time and will be taken in account in the next part for Amdahl's and Gustafson's laws.

1.3.2 Multiple Instructions, Single Data: MISD

This model can correspond to a pipelined computer. Different operations are applied to the datum, which is transfered to the next computational unit and so on. This is the least common execution model.

1.3.3 Multiple Instructions, Multiple Data: MIMD

In MIMD every element executes its own instructions on its own data set. This can represent the behavior of a processor using several cores, threads or even the different nodes of a supercomputer cluster. Two subcategories are identified in this model:

SPMD

The Single Program Multiple Data model, SPMD, is the most famous parallelism way for HPC purpose: each process execute the same program. At opposite to SIMD the programs are the same but does not share the same instruction counter. This model was proposed for the first time in [DGNP88] in 1988 using Fortran. This is the common approach working with runtime like MPI. The programs are the same and the execution similar but based on their ID the processes will target different data.



Figure 1.3: MIMD memory models

MPMD

The Multiple Program Multiple Data model is also known for HPC. Generally with a separation between a main program generating data for sub-programs. This is the model on which we work in part II regarding the Langford problem resolution using split of the resolution tree.

1.3.4 Single Instruction, Multiple Data: SIMD

This execution model corresponds to a many-core architecture like a GPU. SIMD can be extended from 2 to 16 elements for classical CPUs to hundreds and even thousands of core for GPGPUs. In the same clock, the same operation is executed on every process on different data. The best example stay the work on matrices like a stencil, same instruction executed on every element of the matrix.

1.3.5 SIMT

We find another characterization to describe the new GPUs architecture: Single Instruction, Multiple Threads. This appears in one of NVIDIA's company paper [LNOM08]. This model describes a combination of MIMD and SIMD architectures, every block of threads is working with the same control processor on different data and every block has its own instruction counter. This is the model we describe in part 3.3.3 used for the *warps* model in NVIDIA CUDA.

1.4 Memory

In addition of the execution model and parallelism the memory access patterns have a main role on performances especially in SIMD and MIMD. In this classification we identify three categories: UMA, NUMA and NoRMA for shared and distributed cases. This model have been pointed out in the Johnson's taxonomy[Joh88].

Those different types of memory for SIMD/MIMD model are summed up in figure 1.3.

1.4.1 Shared memory

When it comes to multi-threaded and multi-cores like MIMD or SIMD execution models, several kind of memory models are possible. We give a description of the most common shared memories architectures.



Figure 1.4: UMA vs NUMA memory models

UMA

The Uniform Memory Access is a global memory shared by every threads or cores. In UMA every processor uses its own cache as local private memory. The addresses can be accessed directly by each processor which makes the access time ideal. The downside is that more processors require more buses and thus UMA is hardly scalable. The cache consistency problem also appears in this context and will be discussed in next part. Indeed, if a data is loaded in one processor cache and modified, this information need to be spread to the memory and maybe other processes cache.

With the arising of accelerators like GPUs and their own memory, some constructors found ways to create UMA with heterogeneous memory. AMD creates the heterogeneous UMA, hUMA [RF13], in 2013 allowing CPU and GPU to target the same memory area.

NUMA

In Non Unified Memory Access every processor have access to its own private memory but allows other processors to access those area though Lightning Data Transport, LDT or Quick Path Interconnect, QPI, for Intel architectures.

As we mention for the UMA memory, even if the processors does not directly access to the memory cache coherency is important. Two methods are possible: on one hand, the most used is Cache-Coherent NUMA (CC-NUMA) were protocols are used to keep data coherency through the memory. On the other hand No Cache NUMA (NC-NUMA) forces the processes to avoid cache utilization and write results in main memory losing all the benefits of caching data.

COMA

In Cache-Only Memory Accesses, the whole memory is see as a cache from every processes. Attraction memory is setting up and will attract the data near the process that will use those data. This model is less commonly use and lead to, in best cases, same results as NUMA.

1.4.2 Distributed memory

The previous models are based on shared memory, in the case where the processes can access memory of their neighbors processes. In some cases, like supercomputers, it would be too heavy for processors to handle the requests of all the others through the network. Each process or node will then possess its own local memory, that can be share with local processes. Then, in order to access to other nodes memory, communications through the network have to be done and copied in local memory. This distributed memory is called No Remote Memory Access (NoRMA).

Name	FLOPS	Year	Name	FLOPS	Year
kiloFLOPS	10^3		petaFLOPS	10^{15}	2005
megaFLOPS	10^6		exaFLOPS	10^{18}	2020 ?
gigaFLOPS	10^9	≈ 1980	zettaFLOPS	10^{21}	
teraFLOPS	10^{12}	1996	yottaFLOPS	10^{24}	

Table 1.2: Floating-point Operation per Second and years of reach in HPC.

1.5 Performances characterization in HPC

In the previous parts we described the different executions models, characterizations and memory models for HPC. Based on those tools we need to be able to emphasis the performances of a computer and a cluster.

The performance can be of several kind. It can first be define by the speed of the processor itself with the frequency defined in GHz. This information is not perfect because the ALU is not busy all the time due to memory accesses, communications or side effects. It can be used to estimate the highest computational power of a machine. We define the notion of *cycle* to be the number that determine the speed of a processor. This is the amount of time between two pulses of the oscillator. Higher cycles per seconds is better.

1.5.1 FLOPS

The Floating point Operations Per Second considers the number of floating-point operation that the system will executes in a second. They are an unit of performance for computers. Higher FLOPS is better. This is also the scale used to consider supercomputers computational power. For a cluster we can compute the theoretical FLOPS (peak) based on the processor frequency in GHz with:

$$FLOPS_{cluster} = \#nodes \times \frac{\#sockets}{\#node} \times \frac{\#cores}{\#socket} \times \frac{\#GHz}{\#core} \times \frac{FLOPS}{cycle} \quad (1.1)$$

With $\#nodes$ the number of computational node of the system, $\frac{\#sockets}{\#node}$ the number of sockets (= processor) per node, $\frac{\#cores}{\#socket}$ the number of core in the processor, $\frac{\#GHz}{\#core}$ the frequency of each core and finally $\frac{\#FLOP}{\#cycle}$ the number of floating-point operations per cycles for this architecture.

On figure 1.2, the scale of FLOPS and the year of the first world machine is presented. The next milestone, the exascale, is expected to be reach near 2020.

FLOPS is the main way to represent a computer's performance but other ways exists like Instructions Per Seconds (IPS), Instructions per Cycle (IPC) or Operations Per Second (OPS). Some benchmarks also provide their own metrics.

1.5.2 Power consumption

Another way to consider machine performance is to estimate the number of operations regarding the power consumption. It can consider all the previous metrics like FLOPS, IPS, IPC or OPS. Benchmarks, like the Green500, consider the FLOPS delivered over the watts consumed. For nowadays architectures the many-cores architectures like GPUs seems to deliver the best FLOPS per watt ratio.



Figure 1.5: Observed speedup: linear, typical and hyper-linear speedups

1.5.3 Scalability

The scalability express the way a program react to parallelism. When an algorithm is implemented on a serial machine and is ideal to solve a problem, one may consider to use it on more than one core, socket, node or even cluster. Indeed, one may expect less computation time, bigger problem or a combination of both while using more resources. This completely depend on the algorithm parallelization and is expressed through scalability. A scalable program will scale on as many processors as we give, whereas a poorly scalable one will give same of even worst results as the serial code. Scalability can be approach using speedup and efficiency.

1.5.4 Speedup and efficiency

The latency is the time required to complete a task in a program. Lower latency is better.

The speedup compare the latency of both sequential and parallel algorithm. In order to get relevant results, one may consider the best serial program against the best parallel implementation.

Considering n , the number of processes, and $n = 1$ the sequential case with T_n the execution time working on n processes and T_1 working on one process, the sequential execution time. The speedup can be defined using the latency by the formula:

$$\text{speedup} = S_n = \frac{T_1}{T_n} \quad (1.2)$$

As shown on figure 1.5 several kind of speedup can be observed.

Linear: reference The linear speedup usually represents the target for every program in HPC. Indeed, having the speedup growing linearly as the number of processors grows is the ideal case. Codes fall typical into two cases, typical and hyper-linear speedup.

Typical speedup This represents the most common observed speedup. As the number of processors grows, the program face several of the HPC walls like communications wall or memory wall. The increasing number of computational power is reduced to the sequential part or lose time in communications/exchanges.

Hyper-linear speedup In some cases we can observe an hyper-linear speedup, meaning that the results in parallel are even better than the ideal case. This can occur if the program can fit exactly in memory for less data on each processor or even fit perfectly for the cache utilization. The parallel algorithm can also be way more efficient than the sequential one.

In addition to speedup, the efficiency is defined by the speedup divided by the number of workers:

$$\text{efficiency} = E_n = \frac{S_n}{n} = \frac{T_1}{nT_n} \quad (1.3)$$

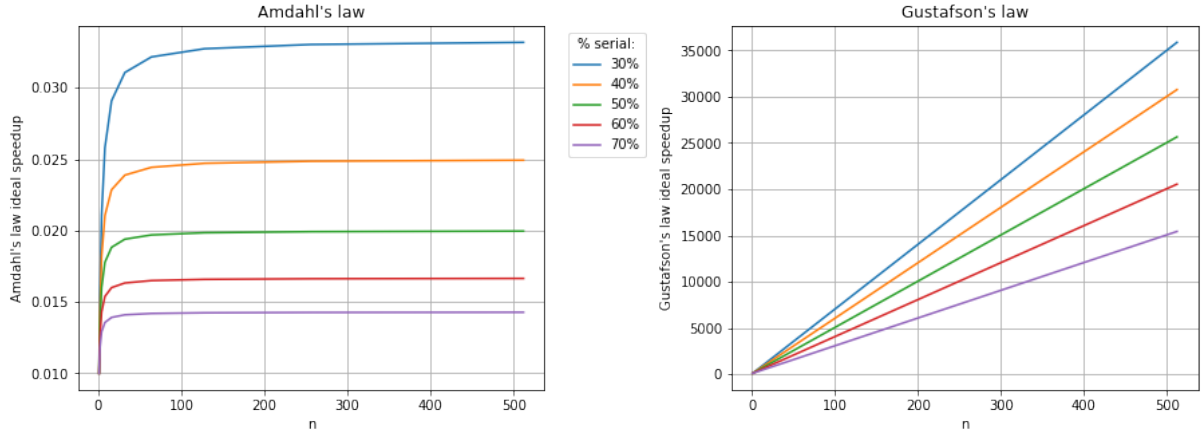


Figure 1.6: Theoretical speedup for Amdahl's (left) and Gustafson's (right) law

The efficiency, usually expressed in percent, represents the evolution of the code stability to growing number of processors. As the number of processes grows, a scalable application will keep an efficiency near 100%.

1.5.5 Amdahl's and Gustafson's law

The Amdahl's and Gustafson's laws are ways to evaluate the maximal possible speedup for an application taking in account different characteristics.

Amdahl's law

The Amdahl's law[Amd67] is used to find the theoretical speedup in latency of a program. We can separate a program into two parts, the one that can be execute in parallel and the one that is sequential. The law states that even if we reduce the parallel part using an infinity of processes the sequential part will reach 100% of the total computation time.

Extracted from the Amdahl paper the law can be written as:

$$S_n = \frac{1}{Seq + \frac{Par}{n}} \quad (1.4)$$

Where $Seq + Par = 1$ and Seq and Par respectively the sequential and parallel ratio of a program. Here if we use up to $n = \inf$ processes, $S_n \leq \frac{1}{Seq}$ the sequential part of the code become the most time consuming.

And the efficiency become:

$$E_n = \frac{1}{n \times Seq + Par} \quad (1.5)$$

A representation of Amdahl's speedup is presented on Fig. 1.6 with varying percentage of serial part. The parallel part is like $Par = (100 - Ser)\%$.

Gustafson's law

The Amdahl's law is focused on time with problem of the same size. John L. Gustafson's idea is that using more computational units, the problem size can grow accordingly. He considered a constant computation time with evolving problem, growing the size accordingly to the number of processes. Indeed the parallel part grows as the problem size do, reducing the percentage of the serial part for the overall resolution.

The speedup can now be estimated by:

$$S_n = Seq + Par \times n \quad (1.6)$$

And the efficiency:

$$E_n = \frac{Seq}{n} + Par \quad (1.7)$$

Both Amdahl's and Gustafson's law are applicable and they represent two solution to check the speedup of our applications. The strong scaling, looking at how the computation time vary evolving only the number of processes, not the problem size. The weak scaling, at opposite to strong scaling we look how the computation time evolve varying the problem size keeping the same amount of work per processes.

1.6 Conclusions

In this chapter we presented the different basic tools to be able to understand HPC: the Von Neumann model that is implemented in every nowadays architecture; the Flynn taxonomy that is in constant evolution with new paradigms like recent SIMT from NVIDIA. We also presented the memory types that will be use at different layers in our clusters, from node memory, CPU-GPGPU shared memory space to global fast shared memory. We finished by presenting the most important laws with Amdahl's and Gustafson's laws. We introduced the concept of strong and weak scaling that will lead our tests through all the examples in Part II and Part III.

Those models have now to be confronted to the reality with hardware implementation and market reality, the vendors. The next part will introduce chronologically hardware and their optimization but always keeping a link with the models presented in this part. As there is always a gap between models and implementation we will have to find way to rank and characterize those architecture. This will be discuss in the last chapter.

Chapter 2

Hardware in HPC

2.1 Introduction

The knowledge of hardware architecture is essential to reach performances through optimizations. Even if the nowadays software, API, framework or runtime already handle most of optimizations, the last percents of gain are architecture's dependent. In this chapter we describe the most important devices architectures from classical processors, General Purpose Graphics Processing Units (GPGPUs), Field Programmable Gate Arrays (FPGAs) and Application-Specific Integrated Circuits (ASICs). This study keeps a focus on multi-core processors and GPUs as we based our tests on those devices.

This chapter also details the architecture of some remarkable supercomputers. This has to go with the description of interconnection network with the most famous interconnection topologies.

We choose to present the architectures in a chronological order following the models presented in the previous chapter with: SISD, MIMD and SIMD/SIMT and presenting the last released technologies. We also present the optimizations of technologies with the arising of parallelism and new types of memories.

2.2 Early improvements to Von Neumann machine

In this section we present the different hardware from 1970s single core processors to nowadays multi-core and many-core architectures. We see the most important optimizations that are always implemented in the most recent machines like in/out of order processors, pre-fetching strategies, vectorization and the memory technologies. They are the milestones, the basic units, to build supercomputers.

2.2.1 Single core processors

The first processors, around the 1970s, were built using a single computation core like described in the Von Neumann model. Many evolutions were made on those single cores processors from the memory, the order of instruction and the frequency.

Transistor shrink and frequency

A lot of new ways to produce smaller transistors had been discovered from the $10\mu m$ of 1971 to nowadays $10nm$. This allows the vendor to add more transistors on the same die, allowing more complex ISA and features for the CPU.

In parallel of the shrink of transistors, the main feature for better performances with the single core architecture came from the frequency augmentation, the clock rate. Indeed, as fast as the clock rate get, more operations can be computed in a second on the core. In 1970s the frequency was about 4 MHz allowing a maximum of 4 millions of cycles per seconds. Nowadays

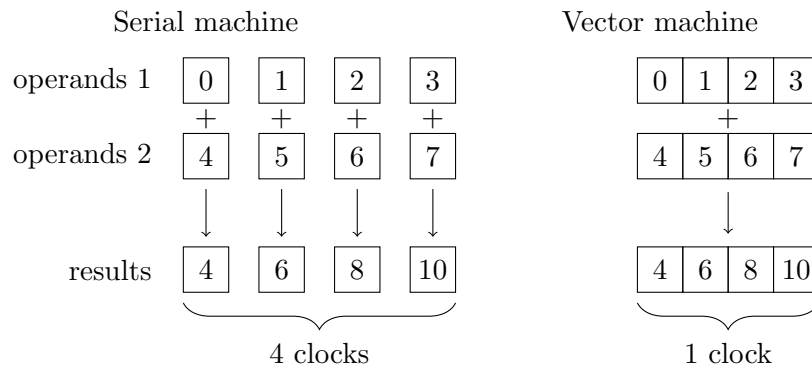


Figure 2.1: Vectorized processeur example on 4 integer addition: 128 bits wide bus

core can work at a frequency of 4GHz and even 5GHz performing billions of operations per cycles.

In/Out-Of-Order

In-order-process is the one describes in previous chapter. The control unit fetches instruction in memory and the operands. The ALU computes the operation, and finally the result is stored in memory.

In this model the time to perform an instruction is the cumulation of: instruction fetching + operand(s) fetching + *computation* + store the result. This time can be high regarding the time when the ALU itself is busy for *computation*, technically just one clock cycle. The idea of Out-of-order is to compute the instructions without following the Program Counter order. Indeed, for independent tasks (pointed out with dependency graphs) while the process fetches the next instructions data, the ALU can perform another operation with already available operands. This leads to better usage of computational resources in the CPU and thus better overall performances.

Vectorization

Vector processors allows the instructions to be executed at the same time in a SIMD manner. If the same instruction is executed on coalescent data they can be executed in the same clock cycle. We can, as an example, execute operations simultaneously on 4 to 8 floats with a bus size of 128 or 256 bits in the same cycle. This tool requires specific care during coding with *unrolling* and *loop tiling* and will be address later in this study. The main downside on latest architectures is that vectorization slightly lower the frequency of processors to operate. This requires even more care during programming to avoid bad behavior leading to bad performances.

The Cray-1 supercomputer[Rus78], installed in 1975 in the Los Alamos National Laboratory, is a perfect example of vector processor supercomputer. This supercomputer was designed by Seymour Cray the founder of Cray Research. Based on vector processor it was able to deliver up to 160 MFLOps. It was the fastest supercomputer of 1978 and due to its shape and price he was humorously called *the world's most expansive love-seat*.

The behavior of vector machine is presented with a 16 bytes vector machine (4 integer of 4 bytes = 128 bits bus) on figure 2.1. We see on left that performing the 4 operations requests 4 cycle and, at opposite, 1 cycle on the right with the vectorized machine.

Linked with the CPU optimizations, the memory optimizations also have to be considered. Indeed, even if the ALU can perform billions of operations per second it needs to be fed by fast transfers.

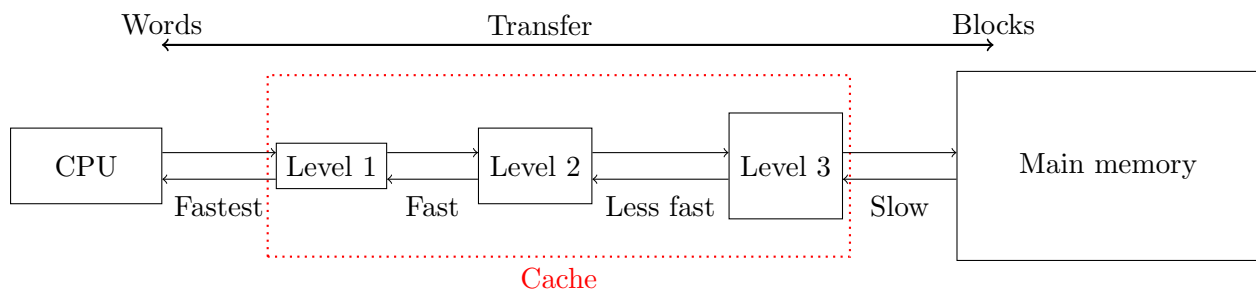


Figure 2.2: Cache memory technology on three levels L1, L2 and L3

Memory technology evolution

The memories technologies optimizations contain several aspect. In early 1980s the augmentation of bus size from 4 bits, 8 bits, 16 bits for nowadays 32 bits for single precision and 64 bits for double precision. 128 bits, 256 bits, etc. buses can also be find allowing technologies we just presented, vectorization. Different kind of technologies are considered: the SRAM and DRAM.

SRAM: The Static Random Access Memory is built using so called "flip-flop" circuits that can store the data as long as the machine is powered. This kind of memory is very expensive to produce due to the number of transistor by memory cell needed and the size of the memory. Therefore it is usually limited for small amount of storage. The SRAM is mainly used for cache memory. Cache is a memory mechanism that is useful to consider when targeting performance.

Cache memory: The main idea of cache technology is presented on figure 2.2. This little memory is built over several levels. The closer to the CPU is L1, then L2 and generally no more than L3 except on specific architecture. When looking for a data the CU will first check the L1 cache, otherwise L2 and L3 to get the data to higher level. From the main memory to the L3 cache *blocks* are exchanged, they are chunks of memory. With the smaller level L2 and L1 line of informations usually called *words* are exchanged. This is based on the idea that if a data is used, it shall be use again in the near future. Many cache architectures exist like direct, associative, fully associative, etc. In a program the ratio of cache accesses *cache-hits* and *cache-miss* respectively when a data is present in cache and when a data have to be retrieved from lower level or main memory can be very important to reach performances.

DRAM: The Dynamic Random Access Memory is based on transistors and capacitors to store the binary information. This memory is less expansive to produce but needs to be refresh at a determined frequency however the data are lost. This refresh step is in fact a reading-writing operation on the whole memory at a specific frequency. There is several sub categories of DRAM used in different devices.

Depending on the way the bus are used we can find Single Data Rate, SDR, Double Data Rate, DDR and QDR, Quad Data Rates DRAM memories. The number of data carried can go from 1x to 4x but the limitation of those products is the price of memory constantly rising.

Pre-fetching

Based on memory optimization and especially the cache, pre-fecthing was developed. When a data is not available in L1 cache, it has to be moved from either L2 to L1 or L3 to L2 to L1 or in the worst case RAM to L3 to L2 to L1. Pre-fecthing technology is a way to, knowing the next instructions operands, pre-fetch the data in closer cache. The pre-fetch can either be hardware or software implemented and can concern data and even instructions.



Figure 2.3: Multi-core CPU with 4 cores based on Von Neumann Model presented on figure ??

2.2.2 Multi-core processors

Around the beginning of 2000s the limitations of single core processors were too important. The frequency was already high and requested more power consumption and caused more heat dissipation. Unable to answer the constant augmentation of computational power needed for research and HPC, IBM was the first company to create a multi-core CPU in 2001, the Power4.

The first idea was to provide multi-CPU devies, embedding several CPU on the same motherboard and allowing them to share memory. The evolution of that is multi-core, having several CPUs on the same die directly allowing more optimization inside the die combining all the advantages of single core processors. We note here that in nowadays language the CPU, as describe in the Von Neumann model, is also the name of the die containing several CPUs. This is the architecture of most of nowadays processors. They are called multi-cores and provide up to 2 to 32 cores. Those processors are called "Host" because they are usually bootable and most of the accelerators need to be attached to them in order to work.

This architecture is presented on figure 2.3. The memory, like presented in the previous chapter, is now shared between the cores. The registers and cache are different: another layer is added to the cache and consistency have to be maintain over all the cores. If a process modify a data in the memory this information have to be spread over all the other users of this data, even in their local cache.

2.3 21th century architectures

After years of developpement and research on hardware for Computer Science and specifically HPC, we present here the latest and best technologies to product efficient and general purpose supercomputers.

We present the latest architectures with multi-core, many-core and specific processors and the most famous vendors.

2.3.1 Multi-core

The most world spread architecture in public and high performance computing is the multi-core processors. Most of nowadays accelerators require a classical processor to offload tasks and data on it. We start from the most present processors in HPC world, the Intel company ones. We also present ARM which is another multi-core architecture base on RISC instructions set.



Figure 2.4: Intel Tick-Tock model

Intel

Intel was created in 1968 by a chemist and a physicist, Gordon E. Moore and Robert Noyce, in Mountain View, California. Nowadays processors are mostly Intel ones, this world leader equips around 90% of the supercomputers (November 2017 TOP500 list).

In 2007 Intel adopted a production model called the "Tick Tock", presented on figure 2.4. Since its creation this model followed the same fashion, a new manufacturing technology like shrink of the chip with better engraving on a "Tick" and a new micro-architecture delivered on a "Tock". The Intel processors for HPC are called Xeon and features ECC memory, higher number of cores, large RAM support, large cache-memory, Hyper-threading, etc. compared to desktop processors. Every new processor have a code name. The last generations are chronologically called Westmere, Sandy Bridge, Ivy Bridge, Haswell, Broadwell, Skylake and Kaby lake. Kaby Lake, the last architecture of processor, does not exactly fit the usual "Tick-Tock" process because it is just based on optimizations of the Skylake architecture. It is produce like Skylake in 14nm. This model seems to be hard to maintain due to the difficulties to engrave in less than 10nm with quantum tunneling. This leads to using more many-cores architecture and base next supercomputer generations on hybrid models.

Hyper-threading Another specificity of Intel processor is Hyper-threading (HT). This technology makes a single physical processor appearing as two logical processors for user's level. In fact a processor embedding 8 cores appears as a 16 cores for user. Adding more computation per node can technically allows the cores to switch context when data are fetched from the memory using the processor 100% during all the computation. A lot of studies have been released on HT from Intel itself [Mar02] to other studies [BBDD06, LAH⁺02]. This optimization does not fit to all the cases and can be disable for normal use of the processors.

ARM

Back in 1980s, ARM stood for Acorn RISC Machine in reference of the first company implementing this kind of architecture, Acorn Computers. This company later changed the name to Advanced RISC Machine (ARM). ARM is a specific kind of processor based on RISC architecture as its ISA despite usual processors using CISC. The downside of CISC machines makes them hard to create and they require way more transistor and thus energy to work. The ISA from the RISC is simpler and requires less transistors to operate and thus a smaller silicon area

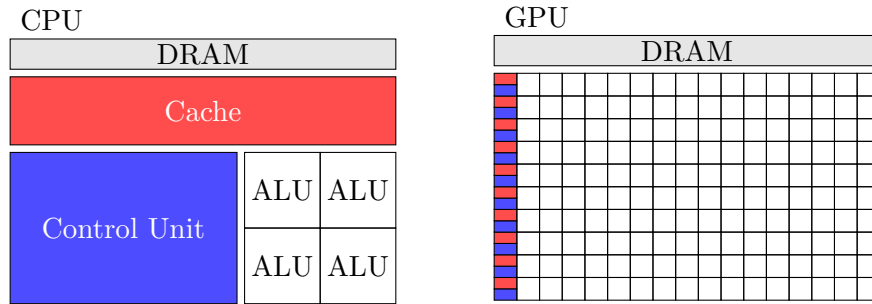


Figure 2.5: Multi-core versus Many-core architecture, case of GPUs

on the die. Therefore, the energy required and the heat dissipated is less important. It would then be easier to create massively parallel processors based on ARM. On the other hand, simple ISA impose more work on the source compilation to fit the simple architecture. That makes the instructions sources longer and therefore more single instructions to execute.

The ARM company provide several version of ARM processors named Cortex-A7X, Cortex-A5X and Cortex-A3X respectively balancing highest-performances, performances and efficiency and less power consumption. We find here the same kind of naming as Intel processors.

The new ARMv8 architecture starts to have the tools to target HPC context [RJAJVH17]. The European approach towards energy efficient HPC, Mont-Blanc project¹, already constructs ARM based supercomputers. For the exascale project in Horizon 2020 this project focus on using ARM-based systems for HPC with many famous contributors with Atos/Bull as a project coordinator, ARM, French Alternative Energies and Atomic Energy Commission (CEA), Barcelona Supercomputing Center (BSC), etc. The project is decomposed in several steps to finally reach exascale near 2020. The third step, Mont-Blanc 3, is about to work on a pre-exascale prototype powered by Cavium's ThunderX2 ARM chip based on 64-bits ARMv8.

2.3.2 Many-cores, SIMT

Several architectures can be defined as many-cores. Those devices integrate thousands of cores that are usually control by less control units. We can consider those cores as "simpler" since they have to work synchronously and under the coordination of a control unit. They are based on SIMD Flynn taxonomy. Some devices are specific like the Xeon Phi of Intel integrating a hundred of regular processor cores which can work independently.

GPU

When a CPU can usually have 2 to 32 computation cores that can operate on different instruction streams, the SIMT architecture of the GPU is slightly different. The cores are grouped and have to share the same instruction at the same clock time but all the groups can have their own instruction.

Figure 2.5 present the vision between CPU and GPU processors. We see on that figure the usual topology with the ALU lined up in front of their control unit and shared cache memory. Every ALU also have its own memory and registers to operate local computations.

Those devices are called General Purpose Graphics Processing Units (GPGPUs). They are derivative from classical GPUs used for graphics purpose. Pioneer shows that they can be use efficiently for classical scientific computations. The vendor provides then specific GPU for general purpose computing. We present here the two main companies providing GPGPUs for HPC world: NVIDIA and AMD.

¹<http://montblanc-project.eu/>



Figure 2.6: NVIDIA Tesla Kepler architecture. Single-precision in green and double-precision in yellow

NVIDIA GPU architecture The NVIDIA company was founded in April 1993 in Santa Clara, Carolina, by three persons in which Jensen Huang, the actual CEO. The company name seems to come from *invidia* the Latin word for Envy and vision for graphics rendering.

Known as the pioneer in graphics, cryptocurrency, portable devices and now Artificial Intelligence (IA), it seems to be even the creator of the name "GPU". NVIDIA's GPUs, inspired from visualization and gaming at a first glance, are available as a dedicated device for HPC purpose since the company released the brand named *Tesla*. The public GPUs can also be used for dedicated computation but does not feature ECC memory, double precision or special functions/FFT cores. The different versions of the architecture are named following famous physicists, chronologically: Tesla, Fermi, Kepler, Maxwell, Pascal and Volta.

We describe here the Kepler brand GPU and more specifically the K20Xm GPU on which we based our study. This NVIDIA Tesla Kepler GPU is based on the GK110 graphics processor described in the white-paper[Nvi12] on 28nm process. The figure 2.6 is a representation of the physical elements of this graphics processor. The K20X comes in active and passive cooling mode with respectively K20Xc and K20Xm. This GPU embeds 2688 CUDA cores distributed in 14 SMX (we note that GK110 normally provides 15 SMX but only 14 are present on the K20X). In this model each SMX contains 192 single precision cores, 64 double precision cores, 32 special function units and 32 load/store units. In a SMX the memory provides 65536 32-bits registers, 64KB of shared memory L1 cache, 48KB of read-only cache. The L2 cache is 1546KB shared by the SMX for a total of 6GB of memory adding the DRAM. The whole memory is protected using Single-Error Correct Double-Error Detect (SECCDED) ECC code. The power consumption is estimated to 225W. This GPGPU is expected to produce 1.31 TFLOPS for double-precision and 3.95 TFLOPS of single-precision.

AMD Another company is providing GPUs for HPC, Advanced Micro Devices (AMD). In front of the huge success of NVIDIA GPU that leads from far the HPC market, it is hard for AMD to find a place for its GPGPUs in HPC. Their HPC GPUs are called FirePro. They are targeted using a language near CUDA but not held by a single company called OpenCL. An

interesting creation of AMD is the Accelerated Processing Units (APUs) which embedded the processor and the GPU on the same die since 2011. This solution allows them to target the same memory.

In the race to market and performances, AMD found an accord with Intel to provide dies featuring Intel processor, AMD GPU and common HBM memory. The project is called Kaby Lake-G and announced for first semester of 2018 but for public, not HPC itself.

Intel Xeon Phi

Another specific HPC product from Intel is the Xeon Phi. This device can be considered as a Host or Device/Accelerator machine. Intel describes it as "a bootable host processor that delivers massive parallelism and vectorization". This architecture embedded multiple multi-cores processors interconnected. This is called Intel's Many Integrated Core (MIC). The architectures names are Knights Ferry, Knights Corner and Knight Landing [SGC⁺16]. The last architecture, Knight Hill, was recently canceled by Intel due to low performances and to focus the Xeon Phi for Exascale. The main advantage of this architecture compared to GPGPUs is the x86 compatibility of the embedded cores and the fact this device can boot and use to drive other accelerators. They also feature more complex operations and handle double precision natively. We considered the Xeon Phi in the many-cores architecture despite the fact that it is composed of completely independent processors. This is due to the number of cores that is very high and the fact it can be used as an accelerator instead of the host.

PEZY

Another many-core architecture just appears in the last benchmarks. The PEZY Super Computer 2, PEZY-SC2, is the third many-core microprocessor developed by the company PEZY. The three first machines ranked in the GREEN500 list are accelerators using this many-core die. We also note that in the November 2017 list the 4th supercomputer, Gyoukou, is also powered by PEZY-SC2 cards.

2.3.3 Other architectures

Numerous architectures have not been presented because out of scope in this study. We present here two technologies we have been confronted in our researches and that can be tomorrow's solution for exascale in HPC.

FPGA

Field Programmable Gate Array are devices that can be reprogrammed to fit the needs of the user after their construction. The leader was historically Altera with the Stratix, Arria and Cyclone FPGAs and is now part of Intel. With the FPGAs the user has access to the hardware itself and can design its own circuit. Nowadays FPGA can be targeted with OpenCL programming language. The arrival of Intel in this market promises the best hopes for HPC version of FPGAs. The main gap for users is the circuit building itself, perfect to respond to specific needs but hard to setup.

ASIC

Application Specific Integrated Circuits are dedicated devices constructed for one purpose. An example of ASIC can be the Gravity Pipe (GRAPE) which is dedicated to compute gravitation given mass/positions. Google leads the way for ASIC and just created its dedicated devices to boost AI bots. We also find ASIC in some optimized communication devices like in fast interconnection networks in HPC.

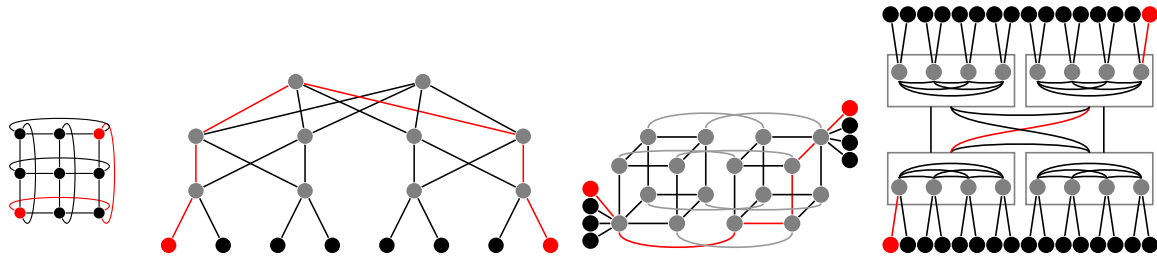


Figure 2.7: Torus, Fat-Tree, HyperX, DragonFly

2.4 Distributed architectures

The technologies presented in previous part is the milestone of supercomputers. They are used together in a whole system to create machine delivering incredible computational power.

2.4.1 Architecture of a supercomputer

From the hardware described before we can create the architecture of a cluster from the smallest unit, cores, nodes, to the whole system:

Core: A core is the smallest unit in our devices. It can refer to the Von Neumann model in case of core with ALU and CU. We can separate core from CPU to GPU, the first one able to be independent whereas the second ones working together and sharing the same program counter.

Socket/Host: A socket is mistakenly called a CPU in nowadays language. It is, for multi-cores sockets, composed of several cores. The name Host comes from the Host-Device architecture using accelerators.

Accelerators/Devices: Accelerators are devices that, when attached to the Host, provide additional computational power. We can identify them as GPUs, FPGAs, ASICs, etc. A socket can have access to one or more accelerators. They can also share the accelerator usage.

Computation node: The next layer of our HPC system is the computation node. Grouping together several socket and accelerators sharing memory;

Rack: A rack is a set of computation nodes, generally a vertical stack. It can also include specific nodes dedicated to the network or the Input/Output.

Interconnection: The nodes are grouped together with hard wire connection following a specific interconnection topology with very high bandwidth.

System/Cluster/Supercomputer The cluster group several racks through an interconnection network.

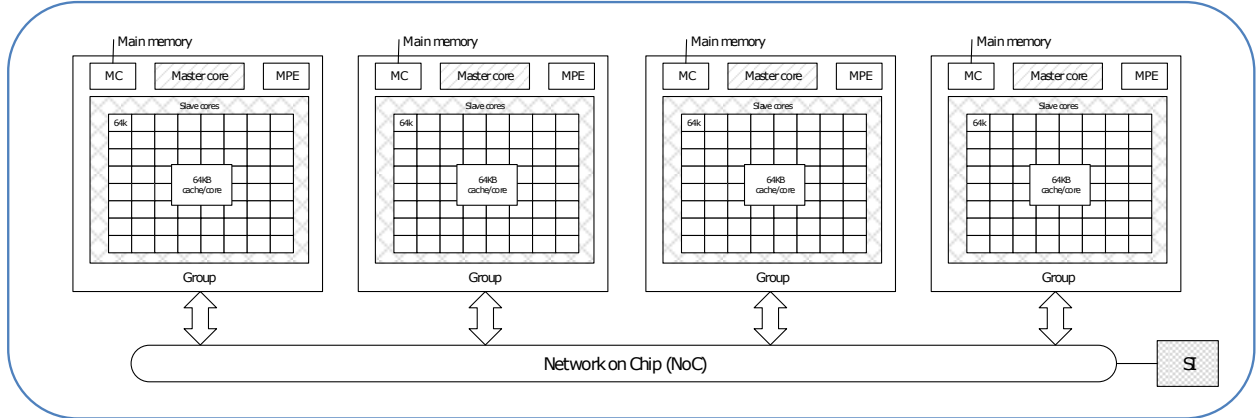
In order to connect node together and allow distributed programming an interconnect technology is required. Interconnection network is the way the nodes of a cluster are connected together.

2.4.2 Interconnection topologies

Several topologies exist from point to point to multi dimensional torus. The figure 2.7 is a representation of famous topologies. Each interconnect technology has its own specificity. These networks take in account the number of nodes to interconnect and the targeted bandwidth/budget. Several declination of each network are not detailed here. The Mesh and the Torus are used as a basis in lower layers of others more complex interconnection networks. A perfect example is the supercomputer called K-Computer described in the next section. The Fat Tree presented here is a k-ary Fat Tree, higher the position in the tree more connections are found and the bandwidth is important. The nodes are available as the leaves, on the middle level we find the switches and on top the routers. Another topology, HyperX[ABD⁺09], is based

Name	Gbs	Year	Name	Gbs	Year
Single DR	2.5	2003	Enhanced DR	25	2014
Double DR	5	2005	Highg DR	50	2017
Quad DR	10	2007	Next DR	100	2020
Fourth DR	14	2011			

Table 2.1: InfiniBand technologies name, year and bandwidth

Figure 2.8: Sunway Taihulight node architecture from *Report on the Sunway TaihuLight System*, Jack Dongarra, June 24, 2016.

on Hyper-Cube. The DragonFly[KDSA08] interconnect is recent, 2008, and use in nowadays supercomputers.

InfiniBand (IB) is the most spread technology used for interconnect with different kind of bandwidth presented in figure 2.1. It provides high bandwidth and small latency and companies like Intel, Mellanox, etc provide directly adapters and switches specifically for IB.

Unfortunately this augmentation of clock rate is not sustainable due to the energy required and the heat generated by the running component. Another idea came in 19th century with the first multi-core processors.

2.4.3 Remarkable supercomputers

The TOP500 is the reference benchmarks for the world size supercomputers. Most of the TOP10 machines have specific architectures and, of course, the most efficient ones. In this section we give details on several supercomputers about their interconnect, processors and specific accelerators.

Sunway Taihulight

Sunway Taihulight is the third Chinese supercomputer to be ranked in the first position of the TOP500 list. A recent report from Jack J. Dongarra, a figure in HPC, decrypt the architecture of this supercomputer[Don16]. The most interesting point is the conception of this machine, completely done in China. The Sunway CPUs were invented and built in China. The Vendor is the Shanghai High Performance IC Design Center.

The SW26010, a many core architecture processor, features 260 cores based on RISC architecture and a specific conception depicted on figure 2.8. The processor is composed of the master core, a Memory Controller (MC), a Management Processing Element (MPE) that manages the Computing Processing Elements (CPE) which are the slaves cores.

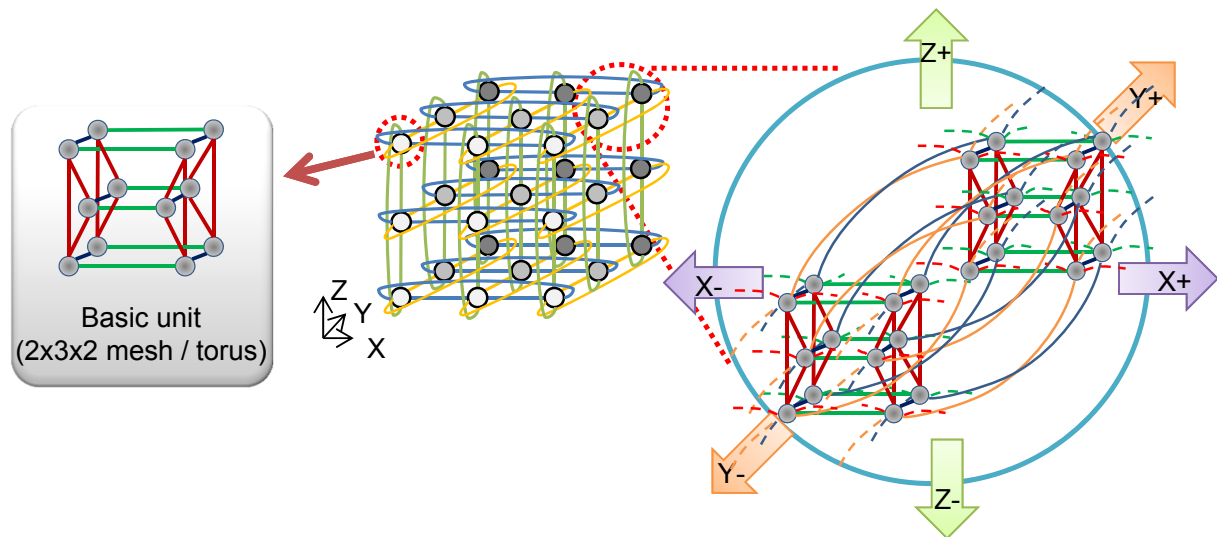


Figure 2.9: TOFU Interconnect schematic from *The K-Computer: System Overview*, Atsuya Uno, SC11

The interconnect network is called Sunway Network and connected using Mellanox Host Channel Adapter (HCA) and switches. This is a five level interconnect going through computing nodes, computing board, super-nodes and cabinets to the complete system. The total memory is 1.31 PB and the number of cores available is 10,649,600. The peak performance is 125.4 PFLOPS and the Linpack is 93 PFLOPS which induce 74.16% of efficiency.

Piz Daint

The supercomputer of the CSCS, Swiss National Supercomputing Center, is currently ranked 2nd of the November 2017 TOP500 list. This GPUs accelerated supercomputer is a most powerful representative of GPU hybrid acceleration. This is also the most powerful European supercomputer. He is composed of 4761 hybrids and 1210 multi-core nodes. The hybrids nodes embedded an Intel Xeon E5-2690v3 and an NVIDIA Tesla Pascal P100 GPGPU. The interconnect is based on a Dragonfly network topology and Cray Aries routing and communications ASICs. The peak performance is 25.326 TFLOPS using only the hybrid nodes and the Linpack gives 19.590 TFLOPS. The low power consumption rank Piz Daint as 10th in the GREEN500 list.

K-Computer

K-Computer was the top 1 supercomputer of TOP500 2011 list. The TOFU interconnect network makes the K-Computer unique [ASS09] and stands for TORus FUsion. This interconnect presented in figure 2.9 mixes a 6D Mesh/Torus interconnect. The basic units are based on a mesh and are interconnected together in a 3 dimensional torus. In this configuration each node can access to its 12 neighbors directly. It also provide a fault tolerant network with many routes to reach distant node.

Sequoia/Mira

Sequoia supercomputer was top 1 of the TOP500 2012 list. It is based on BlueGene from IBM. The BlueGene project made up to three main architectures with BlueGene/L, BlueGene/P and BlueGene/Q. It is very interesting to notice the BlueGene architecture because even in the last GRAPH500 list, November 2017, there is 15 of these machines in the TOP20. The algorithm

used on these supercomputers will be our basis in the part II regarding our implementation of the GRAPH500 benchmark.

2.5 ROMEO Supercomputer

The ROMEO supercomputer center is the computation center of the Champagne-Ardenne region in France. Hosted since 2002 by the University of Reims Champagne-Ardenne, this so called meso-center (French name for software and hardware architectures) is used for HPC for theoretic research and domain science like applied mathematics, physics, biophysics and chemistry.

This project is support by the Champagne-Ardenne region and the CEA (French Alternative Energies and Atomic Energy Commission), aim to host research and production codes of the region for industrial, research and academics purposes.

We are currently working on the third version of ROMEO, installed in 2013. As many of our tests in this study have been done on this machine, we will carefully describe its architecture.

This supercomputer was ranked 151st in the TOP500 and 5th in the GREEN500 list.

2.5.1 ROMEO hardware architecture

ROMEO is a Bull/Atos supercomputer composed of 130 BullX R421 computing nodes.

Each node is composed of two processors Intel Ivy Bridge 8 cores @ 2,6 GHz. Each processor have access to 16GB of memory for a total of 32GB per node, the total memory if 4.160TB. Each processor if linked, using PCIe-v3, to an NVIDIA Tesla K20Xm GPGPU. This cluster provide then 260 processors for a total of 2080 CPU cores and 260 GPGPU providing 698880 GPU cores. The computation nodes are interconnected with an Infiniband QDR non-blocking network structured as a FatTree. The Infiniband is a QDR providing 10GB/s.

The storage for users is 57 TB and the cluster also provide 195 GB of Lustre and 88TB of parallel scratch file-system.

In addition to the 130 computations nodes, the cluster provides a visualization node NVIDIA GRID with two K2 cards and 250GB of DDR3 RAM. The old machine, renamed Clovis, is always available but does not features GPUs.

The supercomputer supports MPI with GPU Aware and GPUDirect.

2.5.2 New ROMEO supercomputer, June 2018

[Avoir les info et decire le nouveau ROMEO](#)

2.6 Conclusion

In this chapter we reviewed the most important nowadays hardware architectures and technologies. In order to use the driver or API in the most efficient way we need to keep in mind the way the data and instructions are proceed by the machine.

As efficiency is based on computation power but also communications we showed different interconnection topologies and their specificities. We presented perfect use cases of the technologies in nowadays top ranked systems. They also show that every architecture is unique in its construction and justify the optimization work dedicated to reach performance.

We can see through the new technologies presented here that every one is moving toward hybrids architectures featuring multi-core processors accelerated by one or more devices, many-core architectures. The exascale supercomputer of 2020 will be shape with hybrid architectures and they represent the best of nowadays technology for purpose of HPC. Combining CPU and GPUs or FPGA on the same die, sharing the same memory space can also be the solution.

Chapter 3

Software in HPC

3.1 Introduction

After presenting the rules of HPC and the hardware that compose the cluster, we introduce the most famous ways to target those architectures and supercomputers with programming models. Then, fitting those models, we present the possible options in the language, the API, the distribution and the accelerators code.

This chapter details the most important programming models and the software options for HPC programming and include the choices we made for our applications. Then it presents the software used to benchmark the supercomputers. We present here the most famous, the TOP500, GRAPH500, HPGC and GREEN500 to give their advantages and weaknesses.

3.2 Parallel and distributed programming Models

The Flynn taxonomy developed in chapter 1 was a characterization of the executions models. This model can be extended to programming models which are an extension of MIMD. We consider here a *Random Access Machine* (RAM). The memory of this machine consists of an unbounded sequence of registers each of which may hold an integer value. In this model the applications can access to every memory words directly in write or read manner. There is three main operations: load from memory to register; compute operation between data; store from register to memory. This model is use to estimate the complexity of sequential algorithms. If we consider the unit of time of each operation (like in cycle) we can have an idea of the overall time of the application. We identify two types of RAM, the Parallel-RAM using shared memory and the Distributed-RAM using distributed memory.

3.2.1 Parallel Random Access Machine

The Parallel Random Access Machine [FW78], PRAM, is a model in which the global memory is shared between the processes and each process have its own local memory/registers. The execution is synchronous, processes execute the same instructions at the same time. In this model each process is identify with its own index enabling to target different data. The problem in this model will be the concurrency in reading (R) and writing (W) data as the memory is shared between the processes. Indeed, mutual exclusion have to be set with exclusive (E) or concurrent (C) behaviors and we find 4 combinations: EREW, ERCW, CREW and CRCW. As the reading is not critical for data concurrency the standard model will be Concurrent Reading and Exclusive Writing: CREW.

3.2.2 Distributed Random Access Machine

For machine that base their memory model on NoRMA the execution model can be qualify of Distributed Random Access Machine, DRAM. It is based on NoRMA memories detailed in

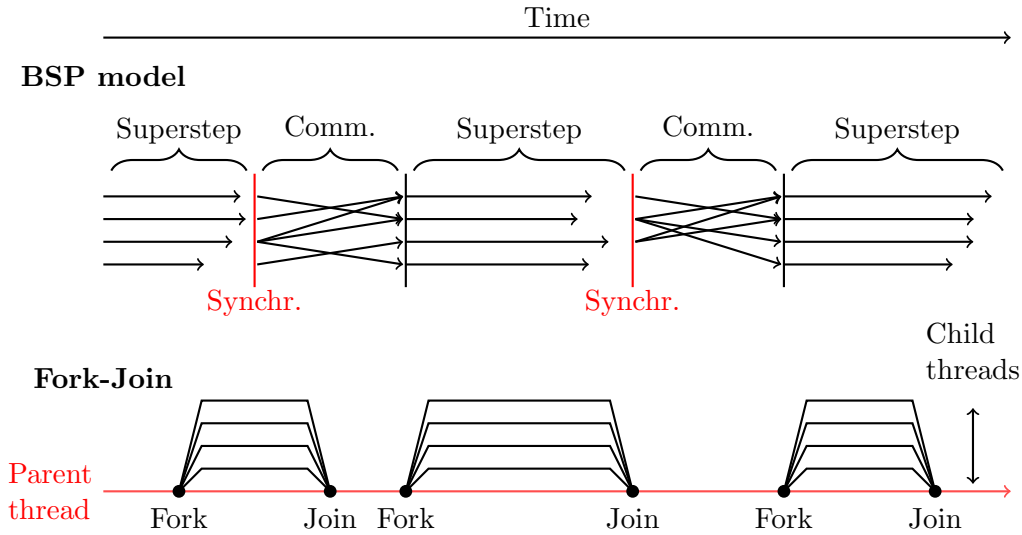


Figure 3.1: Bulk Synchronous Parallel model and Fork-Join model

part 1.4. This model is in opposition to PRAM because the synchronization between processes is made by communications and messages. Those communications can be of several kind and depend of physical architecture, interconnection network and software used.

3.2.3 H-PRAM

A DRAM can be composed of an ensemble of PRAM system interconnected. Each of them working on their own data and instructions. This is an intermediate model between PRAM and DRAM having a set of shared memory and synchronous execution, the overall execution being asynchronous and having distributed memory.

3.2.4 Bulk Synchronous Parallelism

This model was presented in 1990 in [Val90]. Being the link of HPRAM and PRAM The Bulk Synchronous Parallelism model is based on three elements:

- a set of processor and their local memory;
- a network for point-to-point communications between processors;
- a unit allowing global synchronization and barriers.

This model is the most common on HPC clusters. It can be present even on node themselves: a process can be assign on a core or set of cores and the shared memory is separated between the processes. The synchronization can be hardware but in most cases it is handle by the runtime used. A perfect example of runtime, presented later, is MPI.

In this model the applications apply a succession of *supersteps* separated by *synchronizations* steps and data exchanges.

At opposite to H-PRAM which represent the execution as a succession of independent blocks working synchronously, BSP propose independent blocks of asynchronous applications synchronized by synchronization steps.

In a communication/synchronization step we can consider the number of received messages h_r and the number of send ones h_s .

The time lost in communication in one synchronization step is:

$$T_{comm} = hg + I \quad (3.1)$$

With $h = \max(h_s, h_r)$, g the time to transfer data and I the start-up latency of the algorithm. Indeed, the entry points and exit points of communications super-step can be a bottleneck

considered in I .

The time for computing a super-step is:

$$T_{comp} = \frac{w}{r} + I \quad (3.2)$$

With w the maximum number of flops in the computation of this super-step, r the speed of the CPU expressed in FLOPS and I the start-up latency of the algorithm. Indeed, the entry points and exit points of communications super-step can be a bottleneck considered in I .

The BSP model estimates the cost of one super-step with:

$$T_{comm} + T_{comp} = w + gh + 2l \quad (3.3)$$

With T a measure of time, a wall clock that measure elapsed time. We also note that usually g and I are function of the number of processes involved.

It can then be use to compute the overall cost in BSP model summing all super-steps s :

$$T = \sum_s \frac{\max(w_s)}{r} + h_s g + I \quad (3.4)$$

The problem of performances in this model can come from unequal repartitions of work, the load balancing. The processes with less than w of work will be idle.

3.2.5 Fork-Join model

The Fork-Join model or pattern is presented in figure 3.1. A main thread pilot the overall execution. When requested by the application, typically following the idea of *divided-and-conquer* approach, the main thread will fork and then join other threads. The *Fork* operation, called by a logical thread parent, creates new logical threads children working in concurrency. There is no limitations in the model and we find nested fork-join where a child can also call fork to generate sub-child and so on. The *Join* can be called by both parents and child. Children call join when done and the parent join by wait until children completion. The Fork operation increase concurrency and join decrease concurrency.

3.3 Software/API

In this section we present the main runtime, API and frameworks use in HPC and in this study in particular. The considered language will be C/C++, the most present in HPC world along with Fortran.

3.3.1 Shared memory programming

On the supercomputers nodes we find one or several processors that access to UMA or NUMA memory. Several API and language provide tools to target and handle concurrency and data sharing in this context. The two main ones are PThreads and OpenMP for multi-core processors. We can also cite Cilk++ or TBB from Intel.

PThreads

The Portable Operating System Interface (POSIX) threads API is an execution model based on threading interfaces. It is developed by the IEEE Computer Society. It allows the user to define threads that will execute concurrently on the processor resources using shared/private memory. PThreads is the low level handling of threads and the user need to handle concurrency with semaphores, conditions variables and synchronization "by hand". This makes the PThreads hard to use in complex applications and used only for very fine-grained control over the threads management.

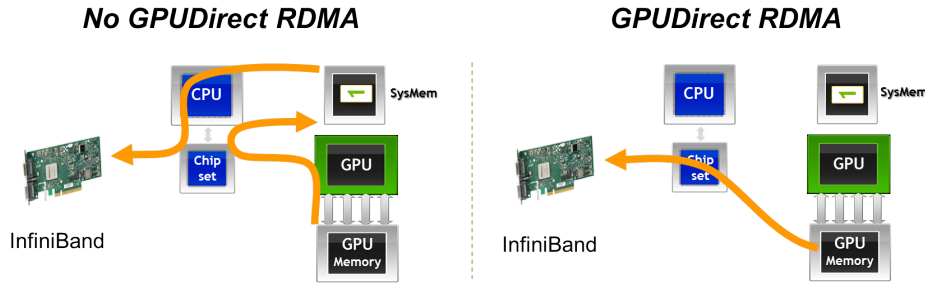


Figure 3.2: GPUDirect RDMA from NVIDIA Developer Blog, *An Introduction to CUDA-Aware MPI*

OpenMP

Open Multi-Processing, OpenMP¹ [Cha08, Sup17], is an API for multi-processing shared memory like UMA and CC-NUMA. It is available in C/C++ and Fortran. The user is provided with pragmas and functions to declare parallel loop and regions in the code. In this model the main thread, the first one before forks, command the fork-join operations.

The last versions of OpenMP 4.0 also allow the user to target accelerators. During compilation the user specify on which processor or accelerator the code will be executed in parallel.

We use OpenMP as a basis for the implementation of our CPU algorithms. Perfect for loop parallelization and parallel sections, we show that we can have the best results for CPU algorithms in most of the case. In our case, OpenMP is always use on the node to target all the processors cores in the shared memory.

We note that the new versions of OpenMP also allows to target directly accelerators like NVIDIA ones.

3.3.2 Distributed programming

In the cluster once the code have been developed locally and using the multiple cores available, the new step is to distribute it all over the nodes of the cluster. This step requires the processes to access NoRMA memory from a node to another. Several runtime are possible for this purpose and concerning our study. We should also cite HPX, the c++ standard distribution library, or AMPI for Adaptive MPI, Multi-Processor Computing (MPC) from CEA, etc.

MPI

The Message Passing Interface, MPI, is the most famous runtime for distributed computing [Gro14, Gro15]. Several implementations exists from Intel MPI² (IMPI), MVAPICH³ by the Ohio State University and OpenMPI⁴ combining several MPI work like Los Alamos MPI (LA-MPI). Those implementation follow the MPI standards 1.0, 2.0 or the latest, 3.0.

This runtime provides directs, collectives and asynchronous functions for process(es) to process(es) communication. A process can be a whole node or one or several cores on a processor.

Some MPI implementations offer a support for accelerators targeting directly their memory through the network without multiple copies on host memory. The data go through one GPU to the other through network and PCIe. This feature is used in our code in part 2 and 3.

Most of our code presented here are based on MPI for the distribution on the cluster. The advantage is its presence on all the cluster and the control over the data transfers.

For NVIDIA this technology is called GPUDirect RDMA and presented on figure 3.2.

¹<http://www.openmp.org>

²<https://software.intel.com/en-us/intel-mpi-library>

³<http://mvapich.cse.ohio-state.edu/>

⁴<http://www.open-mpi.org>

In term of development MPI can be very efficient if use carefully. Indeed, the collectives communications such as *MPI_Alltoall*, *MPI_Allgather*, etc. can be a bottleneck when scaling up to thousands of processes. A specific care have to be taken in those implementation with privilege to asynchronous communications to hide computation than synchronous idle CPU time.

Charm++

Charm++⁵ is an API for distributed programming developed by the University of Illinois Urbana-Champaign. It is asynchronous messages paradigm driven. In contrary of runtime like MPI that are synchronous but can handle asynchronous, charm++ is natively asynchronous. It is based on *chare object* that can be activated in response to messages from other *chare objects* with triggered actions and callbacks. The repartition of data to processors is completely done by the API, the user just have to define correctly the partition and functions of the program. Charm++ also provides a GPU manager implementing data movement, asynchronous kernel launch, callbacks, etc.

A perfect example can be the hydrodynamics N-body simulation code Charm++ N-body Gravity Solver, ChaNGa [JWG⁺10], implemented with charm++ and GPU support.

Legion

Legion⁶ is a distributed runtime support by Stanford University, Los Alamos National Laboratory (LANL) and NVIDIA. This runtime is data-centered targeting distributed heterogeneous architectures. Data-centered runtime focuses to keep the data dependency and locality moving the tasks to the data and moving data only if requested. In this runtime the user defines data organization, partitions, privileges and coherency. Many aspect of the distribution and parallelization are then handle by the runtime itself.

The FleCSI runtime develops at LANL provide a template framework for multi-physics applications and is built on top of Legion. We give more details on this project and Legion on part 3.

3.3.3 Accelerators

In order to target accelerators like GPU, several specific API have been developed. At first they were targeted for matrix computation with OpenGL or DirectX through specific devices languages to change the first purpose of the graphic pipeline. The GPGPUs arriving forced an evolution and new dedicated language to appear.

CUDA

The Compute Device Unified Architecture is the API develop in C/C++ Fortran by NVIDIA to target its GPGPUs. The API provide high and low level functions. The driver API allows a fine grain control over the executions.

The CUDA compiler is called NVidia C Compiler, NVCC. It converts the device code into Parallel Thread eXecution, PTX, and rely to the C++ host compiler for host code. PTX is a pseudo assembly language translated by the GPU in binary code that is then execute. As the ISA is simpler than CPU ones and able the user to work directly in assembly for very fine grain optimizations.

As presented in figure 3.3, NVIDIA GPUs include many *Streaming Multiprocessors* (SM), each of which is composed of many *Streaming Processors* (SP). In the Kepler architecture, the SM new generation is called SMX. Grouped into *blocks*, *threads* execute *kernels* functions synchronously. Threads within a block can cooperate by sharing data on an SMX and synchronizing their execution to coordinate memory accesses; inside a block, the scheduler organizes *warps* of

⁵<http://charmplusplus.org/>

⁶<http://legion.stanford.edu/>

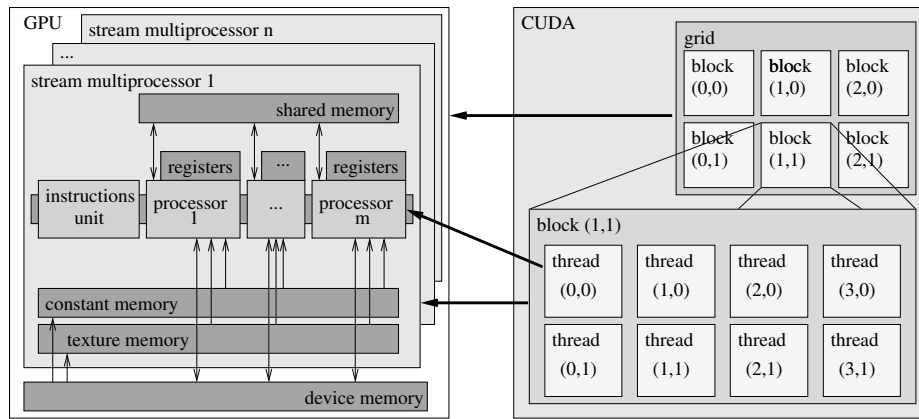


Figure 3.3: NVIDIA GPU and CUDA architecture overview

32 threads which execute the instructions simultaneously. The blocks are distributed over the GPU SMs to be executed independently.

In order to use data in a device kernel, it has to be first created on the CPU, allocated on the GPU and then transferred from the CPU to the GPU; after the kernel execution, the results have to be transferred back from the GPU to the CPU. GPUs consist of several memory categories, organized hierarchically and differing by size, bandwidth and latency. On the one hand, the device's main memory is relatively large but has a slow access time due to a huge latency. On the other hand, each SMX has a small amount of shared memory and L1 cache, accessible by its SPs, with faster access, and registers organized as an SP-local memory. SMs also have a constant memory cache and a texture memory cache. Reaching optimal computing efficiency requires considerable effort while programming. Most of the global memory latency can then be hidden by the threads scheduler if there is enough computational effort to be executed while waiting for the global memory access to complete. Another way to hide this latency is to use streams to overlap kernel computation and memory load.

It is also important to note that branching instructions may break the threads synchronous execution inside a warp and thus affect the program efficiency. This is the reason why test-based applications, like combinatorial problems that are inherently irregular, are considered as bad candidates for GPU implementation.

Specific tools have been made for HPC in the NVIDIA GPGPUs.

Dynamic Parallelism This feature allow the GPU kernels to run other kernels themselves. When more sub-tasks have to be generated this can be done directly on the GPU using dynamic parallelism.

Hyper-Q This technology enable several CPU threads to execute kernels on the same GPU simultaneously. This can help to reduce the synchronization time and idle time of CPU cores for specific applications.

NVIDIA GPU-Direct GPUs' memory and CPU ones are different and the Host much push the data on GPU before allowing it to compute. GPU-Direct allows direct transfers from GPU devices through the network. Usually implemented using MPI.

OpenCL

OpenCL is a multi-platform framework targeting a large part of nowadays architectures from processors to GPUs, FPGAs, etc. A large group of company already provided conform version of the OpenCL standard: IBM, Intel, NVIDIA, AMD, ARM, etc. This framework allows to produce a single code that can run in all the host or device architectures. It is quite similar to NVIDIA CUDA Driver API and based on kernels that are written and can be used in On-line/Off-line compilation meaning Just In Time (JIT) or not. The idea of OpenCL is great by

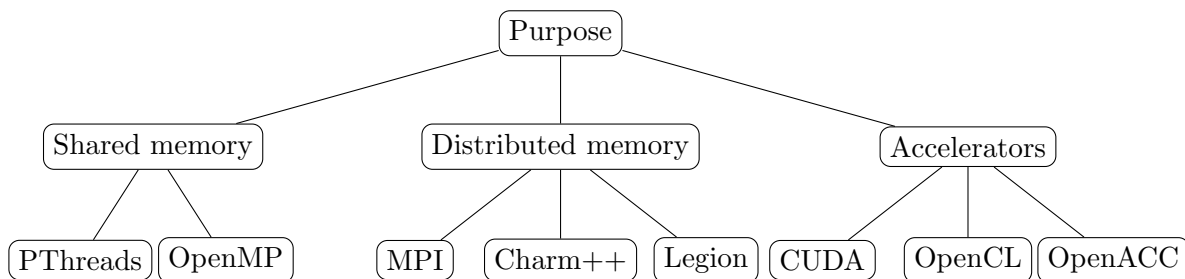


Figure 3.4: Runtimes, libraries, frameworks or APIs

rely on the vendors wrapper. Indeed, one may wonder, what is the level of work done by NVIDIA on its own CUDA framework compare to the one done to implement OpenCL standards? What is the advantage for NVIDIA GPU to be able to be replace by another component and compare on the same level? Those questions are still empty but many tests prove that OpenCL can be as comparable as CUDA but rarely better[KDH10, FVS11].

In this study most of the code had been developed using CUDA to have the best benefit of the NVIDIA GPUs present in the ROMEO Supercomputer. Also the long time partnership of the University of Reims Champagne-Ardenne and NVIDIA since 2003 allows us to exchange directly with the support and NVIDIA developers.

OpenACC

Open ACCelerators is a "user-driven directive-based performance-portable parallel programming model"⁷ developed with Cray, AMD, NVIDIA, etc. This programming model propose, in a similar way to OpenMP, pragmas to define the loop parallelism and the device behavior. As the device memory is separated specific pragmas are use to define the memory movements. Research works[WSTaM12] tend to show that OpenACC performances are good regarding the time spend in the implementation itself compare to fine grain CUDA or OpenCL approaches. The little lack of performances can also be explain by the current contribution to companies in the wrapper for their architectures and devices.

The runtime, libraries, frameworks and APIs are summarized in figure 3.4. They are used in combination. The usual one is MPI for distribution, OpenMP and CUDA to target processors and GPUs.

3.4 Benchmarks

All those models, theory, hardware and software leads to better understanding and characterization of machines to produce better algorithm and solve bigger and harder problems. The question that arise is: how to know if a machine is better than another? We answer that question with FLOPS, IPC, OPS or just the frequency of the machine. The models like BSP or law's like Amdahl and Gustafson ones propose to find the best/worst case during the execution.

In real application the only way to really know what will be the behavior of a supercomputer is to try, test real code on it. This is call benchmarking. Several kind of benchmarks exists and target a specific application of supercomputers. We present here the most famous benchmarks of HPC and their specificities.

3.4.1 TOP500

The most famous benchmark is certainly the TOP500⁸. It gives the ranking of the 500 most powerful, known, supercomputers of the world as its name indicates. Since 1993 the organization

⁷<https://www.openacc.org/>

⁸<http://www.top500.org>

assembles and maintains this list updated twice a year in June and November.

This benchmark is based on the LINPACK[DMS⁺94] a benchmark introduced by Jack J. Dongarra. This benchmark rely on solving dense system of linear equations. As specified in this document this benchmark is just one of the tools to define the performance of a supercomputer. It reflects "the performance of a dedicated system for solving a dense system of linear equations". This kind of benchmark is very regular in computation giving high results for FLOPS.

In 1965 the Intel co-fonder Gordon Moore made an observation[Pre00] on the evolution of devices. He pointed the fact that the number of transistors in a dense integrated circuit doubles approximately every eighteen months. This is know as the Moore's law. Looking at the last TOP500 figure presented on figure ??, in the introduction of this document, we saw that nowadays machines does not fit in the law anymore. This is due to the size of transistor and the energy needed to reach more powerful machines. The Moore's law have been sustains by the arrival of many-cores architectures such as GPU or Xeon Phi. Tomorrow machines architectures will have to be based on hybrid with more paradigms and tools to take part of massive parallelism.

3.4.2 Green500

In conjunction of the TOP500, the Green500⁹ focus on the energy consumption of supercomputers. The scale is based on FLOPS per watts [FC07]. Indeed the energy wall is the main limitation for next generation and exascale supercomputers. In the last list, November 2017, the TOP3 machines are accelerated with PEZY-SC many-core devices. The TOP20 supercomputers are all equipped with many-cores architectures: 5 with PEZY-SC, 14 with NVIDIA P100 and 1 with the Sunway many-core devices. This show clearly that the nowadays energy efficient solutions resides in many-core architecture and more than that, hybrid supercomputers.

3.4.3 GRAPH500

The GRAPH500¹⁰ benchmark[MWBA10] focus on irregular memory accesses, and communications. The authors try to find ways to face the futures large-scale large-data problems and data-driven analysis. This can be see as a complement of the TOP500 for data intensive applications. The aim is to generate a huge graph to fill all the maximum memory on the machine and then operate either:

BFS: A Breadth-First Search which is an algorithm starting from a root and exploring recursively all the neighbors. This requires a lot of irregular communications and memory accesses.

SSSP: A Single Source Shortest Path which is an algorithm searching the shortest path from one node to the others. Like the BFS it has an irregular behavior but also requires to keep more data during the computation.

This benchmark will be detailed in Part II Chapter II in our benchmark suite.

3.4.4 HPCG

The High Performance Conjugate Gradient benchmark¹¹ is a new benchmark created in 2015 and presented for the first time at SuperComputing 2015. The last list, November 2017 contains 115 supercomputers ranked. The list also offer to compare the results of Linpack compared to Conjugate Gradient. This benchmark is a first implementation of having both computation and communications aspects of HPC in the same test.

This benchmark is presented and features:

⁹<https://www.top500.org/green500/>

¹⁰<https://www.graph500.org/>

¹¹<http://www.hpcg-benchmark.org/>

- Sparse matrix-vector multiplication;
- Vector updates;
- Global dot products;
- Local symmetric Gauss-Seidel smoother;
- Sparse triangular solve (as part of the Gauss-Seidel smoother);
- Driven by multigrid preconditioned conjugate gradient algorithm that exercises the key kernels on a nested set of coarse grids;
- Reference implementation is written in C++ with MPI and OpenMP support.

The benchmarks presented in this section are the most famous of HPC world. Indeed, they are not the perfect representative of the nowadays application. The upcoming of big data and artificial intelligence in addition to classical "real life" applications impose HPC to evolve and find new ways to target new architectures. The TOP500 target the computational problem but does not handle a lot of irregularity. Indeed, solving dense linear equation is straight forward and also use the memory in a regular way. The Graph500 is very interesting to focus on communication and does handle irregular behavior for communications and memory. The Green500 does target energy wall but can also be applied to any benchmark. The most interesting one may be the HPCG benchmark. It does create irregularity during computation and communication along to memory traversal.

3.5 Conclusion

In this chapter we presented the most used software tools for HPC. From inside node with shared memory paradigms, accelerators and distributed memory using message passing runtime with asynchronous or synchronous behavior.

The tools to target accelerators architectures tend to be less architecture dependent with API like OpenMP, OpenCL or OpenACC targeting all the machines architectures. Unfortunately the vendor themselves have to be involve to provide the best wrapper for their architecture. In the mean time vendor dependent API like CUDA for NVIDIA seems to deliver the best performances.

We show through the different benchmark that hybrid architecture start to have their place even in computation heavy and communication heavy context. They are the opportunity to reach exascale supercomputers in horizon 2020.

Conclusion

This part detailed the state of the art theory, hardware and software in High Performance Computing and the tools we need to detail our experiences.

In the first chapter we introduced the models for computation and memory. We also detailed the main laws of HPC.

The second chapter was an overview of hardware architectures in HPC. The one that seems to be the most promising regarding computational power and energy consumption seems to be hybrid architectures. Supercomputers equipped with classical processors accelerated by devices like GPGPUs, Xeon Phi or, for tomorrow supercomputers, FPGAs.

In the third section we showed that the tools to target such complex architecture are ready. They provide the developer a two or three layer development model with MPI for distribution over processes, OpenMP/PThreads for tasks between the processor's cores and CUDA/OpenCL/OpenMP/OpenACC to target the accelerator.

We also showed in the last part that the benchmarks proposed to rank those architectures are based on regular computation. They are node facing realistic domain scientists code behavior. The question that arise is: How the hybrid architecture will handle irregularity in term of computation and communication? This question will be developed in the next part through one example for irregular computation and another for irregular communication using accelerators.

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