Corporate Finance: IOE 452/MFG 455

University of Michigan

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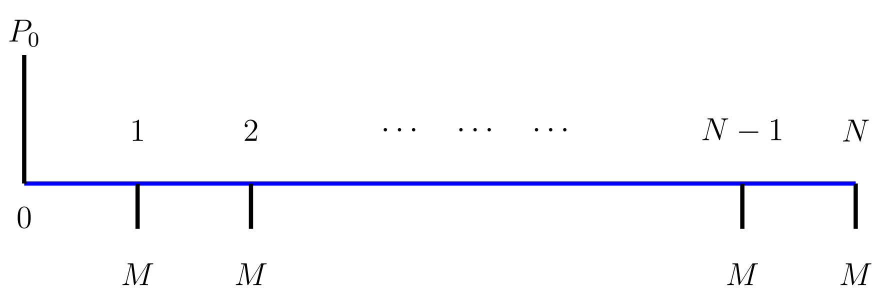
Contents

1 Amortization

- Loan amortization is the process of fully repaying a loan over time through a sequence of scheduled payments.
- A common example of an amortized loan is a home mortgage.
- A mortgage is a legal agreement tied to a debt obligation that specifies the property or other collateral that the borrower pledges. If the borrower fails to make the required payments, the lender has the right to claim this collateral.
- Loan amortization allows us to determine how much of each payment goes toward interest and how much reduces the principal balance.
- Given the amount borrowed, the interest rate (APR), and the number of payments, we can compute the level payment required to fully repay the loan.
- We can also decompose each payment into its interest and principal components. This breakdown is especially useful for tax reporting and financial planning.

1.1 Level Payments

- Suppose an initial loan amount (principal) of P_0 is borrowed at a periodic interest rate of r.
- The borrower makes **equal** payments of amount M at regular intervals until the loan is fully repaid after N periods.
- We first analyze the case of level (constant) payments.
- Let P_n denote the outstanding principal balance at the end of period n.
- Goal: Determine the outstanding loan balance at each payment date, and decompose each payment into interest and principal repayment.



- **Analysis:** At the end of the first payment period ($n = 1$), interest accrues on the initial principal at rate rP_0. A payment of M is then made.

- The outstanding principal at the end of period 1 is

$$P_1 = P_0 + rP_0 - M = P_0(1+r) - M$$

- At the end of the second period ($n = 2$), interest accrues on P_1 and another payment M is made:

$$\begin{aligned} P_2 &= P_1(1+r) - M \\ &= (P_0(1+r) - M)(1+r) - M \\ &= P_0(1+r)^2 - M[1 + (1+r)] \end{aligned}$$

- Continuing in this fashion, at the end of period $n = 3$:

$$P_3 = P_0(1+r)^3 - M\left[1 + (1+r) + (1+r)^2\right].$$

- In general, the outstanding principal after n periods is

$$P_n = P_0(1+r)^n - M\left[1 + (1+r) + \cdots + (1+r)^{n-1}\right]$$

- Recall the geometric series identity:

$$1 + x + x^2 + \cdots + x^{N-1} = \frac{X^N - 1}{x - 1}, \qquad x \neq 1$$

- Applying this result, the outstanding principal after n periods is

$$P_n = \underbrace{P_0\,(1+r)^n}_{\text{Future value of the principal}} - \overbrace{\frac{M}{r} \times \left[(1+r)^n - 1\right]}^{\text{Future value of the annuity payments}} \tag{1}$$

- Equivalently,

$$P_n = \left(P_0 - \frac{M}{r}\right)(1+r)^n + \frac{M}{r} \tag{2}$$

- To ensure the loan is fully repaid at period n, we must have $P_n = 0$. Solving for the payment M yields:

$$\boxed{M = \frac{rP_0(1+r)^n}{(1+r)^n - 1}} \tag{3}$$

- This can also be written as

$$\boxed{M = \frac{rP_0}{1 - (1+r)^{-n}}} \tag{4}$$

Example

Borrow $P_0 = \$100{,}000$ at an annual interest rate of 5%. The loan must be repaid over two years using monthly payments. Determine the level monthly payment.

Solution:

There are $N = 2 \times 12 = 24$ payment periods. The monthly interest rate is

$$i = \frac{.05}{12} = .0042$$

Using the level-payment formula (or Excel):

$$M = \mathbf{PMT}\,(.05/12, 24, -100000, 0, 0) = \$4{,}387.14$$

1.2 Amortization Schedule

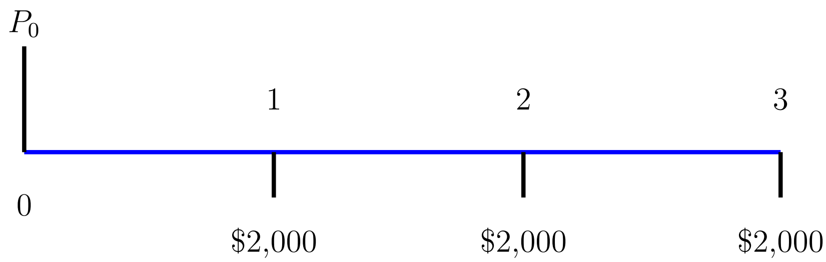
- We introduce the following notation:
    1. I_n: Interest paid in period n
    2. PP_n: Principal repaid in period n
    3. R_n: Total payment in period n
    4. B_n: Loan balance at the end of period n
- Each payment satisfies:
$$R_n = I_n + \mathrm{PP}_n.$$
- The evolution of the loan balance follows:
$$\begin{aligned} I_n &= r \times B_{n-1}, \\ B_n &= B_{n-1} - \mathrm{PP}_n. \end{aligned}$$

Example:

Suppose you obtained a loan at 5%. You must make 3 equal payments of $\$2,000$ each to fully pay the initial loan and its accrued interest. Determine the amortization schedule.

Solution:

The time line of the problem is as follows:



The problem's information is summarized in the following table:

Note that in this case total paid in each period is $R1 = R2 = R3 = \$2,000$.

n	R_n	I_n	$\text{PP}_n = R_n - I_n$	$B_n = B_{n-1} - \text{PP}_n$
0				?
1	$2,000	?	?	?
2	$2,000	?	?	?
3	$2,000	?	?	$0.00
Total				

Note that at the beginning all the known information is highlighted in the blue color.

First, start at period 0. We must determine the balance at period zero, that is B_0. This balance is obtained from:

$$B_0 = \frac{\$2,000}{(1+.05)^1} + \frac{\$2,000}{(1+.05)^2} + \frac{\$2,000}{(1+.05)^3} = \$5{,}446.50$$

In other words, $\$5,446.50$ must be paid in 3 installments in such a way that the balance at the end of period 3 is exactly zero.

Next, consider period 1: the interest payment at period $n = 1$ is $I_1 = r \times B_0 = .05 \times \$5{,}446.50 = \$272.32$. Therefore, the portion going towards the payment of the principal is

$$\text{PP}_1 = R_1 - I_1 = \$2,000 - \$272.32 = \$1{,}727.68$$

Consequently, the balance at the end of period $n = 1$ is

$$B_1 = B_0 - \text{PP}_1 = \$5,446.50 - \$1,727.68 = \$3{,}718.82$$

In a similar manner:

$$\begin{aligned} I_2 &= r \times B_1 = \$185.94 \\ \text{PP}_2 &= \$2,000 - \$185.94 = \$1{,}814.06 \\ B_2 &= \$1{,}904.76 \end{aligned}$$

$$
\begin{aligned}
I_3 &= r \times B_2 = \$95.24 \\
\mathrm{PP}_3 &= \$2,000 - \$95.24 = \$1{,}094.76
\end{aligned}
$$

Remark–The initial balance B_0 must be equal to sum of the principal paid in each period. Similarly, B_1 must be equal to the discounted sum of the remaining total payments (periods 2 & 3).

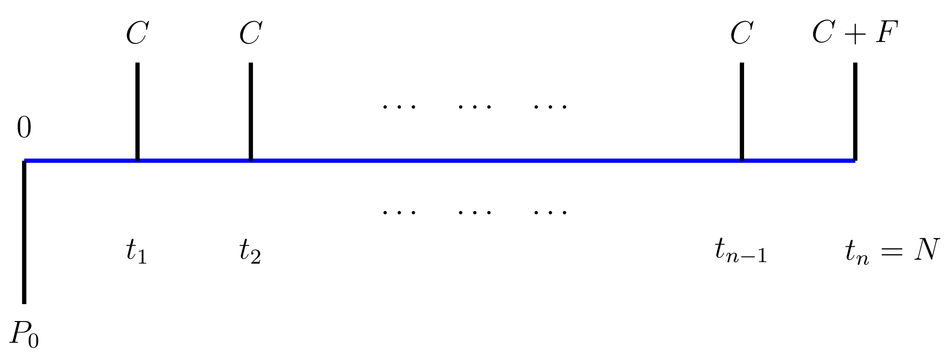
2 Bonds: Basics

- A **bond** is a formal contract between a borrower and a lender. The specific terms of the agreement are set forth in the bond's indenture, a legally binding document that defines the rights and obligations of both parties.
- Bonds are issued and held for two fundamental reasons:
  1. **Issuers** sell bonds to raise capital.
  2. **Investors** purchase bonds to receive future cash flows.
- Under the contract, the borrower agrees to:
  1. Make periodic interest (coupon) payments, and
  2. Repay the principal at maturity.
- Bonds are issued by a wide range of entities, including sovereign governments, government agencies, corporations, and municipalities, to finance expenditures and investments.
- **Corporate bonds** are issued by corporations, while **municipal bonds** are issued by state and local governments.
- Bonds represent the largest segment of the global fixed–income market.
- The term **fixed–income** refers to securities that provide predetermined or contractually specified cash flows.
- By purchasing a bond, an investor becomes a **creditor** of the issuer. The promised interest and principal payments constitute legal obligations of the borrower.
- Failure to meet these obligations may result in default, restructuring, or bankruptcy.
- For corporate issuers, bonds are senior to equity securities: bondholders must be paid before dividends are distributed to shareholders.
- As a result, bond cash flows are generally more predictable than the cash flows of preferred or common stock.
- Most bonds promise:
  1. Periodic coupon payments, and
  2. Repayment of the face value (or par value) at the redemption date.
- Bonds differ significantly in their **risk characteristics**, including default risk, interest rate risk, and liquidity risk.
- Even the safest corporate bonds are riskier than U.S. Treasury securities, which are backed by the full faith and credit of the U.S. government.
- Most bond issues receive **credit ratings** from rating agencies. These ratings reflect both quantitative and qualitative assessments of the issuer's ability and willingness to repay its debt.

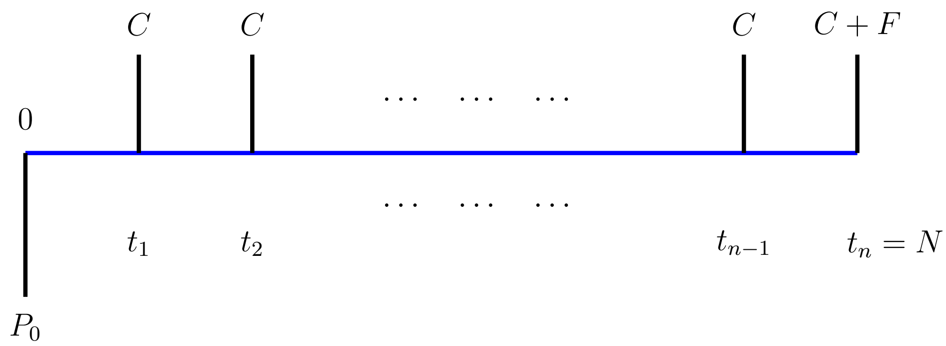
- Many bonds embed callable or puttable features, which give issuers or investors option–like rights that can alter the bond's cash flows.

3 Bond Cash Flows

- A standard bond generates two types of cash flows:
  1. **Coupon payments**: periodic interest payments.
  2. **Principal repayment**: a lump–sum payment at maturity.
- The value of a bond today is equal to the **present value** of its promised future cash flows.
- To value a bond, future cash flows are discounted at a rate that reflects:
  1. The time value of money, and
  2. The risk associated with receiving those cash flows.
- This discount rate is referred to as the bond's yield or yield to maturity (YTM).
- Bonds with more uncertain cash flows must offer higher yields to compensate investors.
- Bond pricing therefore involves two key steps:
  1. Use the coupon rate to determine the bond's cash flows.
  2. Use the yield to maturity to discount those cash flows.



3.1 Pricing a Bond: Valuation



- Bond valuation consists of computing the present value of all expected future cash flows using an appropriate discount rate.
- The traditional valuation approach discounts all cash flows using a single yield.
- Bond cash flows consist of:
  1. Periodic coupon payments C, where typically $C = r \times F$,
  2. Repayment of the face value F at maturity.
- Let:
  - F denote the face (par) value of the bond,
  - r denote the coupon rate per payment period,
  - y denote the yield to maturity,
  - N denote the total number of coupon periods,
  - P denote the bond's market price.
- Bond prices are commonly quoted as a **percentage of face value**.

Example–Suppose the price of a bond with $F = \$1,000$ bond is reported as 98. In this case, $P = 98$. This means that the market value of the bond (the price at which the bond is quoted) is $\$980 = \frac{98}{100} \times \$1,000$.

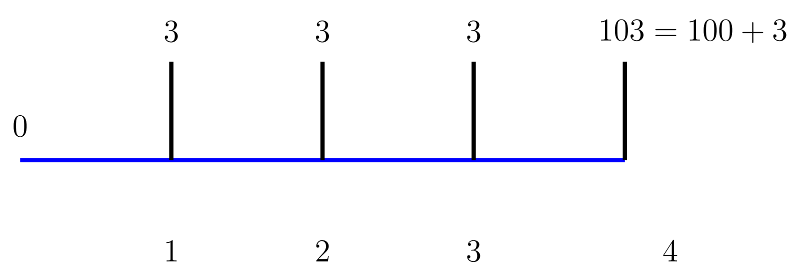
Example– Assume a bond is issued with a face value of \$100. Suppose this bond makes 4 periodic coupons at the end of each period at $r = 3\%$. Suppose investors discount cash–flows at $y = 6\%$. Determine the price of this bond. *For simplicity assume the coupons are paid annually.*

Solution:

In this problem $N = 4$ and $F = \$100$. Each coupon payment is equal to

$$C = r \times F = .03 \times \$100 = \$3.00$$

The bond's cash–flow diagram is as follows:



The present value of the cash flows are:

$$\begin{aligned}
\text{PV (Cash Flows)} &= \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \frac{C+F}{(1+y)^4} \\
&= \frac{3}{(1+y)^1} + \frac{3}{(1+y)^2} + \frac{3}{(1+y)^3} + \frac{103}{(1+y)^4} \\
&= \frac{3}{(1+.06)^1} + \frac{3}{(1+.06)^2} + \frac{3}{(1+.06)^3} + \frac{103}{(1+.06)^4} \\
&= \$89.60
\end{aligned}$$

Therefore, the price of this bond P_0, which is just the sum of the present values of all the cash flows is \$89.60.

It is worth noting that in this case the price of the bond $P_0 = \$89.60$ is less than the face value of the bond $F = \$100$. Moreover, observe that for this situation, $y > r$. This is an example of a bond that is selling below *par*.

Example

Determine the price of a 3 year bond with face value of $\$1,000$ when coupon rate is 3% per year. The coupons are paid semi–annually. Assume a required rate of return of 6%.

Solution

A 3% percent coupon payable semiannually results in a coupon payment of $15 per six–month period, and the 6% annual investor's required rate of return is halved to 3% for 6 semiannual periods until maturity.

Use Excel's **PV** function:

$$\mathrm{PV}(0.03, 6, 15, 1000, 0)$$

4 Bond Types

Bonds are commonly classified based on their market price relative to face value (par):

1. **Premium bond**: $P > F$
2. **Discount bond**: $P < F$
3. **Par bond**: $P = F$

- Regardless of type, all bonds trade at par at maturity. This phenomenon is known as the pull–to–par effect.
- The bond's classification depends on the relationship between the coupon rate r and the yield y.
- If $r > y$, the bond offers more interest than required by the market and trades at a premium.
- If $y > r$, the bond must trade at a discount to offer a competitive return.
- If $r = y$, the bond trades at par.

One–Line Intuition (Premium, Discount, Par):

- A bond trades at a premium when its coupon rate is *too high* relative to the market yield ($r > y$), at a discount when its coupon rate is *too low* ($r < y$), and at par when the coupon rate exactly matches the market yield ($r = y$).

One–Line Intuition (Premium, Discount, Par):

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Example

Consider a 10 year bond with a face value of $\$1,000$. The coupon rate is 4%per year payable on a semi–annual basis. Suppose the required yield in the market is 5%. Determine the price and the type of the bond.

Solution:

In order to determine the price, we must discount all the coupons and face value of the bond at the required yield.

The periodic coupon payments are (semi–annual):

$$\frac{.04 \times \$1,000}{2} = \$20$$

Since we have a 10 year bond, there are 20 periods. In each of these 20 periods. the bond will deliver coupons. To determine the present value of sum of these cash flows, can use the annuity formula!

$$\text{PV of all coupons} = \frac{\frac{.04\times 1000}{2}}{\frac{.05}{2}} \times \left(1 - (1+.025)^{-20}\right) = \$311.78$$

The present value of the face value of the bond is

$$\frac{1000}{(1+.025)^{20}} = \$610.27$$

Thus the price of the bond is $P = \$311.78 + \$610.27 = \$922.05$. Since $\$922.05 < \$1,000$, this is a discount bond.

5 Plain Vanilla Bond Formulas

For a standard bond with constant coupons and yield, the price is given by:

$$P_0 = \frac{F}{(1+y)^N} + \sum_{n=1}^{N} \frac{C}{(1+y)^n} \tag{5}$$

- In the above, i.e., Eq. (5), the yield to maturity y is assumed to be the same for each coupon period. Yield to maturity must be expressed as a decimal.
- The yield to maturity y represents the required rate of return demanded by investors.
- In the U.S., bonds typically pay coupons semiannually; both coupon rates and yields must be adjusted accordingly.
- Variable coupon, constant yield. In this case, the bond price is given by:

$$P_0 = \frac{F}{(1+y)^N} + \sum_{n=1}^{N} \frac{C_n}{(1+y)^n} \tag{6}$$

- Variable coupon, variable yield. In this case, the bond price is given by

$$P_0 = \frac{F}{(1+y_N)^N} + \sum_{n=1}^{N} \frac{C_n}{(1+y_n)^n} \tag{7}$$

- A zero–coupon bond pays no coupons and is priced as:

$$P_0 = P = \frac{F}{(1+y)^N} \tag{8}$$

- The price of a zero–coupon bond is just the present value of its face price.
- Another way of thinking about zero–coupon bonds is to set $C = 0$ or $r = 0$ in $C = r \times F$.
- Bond yields are often quoted in basis points, where 1 basis point equals 0.01%.
- For simple bonds, r is set by the issuer. Therefore, the cash flows of simple bonds are known in advance.
- Treasury yields serve as the benchmark default–free rates for dollar–denominated bonds.

Economic Intuition

- A bond is simply a sequence of promised cash flows. Its price today is the amount an investor is willing to pay in exchange for those future payments.
- Each cash flow is discounted for two reasons:
  1. *Time value of money*: a dollar received in the future is worth less than a dollar today.
  2. *Risk*: the greater the uncertainty about receiving the cash flows, the higher the required yield.
- The yield y summarizes the market's required rate of return for bearing the bond's risk. A higher yield implies heavier discounting and therefore a lower bond price.
- Coupon payments resemble an annuity, while the face value repayment resembles a lump–sum payment at maturity. The bond price is the sum of these two present values.
- When market yields rise, existing bonds become less attractive and their prices fall. When market yields fall, existing bonds become more valuable and their prices rise.

Graphical Intuition: Bond Price vs. Yield

- The relationship between a bond's price and its yield is inverse and nonlinear.
- As the yield y increases, the present value of future cash flows falls, causing the bond price to decline.
- The curve is downward sloping and convex: prices rise more when yields fall than they fall when yields rise by the same amount.
- At the yield where $y = r$, the bond trades at par.

Interpretation: Existing bonds must adjust in price so that their promised cash flows deliver the market-required yield.

Common Student Mistakes with Bonds

- Confusing the coupon rate with the yield to maturity. (Coupon rate determines cash flows; yield determines price.)
- Forgetting to adjust both rates and number of periods when coupons are paid semi-annually.
- Believing that bond prices and yields move in the same direction. (They always move in opposite directions.)
- Thinking that a premium or discount bond stays that way forever. (All bonds pull to par as maturity approaches.)
- Discounting coupon payments at different rates without justification in a plain-vanilla bond.