Corporate Finance: IOE 452/MFG 455
University of Michigan
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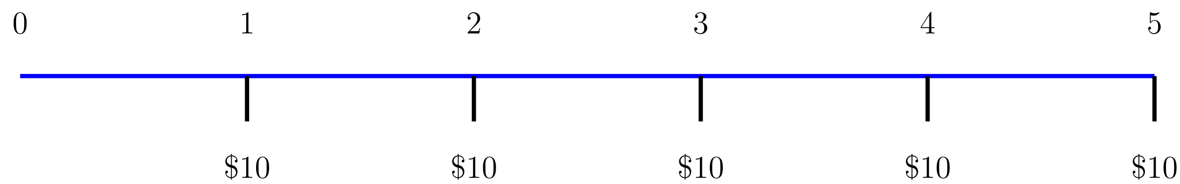
Contents

1 Examples: Present Value, Future Value & Compounding

Example:

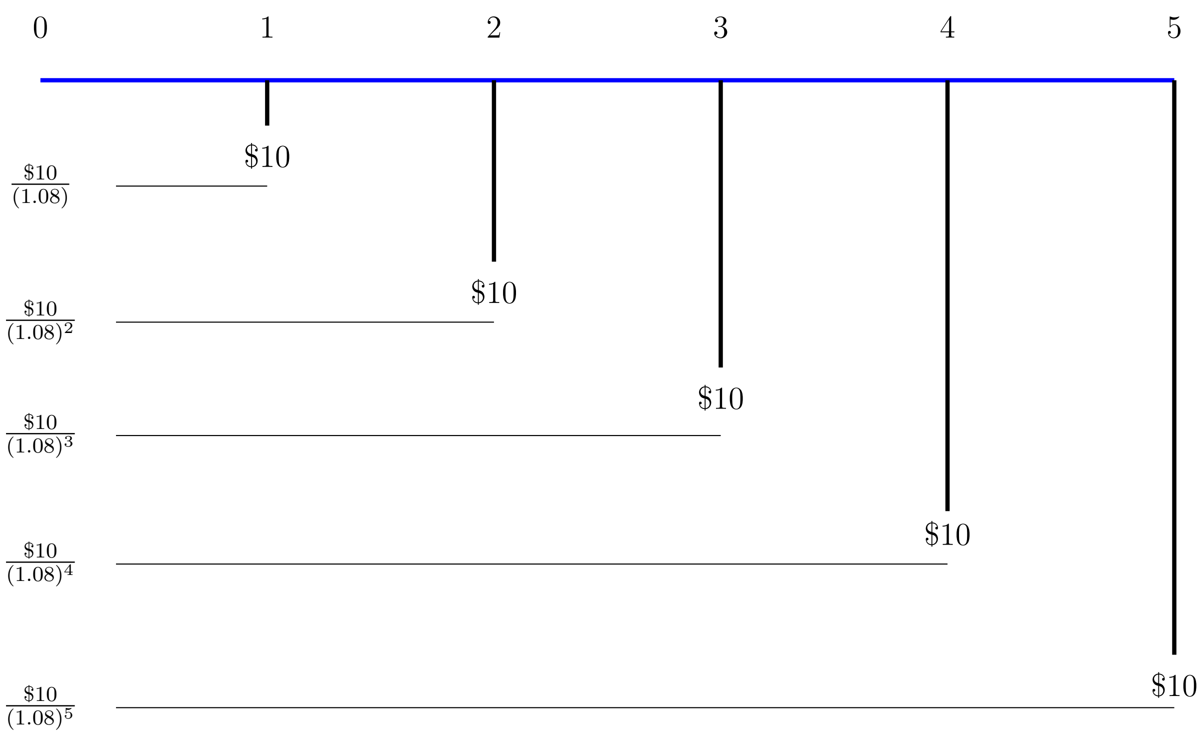
Suppose you receive \$10 for the next 5 periods. How much would you be willing to pay in order to receive the above pattern of cash flows?



Solution:

The answer depends on the level of interest rate! That is the rate at which the incoming payments can be reinvested during the subsequent five periods. For example, if people predict (expect) a reinvestment rate is 8%, then the *present* values of the five coupon payments will be:

$$
\begin{aligned}
\textbf{PV} &= \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \frac{C}{(1+r)^4} + \frac{C}{(1+r)^5} \\
&= \frac{\$10}{(1+.08)^1} + \frac{\$10}{(1+.08)^2} + \frac{\$10}{(1+.08)^3} + \frac{\$10}{(1+.08)^4} + \frac{\$10}{(1+.08)^5} \\
&= \$39.93
\end{aligned}
$$



Example:

Suppose we wanted to calculate the *future* values of the above cash flows.

Solution:

We may proceed as follows:

0	1	2	3	4	5	
	$\$10$	$\$10\,(1.08)$	$\$10\,(1.08)^2$	$\$10\,(1.08)^3$	$\$10\,(1.08)^4$	Reinvest 1 period interest
		$\$10$	$\$10\,(1.08)$	$\$10\,(1.08)^2$	$\$10\,(1.08)^3$	Reinvest 2 period interest
			$\$10$	$\$10\,(1.08)$	$\$10\,(1.08)^2$	Reinvest 3 period interest
				$\$10$	$\$10\,(1.08)^1$	Reinvest 4 period interest
					$\$10$	Reinvest 5 period interest

$$\begin{aligned} \textbf{FV} &= \$10\,(1.08)^4 + \$10\,(1.08)^3 + \$10\,(1.08)^2 + \$10\,(1.08)^1 + \$10 \\ &= \$58.67 \end{aligned}$$

Remark–The **FV** of \$58.67 is indeed identical to $\textbf{PV} \times (1 + .08)^5$. In other words, if you invest \$39.93 in a bank account with per period interest rate of 8% per period, you will end up with \$58.67 after 5 periods.

It is also worth noting that at time $t = 0$, this cash flow can be "stripped" and sold as five distinct cash flows. (Actually, this type of procedure is very common in bond pricing.) For example, the first period cash flow can be sold separately as a one–period asset at price $\frac{\$10}{1.08} = \$9.26$. The second period cash flow can be sold separately as a one–period asset at price $\frac{\$10}{1.08^2} = \$8.57$. And so on.

2 Continuous Compounding

Suppose you initially invest P for N years at a nominal annual rate of r per period.

Suppose interest is compounded m times per year. Hence, P accumulated after N years is:

$$FV = P \times \left(1 + \frac{r}{m}\right)^{m \times N}$$

As the frequency of the compounding increases, that is as $m \to \infty$, the initial investment of P becomes

$$\lim_{m \to \infty} P \times \left(1 + \frac{r}{m}\right)^{m \times N} \longrightarrow P\, e^{r \times N} \tag{1}$$

Remark–Suppose $N = 1$. That is consider investing for 1 year. Therefore, the future value using continuous compounding is given by $P \times e^r$.

It is worth while to compare the above expression for continuous compounding for 1 year, namely, $FV_1 = P \times e^r$, with our previous future value formula for one period, namely

$$FV_1 = P\,(1 + r)$$

Why are the two expressions different? Did we neglect anything?

In fact there is no discrepancy as long as we recognize the range of validity for the exponential function. In fact, using Taylor series expansion of e^r, we get, for “small” r,

$$e^r = 1 + r + \frac{r^2}{2!} + \cdots, \approx 1 + r$$

3 Annuity

- Most investments return a stream of payments rather than a single lump sum payment at some future date.
- The principles of determining the future value or present value of a series of cash flows are the same as for a single cash flow, yet the math becomes a bit more cumbersome.
- When we make future/present value calculations, usually we are working with multiple cash flows paid at regular intervals, called periods.
- To calculate the present value of such a stream, we must add up the present values of each individual payment for the period when it will be received.
- An annuity is a series of payments of fixed amounts for a specified number of periods. Common types of annuities are:

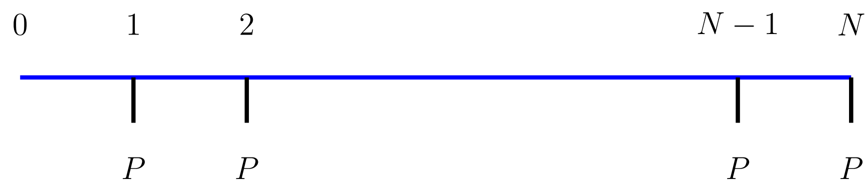
    1. Ordinary annuity: This is an annuity where payments come at the end of the period and the first payment is made one period from now.
    2. Annuity due: This is an annuity where payments are made at the beginning of the period.
    3. Deferred annuity: A deferred annuity is an annuity in which the first cash flow occurs beyond the end of the first period.

- Annuities occur in a variety of different settings:

    1. Saving for retirement by investing a constant amount of money at the end of month in an an account paying a fixed rate.
    2. Estimating the value of a lottery, etc.

3.1 Ordinary Annuity: Future Value

- Let P be the amount invested at the end of each period.
- Let N be the total number of periods. Let r be the interest rate per period.
- Goal: Determine the future value at the end of N^{th} period P_N. That is, want to find a formula for the future value P_N.



- **Fact**–The future value FV_N is given by:

$$\text{FV}_N = P\sum_{k=1}^{N}(1+r)^{k-1} = \frac{P}{r}\Big((1+r)^N - 1\Big) \tag{2}$$

- **Future Value (Proof) : Method I**
- At the end of period 1 we have made 1 payment. Hence future value is:

$$P_1 = P$$

- At the end of period 2 we have made 2 payment. Hence

$$P_2 = P_1(1+r)^1 + P$$

- At the end of period 3 we have made 3 payment. Hence

$$P_3 = P_2(1+r) + P = P(1+r)^2 + P(1+r) + P$$

- The pattern suggests the following recursion:

$$P_{k+1} = P_k(1+r) + P$$

- At the end of period N we have made a total of $(N-1)$ payments, and hence

$$\begin{aligned} P_N &= P(1+r)^{N-1} + P(1+r)^{N-2} + \cdots + P(1+r) + P \\ &= P\Big(1 + (1+r) + \cdots\cdots + (1+r)^{N-2} + (1+r)^{N-1}\Big) \end{aligned}$$

- In order to simplify, let $x = (1 + r)$ and note that we are summing a finite term geometric series!

$$P_N = P \sum_{i=0}^{N-1} x^i$$

- Recall the following result from summing a geometric series:

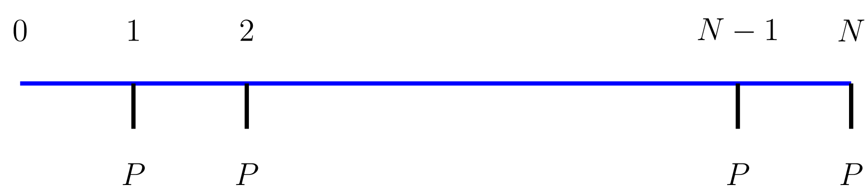
$$1 + x + x^2 + \cdots + x^{N-1} = \frac{X^N - 1}{x - 1}, \qquad\qquad x \neq 1$$

- To conclude, using the above substitution:

$$P_N = P \sum_{i=0}^{N-1} x^i = P \left(\frac{(1 + r)^N - 1}{r} \right)$$

- It is more meaningful to remember the above future value by writing it as

$$P_N = \frac{P}{r} \left((1 + r)^N - 1 \right) \tag{3}$$



- **Future Value: (Proof) : Method II**
- Consider period N: Make 1 payment. Its future value is:

$$P_N = P\,(1+r)^{\,0} = P$$

- Consider period $N-1$: Make 1 payment with future value of

$$P_{N-1} = P\,(1+r)^{\,1}$$

- Consider period $N-2$: Make 1 payment with future value of

$$P_{N-2} = P\,(1+r)^{\,2}$$

- $\cdots$ Continue till period 1. Future value of period 1 payment is

$$P_1 = P\,(1+r)^{\,N-1}$$

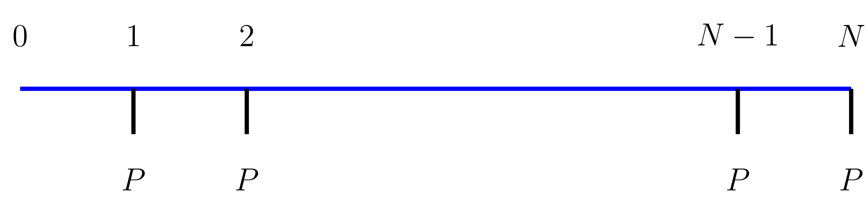
- By the time we reach period 1, made a total of N payments, therefore, their total contribution is

$$P_N = P + P\,(1+r) + \cdots\cdots + P\,(1+r)^{N-2} + P\,(1+r)^{N-1}$$

- Again, using the sum of a finite geometric series (set $r = (1+x)$)

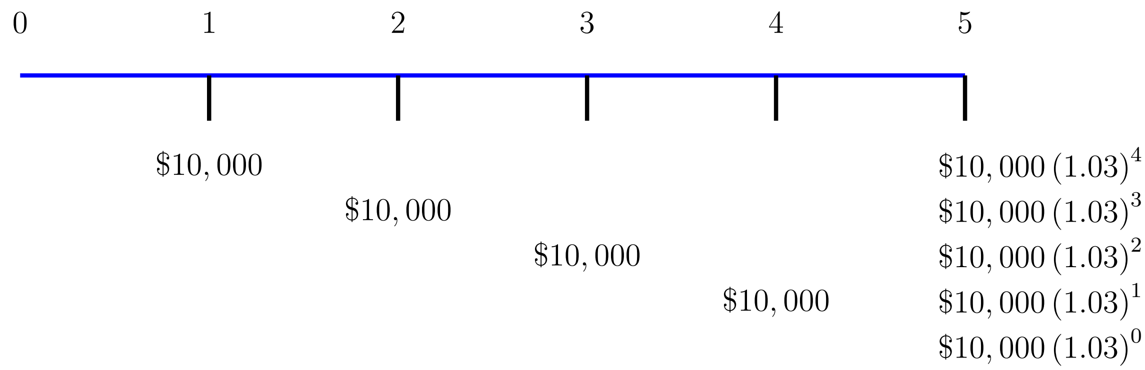
$$1 + x + x^2 + \cdots + x^{N-1} = \frac{1-x^N}{1-x}, \qquad\qquad x \neq 1$$

$$P_N = P \sum_{k=1}^{N} (1+r)^{k-1} = \frac{P}{r}\left((1+r)^N - 1\right) \tag{4}$$



Remark–Since $r > 0$, therefore, $(1+r)^N > 1$. Hence, the future value of an annuity is always greater than $\frac{P}{r}$.

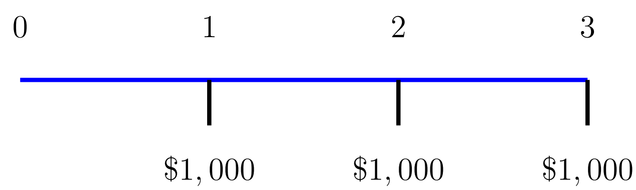
Example–An investor expects to receive \$10,000 a year from now for each of the next five years. The investor can earn an annual interest rate of 3% each time \$10,000 is invested. Determine investor's wealth at the end of the fifth year. *Ans:* \$53,091.36.



$$\textbf{FV}\,(.03, 5, 10000, 0) = 53091.36$$

Example–Find the future value for a three–payment ordinary annuity that has payments of \$1,000 and a 5% interest rate.

Solution:



Check and make sure you get \$3,152.50.

$$\textbf{FV}\,(.05, 3, 1000, 0)$$

Example– A 25 year old recent graduate wants to save for the retirement. This person has decided to save \$20,000 at the end of each year until age 45. If the retirement account earns 7% per year, how much will be in the account at the retirement?

Solution:

$$\text{FV} = \$20,000 \times \frac{1}{.07}\left(1.07^{20} - 1\right) = \$819,909.85$$

3.2 Present Value of an Ordinary Annuity

If you invest P at the end of every period for N periods at a rate of r per period, then P_0 or the present value of the annuity is:

$$P_0 = \frac{P_N}{(1+r)^N} = \frac{P}{r} \times \left(1 - (1+r)^{-N}\right) \tag{5}$$

One way to use Eq. (5) is to receive an initial loan of P and repaying P exactly over N periods. This can be noted by

$$P_0 = P \sum_{k=1}^{N} \frac{1}{(1+r)^{N+1-k}}$$

Example– A person is the winner of a \$20 million lottery. The prize can either be taken as (a) \$10 million paid today, or (b) 15 payments of \$1 million per year starting today. If the interest rate is 5%, which is a better option?

Solution:

The payments of 15 equal sizes suggests an annuity formulation. Since the first payment begins immediately, the last payment will occur in 14 years (a total of 15 payments). The present value of this annuity is

$$\text{PV} = \$1\text{million} \times \frac{1}{.05}\left(1 - \frac{1}{1.05^{14}}\right) = \$9,898,640.94$$

Adding \$1 million we receive upfront, the option (b) has a present value of $\$10,898,640.94$.

Clearly, option (b) is a better one in this case.

3.3 Annuity Due

- An annuity due is like an ordinary annuity, yet the first cash flow occurs *immediately*, instead of one period from today.
- Let P the be the amount invested at the beginning of every period
- Let N be the total number of periods. Let r be the interest rate per period.
- Goal: Determine the future value at the end of N^{th} period P_N.
- **Solution:**
- To find a formula for P_N, let us look at P_1, P_2 and so on to a see a pattern!
- If we make only 1 payment at the end of period 1, its future value is

$$P_1 = P\left(1+r\right)$$

- If we make 2 payments at the end of period 2, its future value is

$$P_2 = \left(P + P_1\right)\left(1+r\right) = P\left(1+r\right) + P\left(1+r\right)^2$$

- If we make 3 payments at the end of period 3, its future value is

$$P_3 = P\left(1+r\right) + P\left(1+r\right)^2 + P\left(1+r\right)^3$$

- The above patterns suggest the following recursion:

$$P_N = P\left(1+r\right) + P\left(1+r\right)^2 + P\left(1+r\right)^3 + \cdots\cdots + P\left(1+r\right)^{N-1} + P\left(1+r\right)^N$$

- Use the sum of finite geometric series of the following form:

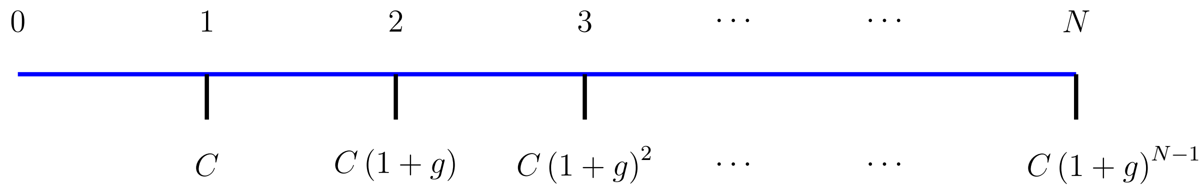
$$1 + x + \cdots + x^{N-1} = \frac{x^N - 1}{x - 1}, \qquad x \neq 1$$

- We conclude that If we invest P at the beginning of every period for N periods at a rate r, then P_0, the present value of the annuity is

$$P_0 = \frac{P_N}{\left(1+r\right)^N} = \frac{P}{r}\left[1 - \left(1+r\right)^{-N}\right]\left(1+r\right) \tag{6}$$

3.4 Ordinary Annuity: Present Value of Growing Cash Flows

- Suppose cash flows are expected to grow at a constant rate in each period.
- The first cash flow arrives in period 1 and it does NOT grow at that period.
- Suppose cash flows C grow at rate g every period until N:



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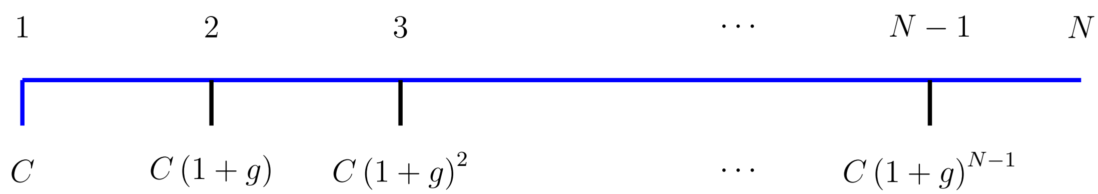
- The present value of a growing ordinary annuity is

$$\text{PV} = \frac{C}{r-g}\left(1-\left(\frac{1+g}{1+r}\right)^{N}\right) \tag{7}$$

3.5 Annuity Due: Present Value of Growing Cash Flows

- If the first cash flow occurs at period 0, the time line is as follows:



- The present value of annuity due can be shown to be:

$$\text{PV} = \frac{C}{r-g}\left(1-\left(\frac{1+g}{1+r}\right)^{N}\right) \times (1+r) \tag{8}$$

Example: HW1

4 Perpetuity

- A particularly interesting case is to calculate the present value of payments that are extended forever!
- Consider a situation that an entity promises to pay $\$C$ dollars once a year forever.
- How much would you pay to receive the above series of cash–flows?
- To compute the value of this stream of payments we have to compute the infinite sum:

$$\mathbf{PV} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots\cdots + \cdots\cdots$$

- The "trick" in evaluating the above infinite series is to factor out $\frac{1}{(1+r)}$ to get

$$\mathbf{PV} = \frac{1}{1+r}\left[C + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots\cdots\right]$$

- Note that the term sin the brackets are just C plus the present value! Substituting

$$\mathbf{PV} = \frac{1}{1+r}\left[C + \mathbf{PV}\right]$$

- Solving for **PV**

$$\mathbf{PV} = \frac{C}{r}$$

- **Remark**– Again, as $r \uparrow$, PV $\downarrow$.

Example–IOE 452 students (Winter 2025) want to endow an annual graduation party for their favorite instructor. They will commit $\$10,000$ per year forever to the department. The department earns 5% per year on its investments. How much will they have to donate? (the first party stars in one year time).

Solution:

$$\text{PV} = \frac{\$10,000}{.05} = \$200,000 \qquad \text{today}$$

5 Growing Perpetuity: Present Value

- Let C be the constant base cash flow per period to be paid forever.
- Assume cash flows grow at a constant rate of g per period.
- Therefore, cash flows follow the following pattern

$$C + C(1+g) + C(1+g)^2 + \cdots\cdots$$

- The present value of the above cash flows are

$$\mathrm{PV} = \frac{C}{1+r} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots\cdots$$

- To simplify: factor the common term, $\frac{C}{1+r}$ to get

$$\mathrm{PV} = \frac{C}{1+r}\left(1 + \frac{1+g}{1+r} + \left(\frac{1+g}{1+r}\right)^2 + \cdots + \cdots\right)$$

- Let $x = \frac{1+g}{1+r}$. Use the expression for the infinite sum of geometric series

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}, \qquad |x| < 1$$

- To get the sum of a growing perpetuity

$$\mathrm{PV} = \frac{C}{r-g}, \qquad g < r \tag{9}$$

- **Remark**– Again, as $r \uparrow$, PV $\downarrow$.

- An analogous expression is used to value shares!

6 Interest Rate Quotes & Adjustments

• Interest rates are quoted in a variety of ways.

• When evaluating cash flows, we must use a discount rate that matches the time period of cash flows. That is, must use a rate that reflects the actual return over the stated period.

6.1 Effective Rates

Corporate and financial data are often quoted using different time frames. It is important to be able to determine equivalent rates. Although generally quoted as an annual rate, the interest payments may materialize at different intervals, e.g., monthly, semiannually, quarterly, etc. In this section we consider cases where interest is paid either more or less often than the period used for measuring time.

Definition: The **effective rate of interest for a given period** is the amount of money that one unit of principal invested at the start of a particular period will earn during that one period. It is assumed that interest is paid at the end of the period in question.

Effective Annual Rate (EAR)

Recall that r is an effective annual rate (EAR). In this notation, this is the rate that will be earned at the end of one year. This is sometimes called the **APR** (annual percentage rate) when expressed as a percentage. For example, an annual effective rate of .05 would correspond to an **APR** of 5%.

Example–First, suppose we want to find an Effective annual rate (EAR) for periods longer than one year. Suppose initial investment is \$100 and $r = 10\%$.

Solution:

Consider investing P at rate r (effective annual rate (EAR)). Then, at the end of one year, the investment grows to

$$P \times (1 + r) = \$100 \times (1 + .1) = \$110$$

After two years, the initial investment becomes

$$P \times (1 + r)^2 = \$100 \times (1 + .1)^2 = \$121$$

Therefore, earning an effective annual rate (EAR) of $r = 10\%$ for two years, is equivalent to earning $21\% = \frac{121}{100}$ in total interest over the entire period. So for 2 years, the effective interest rate is found using the following expression:

$$P \times (1+r)^2 = P \times \left(1 + r_{\text{eff}}\right)$$

In general, by raising the interest factor $(1 + r)$ to the appropriate power, we can compute an equivalent interest rate for a longer time period. That is, we can determine r_{eff} from

$$P \times (1+r)^n = P \times \left(1 + r_{\text{eff}}\right)$$

In other words,

$$\text{Equivalent n–period Discount Rate} = r_{\text{eff}} = (1+r)^n - 1 \tag{10}$$

Example–Suppose you are given the monthly rate of return. Let r_1 be the the investment return in each month. Determine the annualized equivalent rate r_A.

Solution:

Compounding r_1 for one year, the annualized equivalent rate r_A is given by

$$r_A = (1 + r_1)^{12} - 1$$

Example–Determine the average monthly return to compound to a 15% annual rate.

Solution:

Let r_M be the monthly rate. Then

$$(1 + r_A)^{\frac{1}{12}} = (1 + r_M) \Longrightarrow r_M \approx 1.17\%$$

Remark–We can use the same method as above to find the equivalent interest rate for periods shorter than one year. In this case, we raise the interest rate factor $(1 + r)$ to the appropriate fractional power.

Example–Suppose one earns 5% interest based on annual rate. Determine the earning on a semi–annual basis.

Solution:

Earning 5% interest in one year is equivalent to receiving

$$\left(1 + r_{\text{eff}}\right)^{\frac{1}{2}} = 1.05 \Longrightarrow r_{\text{eff}} = 2.47\%$$

That is, a 5% effective annual rate is equivalent to an interest rate of approximately 2.47% earned every six months.

You may verify the above result by noting that if you invest at 2.47% compounded every six months, you will end up with a 5% annual rate:

$$\left(1 + r_{\text{eff}}\right)^2 = (1.0247)^2 = 1.05$$

6.2 Annual Percentage Rates

• Many agencies quote interest rates in terms of an **annual percentage rate (APR).**

• This rate is the amount of *simple interest rate* earned in one year. Therefore, **APR** is the amount of interest earned without the effect of compounding.

• Since **APR** does not include the effect of compounding, the **APR** quote is usually *less* than the actual amount of interest that one earns. Therefore, to compute the actual amount that one will earn in one year, one must convert the **APR** to an **Effective Annual Rate (EAR)**.

Example:

Suppose bank **A** advertises to pay its saving account holders an interest rate of 3% **APR** with monthly compounding. Determine the **EAR**.

Solution:

In this case, one will earn a monthly rate of

$$\frac{.03}{12} = .25\% \qquad \text{every month}$$

In other words, an **APR** with monthly compounding is actually a way of quoting a monthly interest rate, rather than an annual rate. Since interest compounds each month, at the end of one year you have for every \$1 investment

$$\$1 \times (1 + .0025)^{1\times 12} = 1.0304$$

Therefore, the **EAR** is 3.0416%. The 3.0416% that you earn on your deposit is *higher* than the quoted 3% **APR** due to compounding. The reason: in later months, you earn interest on the interest paid in earlier months.

Remark–Converting **APR** to **EAR**:

It is important to note that since **APR** does not reflect the true amount you will earn in one year, you can not use the **APR** as a discount rate.

To see this, note that **APR** with k compounding periods is a way of quoting the actual interest rate each compounding period:

$$\text{Interest Rate per Compounding Period} = \frac{\text{APR}}{k\,\text{periods/year}} \tag{11}$$

Therefore, once you have the compounded interest rate earned per period from the above formula, Eq. (11), you can compute the effective annual rate **EAR** by compounding using using Eq. (10). Therefore, the effective annual rate EAR corresponding to an **APR** with k compounding per periods per year is given by the following:

$$1 + \text{EAR} = \left(1 + \frac{\text{APR}}{k}\right)^k \tag{12}$$

Example–The following Table shows the effective annual rates **EAR**, that correspond to an ARR of 6% with different compounding intervals:

Compounding Interval	**Effective Annual Rate**
Annual	$\left(1 + \frac{.06}{1}\right)^1 - 1 = 6\%$
Semiannual	$\left(1 + \frac{.06}{2}\right)^2 - 1 = 6.09\%$
Monthly	$\left(1 + \frac{.06}{12}\right)^{12} - 1 = 6.1678\%$
Daily	$\left(1 + \frac{.06}{365}\right)^{365} - 1 = 6.1831\%$

6.3 Computing Effective and Nominal Rates in Excel

To calculate the effective annual rate, we use an Excel function called **EFFECT**. The parameters are

- **Nominal rate:** This is the nominal rate of interest per annum.
- **Npery**: This is the frequency of compounding per annum.

Example–Suppose the nominal rate is 5% per year. Determine the equivalent annual rate if interest is paid quarterly.

Solution:

$$\text{EFFECT}(0.05, 4) = 5.0945\%$$