Corporate Finance: IOE 452/MFG 455
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Contents

1 Time Value of Money

- One of the most important tools in corporate finance and investing is the time value of money.
- In general, evaluating financial transactions requires valuing uncertain future cash flows; that is, determining what uncertain cash flows are worth at different points in time.
- We are often concerned about what a future cash flow or a set of future cash flows are worth today.
- One complication is the time value of money: a dollar today is not worth a dollar tomorrow or the next year.
- Another complication is that any amount of money promised in the future is uncertain, some are riskier than others.
- Moving money through time—that is, finding the equivalent value to money at different points in time–involves translating values from one period to another.
- To do a meaningful analysis, it is only possible to compare/combine cash flows at the same point in time.
- Translating money from one period involves interest which is how the time value of money and risk enter into the process.
- Borrowers and lenders meet in *fixed income* markets to trade funds across time.
- First, we cover the case of certain or deterministic cash flows. Later in the course, we will incorporate uncertainty.

1.1 Future Value: Two Time Periods, Single Amount

- Suppose there are only *two* periods to consider: $[t_0, t_1]$.
- Let t_0 designate now and t_1 be some future or next period.
- Suppose we can borrow and lend at a fixed interest rate of r.
- Question: What is the future equivalent of having \$1 today?

- Solution: One dollar today can be transformed into $(1 + r)$ dollars next period; simply lend it to a bank at an interest rate of r. At the end of the period, collect your original loan of \$1 and the interest accrued on it for the amount r.
- Hence, $(1 + r)$ dollars next period is equivalent to \$1 today since that is how much you would get next period by lending \$1 today.
- $1 \Longleftrightarrow (1 + r)$. Clearly, $(1 + r) > 1$, for $r > 0$.
- Rational agents are willing to pay more than \$1 in the future in exchanged for \$1 today.
- A fundamental fact of financial markets where borrowers pay lenders for the use of their funds, is known as the time value of money.
- What if rates are *negative?*

Remark–As $r \uparrow$, $\Longrightarrow$ **FV** $\uparrow$.

Example–Assume interest rate is 10% per year. Then, one dollar invested today has a **FV** of \$1.10 next year.

Solution: Excel Implementation. Use **FV** function. The parameters are:

- **Rate:** Rate is the periodic interest rate, which in the above example is 10% or .10.
- **Nper:** Nper is the number of periods which is 1 in the above example.
- **Pmt:** Pmt stands for the periodic payment, and is not applicable in this case, because there are no periodic cash flows. Thus, either put a zero, or else an extra comma for this example.

- **Pv**: Pv stands for the present value, or the initial investment, which in this case is \$1. We input it as -1 in order to ensure that the answer is positive.

$$\textbf{FV}\,(\text{Rate, Nper, Pmt, -Pv}) = \textbf{FV}\,(.1, 1, , -1) = \$1.10$$

Example–A firm's investment opportunity costs are $\$100,000$ today. If investment is undertaken, it will generate $\$105,000$ for sure at the end of next year. Determine if this is a good investment. Assume a rate of interest of 10% per year.

Solution:

$$\begin{aligned} \text{Cost in 1 year} &= \$100,000 \times 1.1 = \$110,000 \\ \text{Net benefit in 1 year} &= \$105,000 - \$110,000 = -\$5,000 \end{aligned}$$

Clearly, not a good investment!

1. $\$110,000$ is the opportunity cost of spending $\$100,000$ today.
2. Borrowing $\$100,000$ from the bank, the firm would be owed $\$110,000$ in one year.

• The above conclusion is reached by applying a core principle of economics: You should undertake activities only when the benefits exceed the costs. That may seem obvious, so the trick is in the systematic thinking which ensures that you follow through with this obvious advice.

How would your answer change if interest rate becomes 3% per year?

1.2 Future Value: Many Periods & No Compounding: Simple Interest

- Let P_0 be the initial principal invested at rate r. Let n be the number of periods.
- Withdraw the interest each period and leave the original amount to earn interest!
- At the end of the first interest period ($n = 1$) we receive $r\,P_0$ in interest, so the future value of P_0 after one period is:

$$\mathbf{FV_1} = P_1 = P_0 + r\,P_0 = P_0\,(1 + r)$$

- At the end of the second interest period ($n = 2$) we again receive $r\,P_0$ in interest, so the future value after two periods is

$$\mathbf{FV}_2 = P_2 = P_1 + r\,P_0 = P_0\,(1 + r) + r\,P_0 = P_0\,(1 + 2\,r)$$

- Using analogous reasoning at the end of the third interest period ($n = 3$) we again receive $r\,P_0$ in interest, so the future value after 3 periods is

$$\mathbf{FV}_3 = P_3 = P_2 + r\,P_0 = P_0\,(1 + 2\,r) + r\,P_0 = P_0\,(1 + 3\,r)$$

- Hence, we may state the following important result known as *Simple Interest Theorem:*

$$\boldsymbol{FV}_n = P_n = P_0\,(1 + n\,r)$$

In general, with simple interest

$$P_{k+1} = P_k + r\,P_0 \qquad\qquad k = 0, 1, \cdots\cdots, n-1$$

- Simple interest is usually used for short–term transactions, that is, for investments for a period of one year or less. Consequently, simple interest is the norm for money market calculations.

Remark–$P_n = P_0\,(1 + n\,r)$ is a function of 3 variables:

- The quantity $(P_n - P_0)$ is linear in n and r.
- The quantity $(P_n - P_0)$ is known as the principal appreciation.
- The quantity $\frac{(P_n - P_0)}{P_0}$ is called the rate of return.

Example–You borrow \$100 at 7% simple interest and must repay the loan in two years. How much do you have to pay back?

Solution:

Must find P_2. In this example, $P_0 = \$100$, $n = 2$, and $r = 7\%$. Therefore, you must pay

$$\text{Repayment with simple interest} = \$100 + (\$100 \times 2 \times .07) = \$114$$

1.3 Present Value: Two Time Periods, Single Amount

This section explains the procedure for determining how much money must be invested today (called the present value) in order to realize a specific amount in the future.

- Present value is just the *reverse* of the future value: everything is measured in terms of today's dollars.
- Suppose there are only *two* periods to consider: $[t_0, t_1]$.
- Let t_0 designate now and t_1 be some future or next period.
- Suppose we can borrow and lend at a fixed interest rate of r.

- Question: How much is having $1 next period is worth in todays dollar?
- The answer is $\frac{\$1}{(1+r)}$. Therefore, the present value of a dollar to be delivered next period is $\frac{1}{(1+r)}$.
- This is because $\frac{1}{(1+r)}$ can be transformed into a dollar next period simply by saving it at the interest rate of r.
- To see: $\frac{1}{(1+r)} \times (1+r) = 1$.
- A fundamental fact of financial markets where receiving $1 today is better than receiving $1 in the future is known as time value of money.
- We may define the simple one period **discount factor** d as:

$$d = \frac{1}{1+r}$$

Remark– As r $\uparrow \Longrightarrow$ **PV** $\downarrow$.

Example–If the rate of interest is 10% per year, getting $1.10 next year has a **PV** of $1 today.

Solution: Excel Implementation

- The required function in Excel is **PV**. The parameters are:
- **Rate:** Rate is the periodic interest rate.
- **Nper:** Nper is the number of periods.
- **Pmt:** Pmt stands for the periodic payment.
- **Fv:** stands for the future value.
- **Type:** This is a binary variable which is either 0 or 1. It is not required at this stage, and we can just leave it blank.

Example–A firm's investment of $\$100,000$ today will generate $\$105,000$ at the end of one year. Determine if this is a good investment. Assume a rate of interest of 10% per year.

Solution:

$$\begin{aligned}\text{Revenue equivalent now} &= \$105,000 \times \frac{1}{1.10} = \$95,454.55 \\ \text{Net benefit now} &= \$95,454.55 - \$100,000 = -\$4,545.55\end{aligned}$$

Not a good investment!

How would your answer change if interest rate changes to 4%?

• The interest rate that is used in present value calculations is called the **discount rate** or **opportunity cost of capital**.

2 Compound Interest: FV & PV

2.1 Future Value: Compound Interest

- Let P_0 be the initial principal invested at rate r. Let n be the number of periods.
- At the end of the first interest period ($n = 1$) we receive $r\,P_0$ in interest, so the future value of P_0 after one period is

$$\mathbf{FV}_1 = P_1 = P_0 + r\,P_0 = P_0\,(1+r)$$

- Suppose we leave the interest and the original amount to earn interest! (No withdrawal of interest)
- At the end of the second interest period ($n = 2$) we receive $r\,P_1$ in interest, so the future value after two periods is

$$\mathbf{FV}_2 = P_2 = P_1 + r\,P_1 = P_1\,(1+r) = P_0\,(1+r)^2$$

- In general,

$$P_{k+1} = P_k + r\,P_k \qquad\qquad k = 0, 1, \cdots\cdots, n-1$$

Remark–$P_n = P_0\,(1+r)^n$ is a function of 3 variables: P_0, n and r.

- The quantity $(P_n - P_0)$ is non–linear in n and r.
- The quantity $(P_n - P_0)$ is known as the principal appreciation.
- The quantity $\frac{(P_n - P_0)}{P_0}$ is called the rate of return.
- The basic valuation equation given by the following expression is known as : *Future Value: Compound Interest Theorem*

$$\mathbf{FV}_n = (1+r)^n$$

- In other words, when you multiply the value today–the present value–by the compound factor, you get the future value.
- Spreadsheets time value of money programs are set up to perform calculations involving compound interest!

Example–Determine the amount that \$1,000 will grow to at the end of 8 periods if interest is earned at an annual rate of 7%.

Solution–$\mathbf{FV}_8 = \$1,000 \times (1+.07)^8 = \$1,718.19$.

2.2 Frequent Compounding: N Periods

So far, we have computed the future value for whole periods (years). The future value formula, however, is the same if an investment is made for part of a period (year).

- Many investments may pay interest more than one time per year, e.g., interest may be paid semiannually, quarterly, monthly, etc.
  1. Must convert the *annual* interest rate into a periodic interest rate.
  2. Must convert the number of years until the cash flow is to be received into the appropriate number of periods that matches the compounding frequency.
- Let r be the yearly or annual interest rate. Assume this rate is paid m times per year.
- Suppose there are N years. Therefore, there are $n = N \times m$ payments.
- Let $i = \frac{r}{m}$ be the per period interest rate. That is, the annual interest rate divided by m frequency of payments.

Example–Suppose you invest $\$1,000,000$ at a yearly of 7.2% for 8 years. Interest on this investment is paid semiannually. Determine the future value of this investment.

Solution–In this problem, $r = 7.2\%, N = 8$ and $m = 2$. Therefore, $n = 2 \times 8 = 16$ and $i = \frac{r}{2} = \frac{0.072}{2} = 0.036$.

Hence $\mathbf{FV} = \$1,000,000 \times (1 + .036)^{16} = \$1,760,986.74$.

The higher future value when interest is paid semiannually reflects the more frequent opportunity for reinvesting the interest paid.

Moving to quarterly compounding, all else equal, should result in an even *larger* future value.

2.3 Present Value: Compound Interest

- Can rearrange the basic valuation equation to solve for the present value to get

$$\mathbf{PV} = \mathbf{FV}_n \times \frac{1}{(1+r)^n} = \mathbf{FV}\,(1+r)^{-n} \tag{1}$$

- The quantity $\frac{1}{(1+r)^n}$ is known as the discount factor.
- The present value of a single cash flow paid in period n when interest is paid m times per year is given by:

$$\mathbf{PV} = \frac{FV_n}{\left(1+\frac{r}{m}\right)^{(n\times m)}}$$

- This present value is smaller than if interest were compounded annually ($m = 1$).
- The lower present value when interest is paid more frequently means that for a given annual interest rate we can invest a smaller amount today and still have a given future value for a given time horizon.

Example–An investor expects to receive \$1,000, 15 years from today and the relevant interest rate is 4% compounded semiannually. Determine the present value of its cash flow.

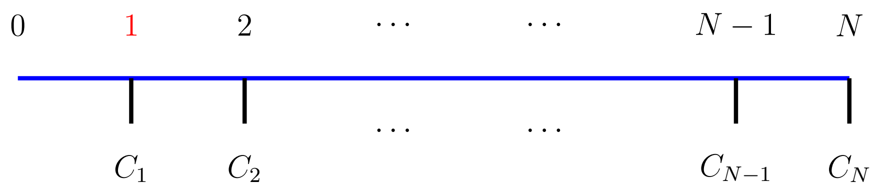
Solution:

$$\begin{aligned}\text{PV} &= \frac{\text{FV}_n}{(1+i)^n} \\ &= \frac{1000}{\left(1+\frac{.04}{2}\right)^{15\times 2}} = \$552.07\end{aligned}$$

- Moving to quarterly compounding, all else equal, should result in an even *smaller* present value.

3 Valuing a Stream of Cash flows

- Suppose you have a business opportunity to invest *now* and receive a *series* of cash flows at regular intervals in the future.
- Suppose cash flows are at each period t are C_t. Take $t = 1, 2, \cdots N$.
- **Note the first cash flow in this formulation starts at the next period, that is at 1!**
- Take interest rate to be r.
- The time line diagram is as follows:

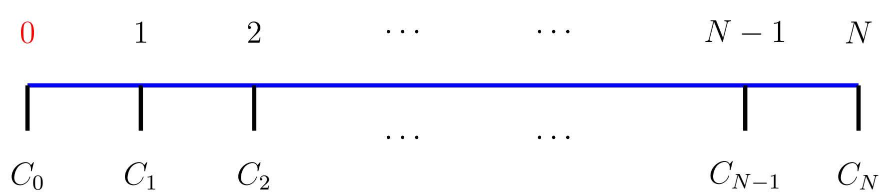


- The same principle of time value for compounding and discounting apply to multiple cash flows.

  The above time line provides us with a general formula for the present value of a cash flow stream that start in period 1 and last for N periods:

$$\text{PV of Stream} = \text{PV} = \sum_{n=1}^{N} \frac{C_n}{(1+r)^n} \tag{2}$$

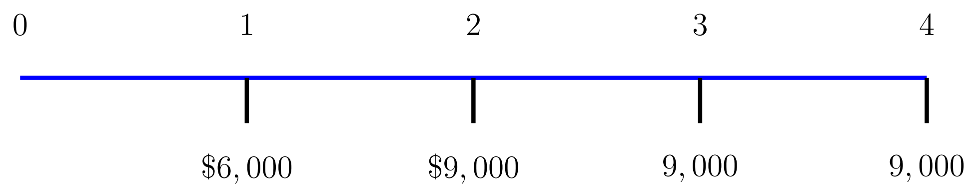
- Had the first cash flow occurred at period 0, the present value of the stream of cash flows will have the following timeline and is given by



$$\text{PV of Stream} = \text{PV} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n} \tag{3}$$

Note that when speaking of cash flows, the numbers $1, 2, \cdots, n$ always refer to the end of the period. That is, C_1 refers to the end of period 1 cash flow. Similarly, C_k refers to the end of period k cash flow for $1 \leq k \leq n$.

Example–Suppose you agree to pay the following cash flows to **RK** at designated time periods:



- Assume $r = 8\%$. Determine the present value of the above receivables for **RK**.

Solution:

$$\begin{aligned} \text{PV} &= \frac{6000}{1.08} + \frac{9000}{1.08^2} + \frac{9000}{1.08^3} + \frac{9000}{1.08^4} \\ &= \$27,031.36 \end{aligned}$$

An interpretation of the above $\$27,031.36$ is that if you agreed to pay the above four cash flows when the appropriate rate of interest was $r = 8\%$, then someone is willing to lend you $\$27,031.36$.

Example–Excel Implementation

Use the function **NPV**.

Example–Another variation or a way of interpreting the above example is as follows: Suppose **RK** wants to lend you money. **RK** assumes you will be paying her back according to the the above payment schedule at the designated time periods. Given $r = 8\%$, how much should **RK** be willing to lend you?

Solution:

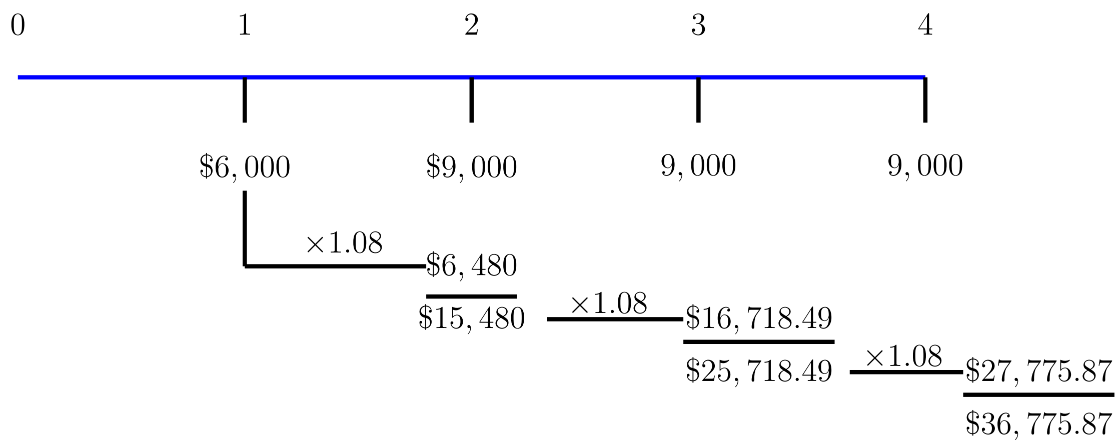
Knowing the schedule and the amount of payments, **RK** knows your promises of payments. Therefore, **RK** will be willing to extend you an amount that is equivalent to the present value of the payments. This amount is $\$27,031.36$.

- Note $\$27,031.36$ is less than the total you promise to pay ($\$6000 + \$9000 + \$9000 + \$9000 = \$33000$) due to the time value of money.

Example–Here is another way of interpreting the set up of our examples: Suppose **RK** deposits $\$27,031.36$ in a bank account which pays 8% per year. After 4 years **RK** would have:

$$\text{FV} = \$27,031.36 \times \left(1.08^4\right) = \$36,775.87$$

Let us assume **RK** hands you a loan of $\$36,775.87$. In return you agree to making 4 payments at the above intervals using the specified amounts. In this case, **RK** will be depositing your payments as they arrive into her bank account. The future value of the yearly deposits must be $\$36,775.87$.



3.1 Future Value of a Stream of Cash flows

For multiple periods and multiple cash flows, the accumulated future value at time N of a series of periodic cash flows (*at the beginning of a period*) for amount $C(t)$ is given by:

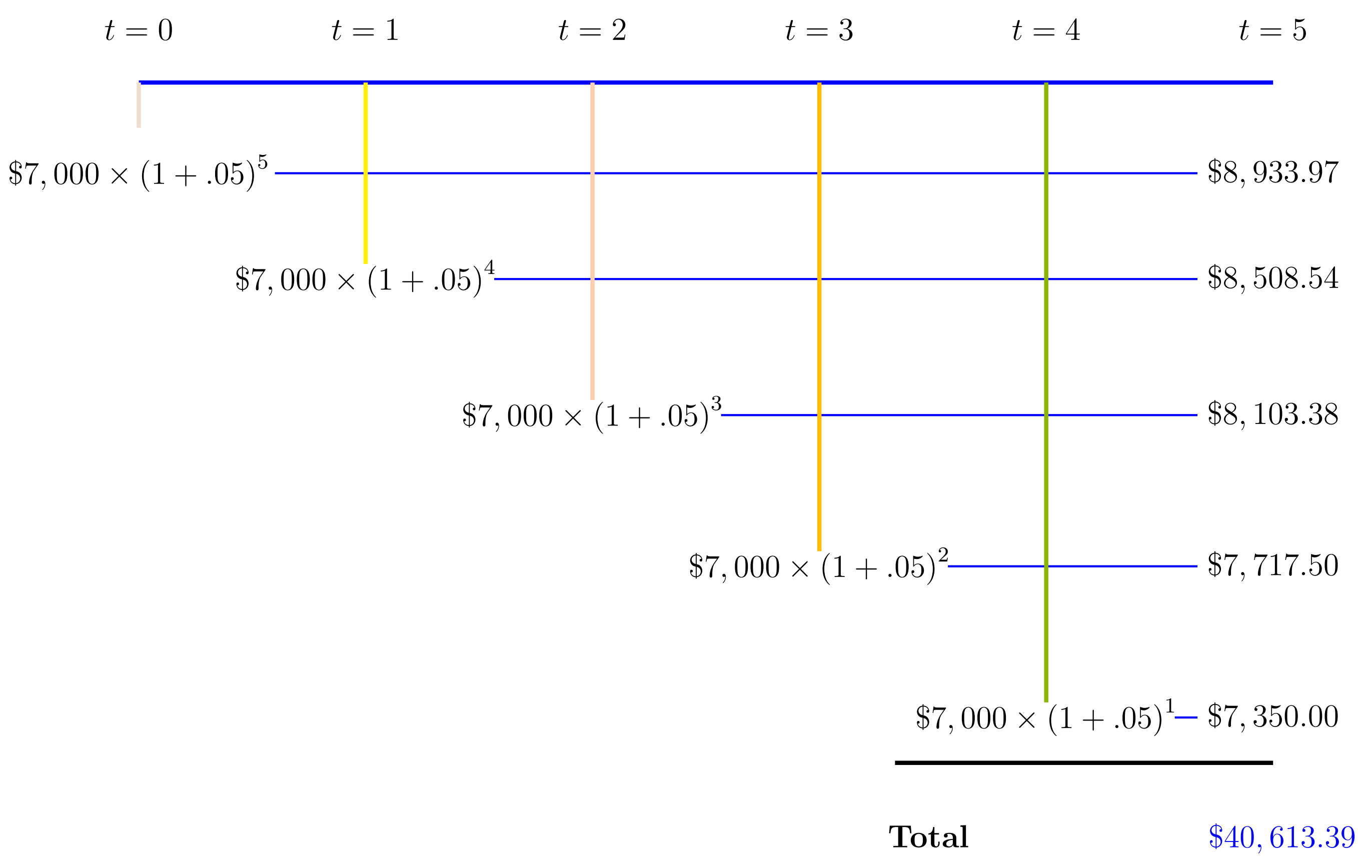
$$\text{PV of Stream} = \text{PV} = C_0\,(1+r)^N + C_1\,(1+r)^{N-1} + \cdots + C_{n-1}\,(1+r) = \sum_{n=0}^{n-1} C(n)\,(1+r)^{N-n} \quad (4)$$

Example–Suppose you want to have \$40,000 five years from now (say to purchase something). You can save for this event by deposing 5 equal contributions of \$7,000 per year at the *start of each year* in a bank that pays you 5% interest per year. Determine if this is a viable strategy.

Solution

Note that it is important to be clear about whether each cash flow occurs at the beginning or end of the period.

In this example we put aside \$7,000 now ($t = 0$) and at the *beginning* of the first year. Therefore, we invest the first \$7,000 for five years, the second \$7,000 for four years, and so on. The last \$7,000 is invested for only one year.



3.2 Calculating NPV

- Now we can compare the costs and benefits of a project to evaluate investment decisions.
- Must compare all cash flows at a common point in time. A *convenient* and standard choice is to use *present values*.
- Managers need an investment decision rule to evaluate projects.
- Which projects add value to the company and which do not?
- **NPV**is a way of determining whether or not it is worth going ahead with a project.
- Define the **net present value** or **NPV** of an investment as follows:

$$\mathbf{NPV} = \mathbf{PV(Benefits)} - \mathbf{PV(Costs)}$$

- The benefits are cash inflows and the costs are cash outflows.
- The basic idea is that cash flows are discounted at their opportunity cost of capital (the best available return on an investment of similar risk.)
- As a rule, an investment must be undertaken if **NPV** > 0. [1]

As an investment criterion, **NPV** is based on a simple premise: it deducts the initial outlay from all the future gains expressed in today's terms.

• A positive result (i.e. **NPV** > 0) means that the future gains is more than the amount put in today, thereby representing the value created by the investment.

• A negative result (i.e. **NPV** < 0), by contrast, means that the investment is value–destroying given that the actualized sum of all the gains made in the future does not cover the amount to be invested today.

• Example

Suppose you can invest in a project that costs $\$1,000$ today. As part of the contract, you can receive $\$1,100$ for sure after 4 years. As an alternative investment, you may deposit your money safely at a bank that pays you 4% annual effective rate. Which of the two alternative would you choose?

Solution

Let us compare the two investments: The **net present value (NPV)** for an investment where you pay $\$1,000$ now and receive $\$1,100$ in 4 years is:

[1] There are three main criteria that managers can use when assessing an investment —net present value, payback and internal rate of return. Payback and internal rate of return are discussed later.

$$\mathbf{NPV} = -\$1,000 + \frac{\$1,100}{(1+.04)^4} = \$25.77 > 0$$

This means that you earn \$25.77 more, measured in today's money, by undertaking the investment project than you would depositing the same amount of money in a bank account earning 4% interest annually.

3.2.1 Payback Method

- **NPV** is not the only tool for making investment decisions. One other method is *payback* method.
- It refers to how long it will take to recover the initial investment.
- The payback method is simple and intuitive to use. This investment criterion looks at how quickly you can get your initial investment money back.
- A draw back of payback method is that unlike **NPV** it ignores the time value of money.
- Another drawback of payback method is that by just focusing on how long it will take to recover the initial outlay is effectively saying that all the values beyond the payback period are irrelevant to the business decision. In other words, this method is biased towards short–term gains.
- It is a very subjective method, given that the decision to accept or reject a project depends heavily on some arbitrary cut–off points set by the decision makers.
- It is because of these shortcomings that companies tend to use this method only for small and less important investments.

3.2.2 Internal Rate of Return

- **NPV** is not the only tool for making investment decisions. One other method is *Internal Rate of Return* (**IRR**) method.
- The **IRR** method after the **NPV** method is the most widely use approach.
- According to the **IRR** method one should take an investment opportunity if the **IRR** exceeds the opportunity cost of capital.
- The **IRR** implicitly assumes that interim cash flows can be reinvested at the same return until the end period.
- It is easier to explain this method by first demonstrating how **IRR** is calculated.

- **Example**

Suppose you can invest in a project that costs \$100 today (2025). For the next 7 years, the project will produce \$25 per year. Determine the **IRR** for this project.

Solution

Note, since the first cash flow comes after one year, its present value is $\frac{\$25}{(1+r)^1}$. Similarly, the second cash flow comes after two years, its present value is $\frac{\$25}{(1+r)^2}$. The last cash flow comes after 7 years, its present value is $\frac{\$25}{(1+r)^7}$.

This calculation is done with the same information as in an **NPV** calculation, but without knowing the the discount rate, r, which is left as the variable to be solved for. In solving for r, one does it by setting the **NPV** equal to 0. That is, solve for r such that the following equation is satisfied:

$$-100 + \frac{\$25}{(1+r)^1} + \frac{\$25}{(1+r)^2} + \frac{\$25}{(1+r)^3} + \frac{\$25}{(1+r)^4} + \frac{\$25}{(1+r)^5} + \frac{\$25}{(1+r)^6} + \frac{\$25}{(1+r)^7} = 0$$

We can solve the above with trial and error! However, we can use the **IRR** formula in Excel. In this case it is found that $r = 16.33\%$. That is, the **IRR** of the above project is 16.33%.

3.3 Simple Bank Account

Suppose you want to know the value of your account balance at each time in the future. For simplicity, assume you deposit some amount at the beginning and will never draw any money from your account. Further, assume that the bank pays a continuously compounding interest of r on your deposit.

• **Formulation**

Mathematically we can formulate the above problem as an **Initial Value Problem (I.V.P)**. Let the time interval of interest be designated by $t \in [t_0, T]$. Let the account balance at any future time $t \in [0, T]$, be denoted by $y(t)$. Suppose you initially (at time t_0) deposit $y(t_0) = y(0)$ into your account. For the bank account problem, **I.V.P** becomes

$$\begin{cases} y' = \frac{dy(t)}{dt} = r \times y(t) \\ \\ y(t_0) = y(0) \end{cases} \tag{5}$$

Remark–The equation (5) is a linear, first order, time–autonomous, constant coefficient differential equation.

The equation (5) is fairly easy to solve. We can solve the equation using the separation of variables method. Rewrite (since $y \neq 0$)

$$\frac{dy}{y} = r \times dt$$

In the above, the left–hand side is a function of $y(t)$ and the right–hand side is a function of t only. Therefore, we can integrate both sides of the above to get

$$\begin{aligned} \int \frac{dy}{y} &= \int r \times dt \\ \ln\left(\frac{y(t)}{y(0)}\right) &= r\,t \end{aligned}$$

Therefore, the solution becomes

$$y(t) = e^{r\,t}\, y(0) \tag{6}$$

According to (6), initial deposit of $y(0)$ has a future value of $y(t)$ under continuous compounding.

Also note that according to (6), the present value of having $y(t)$ in the bank is given by

$$y(0) = y(t)\, e^{-r\,t} \tag{7}$$

Remark–It is worth examining (6) and (7) in light of what we know from discrete–time models of discounting considered earlier. Suppose the time unit is one, that is, set $t = 1$. Then, we can approximate (6) by

$$y(1) = \mathbf{FV} = e^r\, y(0) \approx y(0)\,(1 + r)$$

where we used the Taylor series approximation for the exponential function by keeping the first order term and neglecting all higher order terms:

$$e^x \approx 1 + x + \frac{x^2}{2} + \cdots$$

• As you notice, the above approximation to the continuous time compounding reproduces the discrete time result we had earlier. That is, the future value after one period is the same as $(1 + r)$ times the initial deposit.

Similarly, we can approximate (7) by noting that $e^{-r} = \frac{1}{e^r}$. Hence,

$$y(0) = \mathbf{PV} \approx \frac{y(1)}{(1 + r)}$$

• As you notice, the above approximation to the continuous time compounding reproduces the discrete time result we had earlier. That is, the present value of having $y(1)$ in your account in period one is $\frac{y(1)}{(1+r)}$.