

Milestone 4

AST5220

Julie Thingwall

1 Introduction

In this project we wish to follow in the footsteps of Petter Callin[1] who numerically reproduces the power spectrum obtained by the CMB data. This will be done in several steps, where each step simulates the different physical processes that make up the power spectrum.

The previous milestones have consisted of calculating the background cosmology, the recombination history of the universe and the evolution of the different perturbations that make up the power spectrum. This final milestone consists of actually tusing all these previously calculated qunatities to create the CMB power spectrum.

As with previous milestones, all numerical solutions will be obtained by utilising the C++ code base provided by our lecturer, Hans Winther.

2 Theoretical background

2.1 Spherical harmonics

To understand what the power spectrum represents, we have to start with understanding spherical harmonics. The temperature field that makes up the CMB can be represented using spherical harmonics, which reads as

$$T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}). \quad (1)$$

Here, $T(\hat{n})$ represents the temperature in direction \hat{n} , $a_{\ell m}$ are the spherical harmonic coefficients, and $Y_{\ell m}$ are the spherical harmonic functions themselves. Spherical harmonics are wave functions on the sphere, they are completely analogous to fourier transformations in flat space.

The ℓ s refer to scale, with smaller ℓ s being bigger scales. For each ℓ we have $m = 2\ell + 1$. Now, the CMB power spectrum shows us the expectation value for each $a_{\ell m}$, or

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \langle a_{\ell m} a_{\ell m}^* \rangle. \quad (2)$$

where we, for each ℓ , take the average over all m . This is due to the universe being isotropic.

2.2 The source function and line of sight integration

In Milestone 3, we calculated the photon temperature fluctuations Θ_ℓ for $\ell \in [0, 6]$. But we are really interested in the interval $\ell \in [0, 1200]$, at least! Luckily, we dont have to do the calculations from the last milestone 1200 times. Thanks to Zaldarriaga and Seljak, we can instead do something called line of sight-integration (los-integration)! This integration takes on the form

$$\Theta_\ell(k, x=0) = \int_{-\infty}^0 \tilde{S}(k, x) j_\ell[k(\eta_0 - \eta)] dx, \quad (3)$$

where $\tilde{S}(k, x)$ is the source function, and $j_\ell[k(\eta_0 - \eta)]$ are Bessel functions. The source function looks like

$$\tilde{S}(k, x) = \tilde{g} \left[\Theta_0 + \Psi + \frac{1}{4}\Pi \right] + e^{-\tau} [\Psi' - \Phi'] - \frac{1}{ck} \frac{d}{dx} (\mathcal{H}\tilde{g}v_b) + \frac{3}{4c^2k^2} \frac{d}{dx} \left[\mathcal{H} \frac{d}{dx} (\mathcal{H}\tilde{g}\Pi) \right]. \quad (4)$$

In essence, the source function explains the different physics that affect a photon on its journey from last scattering until we measure it as a CMB photon. The first term, which we can see is relevant at last scattering due to it being weighted by the visibility function, explains how a photon is affected at last scattering, when it climbs out of the gravitational wells created by the baryons before free-streaming towards us. This is called the Sachs-Wolfe term. The next term is the integrated Sachs-Wolfe term, which explains how a photon is affected when traveling through changing gravitational potentials. The third term is a Doppler term, and the fourth term is the term-who-must-not-be-named, apparently!

With line-integration, we don't have to solve all the coupled differential equations for each ℓ , we only need to solve eq. (3) instead, which greatly reduces computational time!

2.3 The temperature and matter power spectrums

For this milestone we are interested in both the temperature power spectrum and the matter power spectrum.

The temperature power spectrum takes on the form

$$C_\ell = \frac{2}{\pi} \int k^2 P_{\text{primordial}}(k) \Theta_\ell^2(k) dk \quad (5)$$

where $P_{\text{primordial}}(k)$, the primordial power spectrum, looks like

$$\frac{k^3}{2\pi^2} P_{\text{primordial}}(k) = A_s \left(\frac{k}{k_{\text{pivot}}} \right)^{n_s-1}. \quad (6)$$

Here, n_s is the spectral index for scalar perturbation, which takes on the value $n_s \approx 0.96$. k_{pivot} is some scale where the amplitude is A_s . For our universe, we have $A_s \approx 2 \times 10^{-9}$ and $k_{\text{pivot}} \approx 0.05/\text{Mpc}$.

The primordial power spectrum sets up the anisotropies from inflation?? idk

Adding this back eq. (5), we get

$$C_\ell = 4\pi \int_0^\infty A_s \left(\frac{k}{k_{\text{pivot}}} \right)^{n_s-1} \Theta_\ell^2(k) \frac{dk}{k}. \quad (7)$$

This is the integral we wish to solve.

For the matter power spectrum, we simply have

$$P(k, x) = |\Delta_M(k, x)|^2 P_{\text{primordial}}(k) \quad (8)$$

where

$$\Delta_M(k, x) \equiv \frac{c^2 k^2 \Phi(k, x)}{\frac{3}{2} \Omega_{M0} a^{-1} H_0^2}. \quad (9)$$

3 Method

3.1 Code structure and parameters

All main coding was done in the `PowerSpectrum.cpp` file. The differential equations were solved using the ODESolver found in the GSL library. All visualisation was done in `Milestone4.plot.py`.

All solutions were found in the interval $x \in [-12, 0]$. Ideally, this interval should have been bigger, but due to limitations from milestone 2, the optical depth was only solved for $x > -12$. We solved the equations for 100 k -values logarithmically spaced, in the interval $k \in [0.00005/\text{Mpc}, 0.3/\text{Mpc}]$. We solve for a set of ℓ 's in the range $\ell \in [2, 2000]$.

3.2 Bessel splines

First of all, we needed to spline bessel functions blabla

3.3 LOS-integration to LOS-ODE

$$\frac{d\Theta_\ell(k, x)}{dx} = \tilde{S}(k, x) j_\ell[k(\eta_0 - \eta)], \quad \Theta_\ell(k, -\infty) = 0 \quad (10)$$

3.4 Temperature power spectrum

3.5 Matter power spectrum

4 Results

References

- [1] Petter Callin. How to calculate the cmb spectrum, 2006.