A heavy-tailed error model

(A) Calculate $P(y_i \mid X, \beta, \omega)$

$$P(y_i \mid X, \beta, \omega) \propto \int P(y_i, \lambda_i \mid X, \beta, \omega)\, d\lambda_i$$

$$\propto \int P(y_i \mid \lambda_i, X, \beta, \omega)\, P(\lambda_i, X, \beta, \omega)\, d\lambda_i$$

remove terms not including y_i

$$\propto \int P(y_i \mid \lambda_i, X, \beta, \omega)\, P(\lambda_i)\, d\lambda_i$$

$$\propto \int \cancel{\omega^{1/2}} \lambda_i^{1/2} \exp\left(-\frac{\omega \lambda_i}{2} (y_i - x_i^T \beta)^2\right) \times$$

$$\lambda_i^{\frac{n}{2}-1} \exp\left(-\lambda_i \frac{n}{2}\right) d\lambda_i$$

Thus is the kernel of a Gamma

$$Ga\left(\frac{n+1}{2}, \frac{\omega (y_i - x_i^T \beta)^2 + n}{2}\right)$$

$$\text{So} \propto \left(\frac{\omega (y_i - x_i^T \beta)^2 + n}{2}\right)^{-\left(\frac{n+1}{2}\right)}$$

$$\propto \left(\frac{\omega (y_i - x_i^T \beta)^2}{n} + 1\right)^{\left(-\frac{n+1}{2}\right)}$$

$$\propto \left(\frac{1}{n} \frac{(y_i - x_i^T \beta)^2}{\frac{\cancel{K}}{\omega K}} + 1\right)^{\left(-\frac{n+1}{2}\right)}$$

ω).

This is a t distribution with mean = $x_i^T \beta$, $s^2 = \frac{1}{\omega}$, dof = n

B) $p(\lambda_i \mid y, \beta, \omega)$

$\propto P(y \mid \lambda_i, \beta, \omega)\, P(\lambda_i, \beta, \omega)$

$\propto P(y \mid \lambda_i, \beta, \omega)\, P(\lambda_i)$

$\propto (\omega \lambda_i)^{1/2} \exp\left(-\frac{1}{2} \omega \lambda_i \, (y - x_i^T \beta)^2\right)$

$\lambda_i^{n/2 - 1} \exp\left(-\frac{\lambda_i n}{2}\right)$

This is a $Ga\left(\frac{n+1}{2}, -\frac{1}{2}\left(n + \omega (y - x_i \beta)^2\right)\right)$