

d) Muestre que vectorialmente, el descenso del gradiente queda definido por:

$$\vec{\theta}_{j+1} = \vec{\theta}_j - \lambda \cdot \left(-2 \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_j)) \nabla_{\vec{\theta}} \eta(x_i, \vec{\theta}_j) \right)$$

Teniendo que la fórmula del gradiente es

$$x_1 = x_0 - \alpha \nabla_x f(x_0) \rightarrow x_{n+1} = x_n - \alpha \nabla_x f(x_n)$$

pono la función de punto $\chi^2(\vec{\theta})$ es decir $\chi^2(\vec{\theta})$ y aplicamos para hallar su gradiente

$$\nabla_{\vec{\theta}} \chi^2(\vec{\theta}) = \left[-2 \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_0)) \frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_0}, -2 \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_1)) \frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_1}, -2 \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_2)) \frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_2} \right]$$

Teniendo

$$\nabla_{\vec{\theta}} \chi^2(\vec{\theta}) = -2 \cdot \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_j)) \nabla_{\vec{\theta}} \eta(x_i, \vec{\theta}_j)$$

Donde

$$\nabla_{\vec{\theta}} \eta(x_i, \vec{\theta}_j) = \left[\frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_0}, \frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_1}, \frac{\partial \eta(x_i, \vec{\theta})}{\partial \theta_2} \right]$$

Quedando demostrado que el descenso del gradiente da

$$\vec{\theta}_{j+1} = \vec{\theta}_j - \lambda \cdot \left(-2 \sum_{i=1}^N (y_i - \eta(x_i, \vec{\theta}_j)) \nabla_{\vec{\theta}} \eta(x_i, \vec{\theta}_j) \right)$$