

Julio Brandasse de Abreu Lima CTII 350 –
Determinantes - Matriz de Ordem 1, 2 e 3 - Tarefa Básica

Matemática

Exercício Básico: Determinantes.

a) $\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}$ Det = $10 - 3 = 7 //$

b) $\begin{vmatrix} -2 & -4 \\ 3 & 6 \end{vmatrix}$ Det = $-12 - (-12) = -12 + 12 = 0 //$
R: 0 //

c) $\begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 4 & -2 & 1 & 4 \end{vmatrix}$ $\begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 4 & -2 & 1 & 4 \end{vmatrix}$
 $\downarrow \quad \downarrow \quad \downarrow$
 $-6 + 1 + 8 = 3.$ $1 - 12 + 4 = -7$

Det = $3 - (-7) \times 3 + 7 = 10 //$

d) $\begin{vmatrix} 3 & 2 & -1 & 3 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 1 & 1 & 4 & 1 & 1 \end{vmatrix}$ $\begin{vmatrix} 3 & 2 & -1 & 3 & 2 \\ 2 & 3 & 1 & 2 & 3 \\ 1 & 1 & 4 & 1 & 1 \end{vmatrix}$
 $\downarrow \quad \downarrow \quad \downarrow$
 $36 + 2 = 38,$ $-3 + 3 + 16 = 16,$
Det = $38 - 16 = 22 //$

2) 4×3

$$ay = \begin{cases} -3 \rightarrow i=j \\ 0 \rightarrow i \neq j \end{cases}$$

det = ?

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$i=j$ on row $a_{11}, a_{22}, a_{33} = -$
 $i \neq j$ on row $i \neq j = 0$.

$$\begin{bmatrix} -3 & 0 & 0 & -3 & 0 \\ 0 & -3 & 0 & 0 & -3 \\ 0 & 0 & -3 & 0 & 0 \end{bmatrix}$$

$0 + 0 + 0 = 0 //$

$-27 + 0 + 0 = -27 //$

Det = $-27 - 0 \rightarrow -27 //$

alternativa (A) ✓

$$3) \begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix} = -3$$

$$\begin{vmatrix} x & 1 & x \\ 3 & x & 4 \\ 1 & 3 & 3 \end{vmatrix} \begin{matrix} \downarrow 3x^2 + 4x + 9x \\ \downarrow x^2 + 12x + 9 \end{matrix}$$

$$\text{Det} = 3x^2 + 4x + 9x - (x^2 + 12x + 9)$$

$$\text{Det} = 3x^2 + 4x + 9x - x^2 - 12x - 9$$

$$\text{Det} = 2x^2 - 3x - 9$$

$$-3 = 2x^2 - 3x - 9$$

$$0 = 2x^2 - 3x - 2$$

$$\Delta = (-3)^2 - 4 \cdot (2) \cdot (-2)$$

$$\Delta = 9 + 16 = 25$$

$$x' = \frac{+3 + \sqrt{25}}{2 \cdot 2} = \frac{3+5}{4} = 2$$

$$x'' = \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}$$

$$E) \left\{ -\frac{1}{2}; 2 \right\} \checkmark$$

4)
$$\begin{array}{c|cc|cc} \text{I} & \text{I} & \text{I/II} & \text{II} & \text{II} \\ \hline (x-1) & -1 & 0 & (x-1) & -1 \\ 0 & x+1 & -1 & 0 & x+1 \\ 2 & 1 & x+1 & 2 & -1 \end{array} \quad \text{Det}=2$$

$$\text{I} \quad (x+1)^2 \cdot (x-1) + 2 + 0 = (x+1)(x+1)(x-1) + 2$$

$$\text{II} \quad x-1$$

$$\text{Det} = (x-1)(x+1)(x+1) + 2 = (x-1)$$

$$2 = (x^2 - 1)(x+1) + 2 - x + 1$$

$$2 = x^3 + x^2 - x - 1 + 2 - x + 1$$

$$0 = x^3 + x^2 - 2x - 1$$

$$0 = x^3 + x^2 - 2x - 1 \quad (a=1, b=1, c=-2)$$

$$x' + x'' + x''' = -\frac{b}{a} = -\frac{1}{1} = -1$$

C) Alternative C.

5) $A_{3 \times 2} \rightarrow$ Det $A \cdot B = ?$

$$a_{ij} = 2i - 3j$$

$$B_{2 \times 3} \rightarrow b_{jk} = k - j$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix}$$

$$a_{11} = 2 \cdot 1 - 3 \cdot 1 = -1$$

$$a_{12} = 2 \cdot 1 - 3 \cdot 2 = -4$$

$$a_{21} = 2 \cdot 2 - 3 \cdot 1 = 1$$

$$a_{22} = 2 \cdot 2 - 3 \cdot 2 = -2$$

$$a_{31} = 2 \cdot 3 - 3 \cdot 1 = 3$$

$$a_{32} = 2 \cdot 3 - 3 \cdot 2 = 0$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$b_{11} = 1 - 1 = 0$$

$$b_{12} = 2 - 1 = 1$$

$$b_{13} = 3 - 1 = 2$$

$$b_{21} = 1 - 2 = -1$$

$$b_{22} = 2 - 2 = 0$$

$$b_{23} = 3 - 2 = 1$$

$$B = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A.B = \begin{bmatrix} (0+4) & (-1-0) & (-2-4) \\ (0+2) & (1-0) & (2-2) \\ (0-0) & (3+0) & (6+0) \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$$

$0 + 0 - 12 = -12_{11}$
 $24 - 0 - 36 = -12_{11}$

$$\text{Det } A.B = -12 - (-12) \rightarrow -12 + 12 = 0_{11}$$

Alternativa C) ✓

$$6) A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$A.B = ?$

$$A.B = \begin{bmatrix} (2+0+0) & (-2+0-2) \\ (-1-1+0) & (1+1+0) \end{bmatrix}$$

$$A.B = \begin{bmatrix} 2 & -4 \\ -2 & 2 \end{bmatrix} \quad \rightarrow \text{Det } A.B = 4 - 8 = (-4)$$

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D) -4 ✓