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Tarefa Básica - Sistemas Lineares Regra de Cramer

$$1) a) \begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$$

$$\rightarrow 6 - 1 = 5$$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$$

$$x = \frac{D_x}{D} = \frac{3}{5}$$

$$y = \frac{D_y}{D} = \frac{-4}{5}$$

$$R: V = \left\{ \left( \frac{3}{5}, -\frac{4}{5} \right) \right\}$$

$$b) \begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = -7 \end{cases}$$

$$D = \begin{cases} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{cases} \begin{cases} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{cases} \quad -10 - 13 = -23$$

$$I. 0 + 9 + 4 = 13$$

$$II. 0 + 12 + 2 = 14$$

$$D_x = \begin{cases} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & 2 \end{cases} \begin{cases} 1 & -1 \\ -1 & 0 \\ 7 & 1 \end{cases} \quad (-22) - 1 = -23$$

$$I. 0 + 3 - 2 = 1$$

$$II. 0 - 21 - 1 = -22$$

$$D_y = \begin{cases} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & 2 \end{cases} \begin{cases} 3 & 1 \\ 2 & -1 \\ 4 & 7 \end{cases} \quad -032 - 55 = -23$$

$$I. 6 + 12 + 14 = 32$$

$$II. -4 + 63 - 4 = 55$$

$$Dz = \begin{Bmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{Bmatrix} \begin{Bmatrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{Bmatrix}$$

$$\begin{array}{l} \text{I. } 0 + 4 + 2 \\ \text{II. } 0 - 3 - 14 \end{array} \left. \vphantom{\begin{array}{l} \text{I. } 0 + 4 + 2 \\ \text{II. } 0 - 3 - 14 \end{array}} \right\} 6 - (-17) = 23$$

$$X = \frac{-23}{-23} = 1$$

$$Z = \frac{23}{-23} = -1$$

$$y = \frac{-23}{-23} = 1$$

$$y = \frac{-23}{-23} = 1$$

$$2. \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{vmatrix}$$

$$\text{I. } -12 + 48 + 12 = 61 - 31 = 30$$

$$\text{II. } -5 - 12 + 48 = 31$$

$$Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{vmatrix}$$

$$78 - 48 = 30$$

$$-1 -$$

$$y = \frac{30}{30} \rightarrow y = 1$$

Alternative A

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$$3. \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$$

$$D = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{aligned} \text{I. } 2 - 33 + 6 &= (-36) - (-37) = 1 \\ \text{II. } -1 - 44 + 9 &= -35 \end{aligned}$$

$$D_y = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & -2 \\ 2 & 1 \end{bmatrix} = (-17) - (-13) = -4$$

$$D_x = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -2 & 11 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & -2 \\ 2 & 1 \end{bmatrix} = (-17) - (-18) = 1$$

$$\begin{aligned} \text{I. } -4 - 11 - 3 &= -18 \\ \text{II. } 2 - 22 + 3 &= -17 \end{aligned}$$

$$D_z = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix} = 2 - 2 = 0$$

$$\begin{aligned} \text{I. } 1, -8, 9 \\ \text{II. } 2, -6, 6 \end{aligned}$$



$$\left. \begin{array}{l} x = -\frac{1}{1} = -1 \\ y = \frac{1}{1} = 1 \\ z = \frac{0}{1} = 0 \end{array} \right\} \begin{array}{l} a+b+c = \\ 1+(-1)+0 = 0 \\ \hline \text{Alternativa C} \end{array}$$

$$4) \begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 9 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} = 1 - (-15) = 16$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 9 & -1 & -2 \end{vmatrix} = (-30) - (-146) = 116$$

$$I. = 174 + 32 + 12$$

$$II. = -72 - 58 - 16$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} = 26 - (-64) = 90$$

$$I. = -12 + 16 - 58$$

$$II. = -8 + 58 - 24$$

$$Dz = \begin{bmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{aligned} \text{I. } 24 + 8 - 29 &= 3 \\ \text{II. } 87 - 4 + 16 &= 99 \end{aligned} \quad \begin{cases} 3 - 99 = -96 \end{cases}$$

$$x = \frac{16}{16} = 1$$

$$z = \frac{-96}{16} = -6$$

$$y = \frac{90}{16} = 5$$

$$x + y + z = 1 + 5 + (-6) \rightarrow x + y + z = 0$$

[Alternative A]

$$x+y+z = 0 \quad 1+5+(-6) = 0 \quad x+y+z = 0$$

[Alternative A]

$$5. \begin{cases} 2x+y=5 \\ 2y+z=3 \\ 3x+2y+z=7 \end{cases}$$

$$D = \begin{bmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{bmatrix} \begin{matrix} 5 \\ 3 \\ 7 \end{matrix}$$

$$\text{I. } 6+10+3 = 19-13=6$$

$$\text{II. } 10+7+0 = 17$$

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$$Dy = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{bmatrix}$$

$$\begin{array}{l} \text{I. } 0 + 14 + 0 \\ \text{II. } 6 + 15 + 0 \end{array} \left. \vphantom{\begin{array}{l} \text{I. } 0 + 14 + 0 \\ \text{II. } 6 + 15 + 0 \end{array}} \right\} 21 - 14 = 7$$

$$Dz = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{bmatrix} \begin{bmatrix} 37 & -42 \\ -5 & \end{bmatrix}$$

$$\begin{array}{l} \text{I. } 28 + 9 + 0 \\ \text{II. } 10 + 12 + 0 \end{array}$$

$$Dx = \begin{bmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 3 & 2 \\ 7 & 1 \end{bmatrix} = 17 - 13 = 4$$

$$\begin{array}{l} \text{I. } 10 - 7 - 3 \\ \text{II. } 0 + 10 + 0 \end{array}$$

$$x = \frac{4}{3} \quad y = \frac{7}{3}$$

$$z = -\frac{5}{3}$$

Alternativa D

$$6. \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$(1 \cdot x) + (0 \cdot y) + (0 \cdot z) = 3$$

$$|x=3|$$

$$(2 \cdot x) + (1 \cdot y) + (0 \cdot z) = 7$$

$$6 + y = 7 \Rightarrow y = 7 - 6 = 1$$

$$(-1 \cdot x) + (2 \cdot y) + (2 \cdot z) = -1$$

$$\downarrow \quad \downarrow$$

$$-3 + 2 + 2z = -1$$

$$-1 + 2z = -1$$

$$2z = -1 + 1$$

$$z = \frac{0}{2}$$

$$z = 0$$

$$x = 3$$

$$y = -1$$

$$z = 0$$

Alternativa E

# Tarefa Basica II

## Tarefa Basica II

$$1. \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\left( \begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \begin{array}{l} \cdot \frac{1}{2} \\ \cdot \frac{1}{2} \\ \cdot \frac{1}{2} \end{array}$$

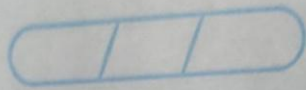
$$\left( \begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & 7/2 & 1/2 & 27/2 \\ 0 & 1/2 & -7/2 & 11/2 \end{array} \right) \begin{array}{l} \cdot \frac{1}{7} \\ \cdot \frac{1}{7} \\ \cdot \frac{1}{7} \end{array}$$

$$\left( \begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & 7/2 & 1/2 & 27/2 \\ 0 & 0 & -25/7 & 25/7 \end{array} \right)$$

$$\frac{-25}{7} z = \frac{25}{7} \quad \left\{ \begin{array}{l} z = \frac{25}{7} \cdot \left( \frac{-7}{25} \right) \\ z = -\frac{25}{25} \rightarrow \boxed{z = -1} \end{array} \right.$$

$$\frac{7}{2} \cdot y + \frac{1}{2} \cdot (-1) = \frac{27}{2}$$

$$\frac{7}{2} y = \frac{27}{2} + \frac{1}{2} \rightarrow y = \frac{14 \cdot 2}{7} = y = 4$$



$$x - 5 \cdot (-1) = 3$$

$$x = 3 - 5$$

$$\boxed{x = -2}$$

$$2. \begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$$

$$\begin{cases} x - 2y = 0 \\ 2y - 3z = 0 \\ x + y + z = 11 \end{cases}$$

$$\left( \begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 1 & 1 & 1 & 11 \end{array} \right) \xrightarrow{-1}$$

$$\left( \begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 3 & 1 & 11 \end{array} \right) \xrightarrow{-2}$$

$$\left( \begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 4/2 & 11 \end{array} \right)$$

$$\begin{cases} 11 \cdot 2 = 11 \\ 2 \\ z = 11 \cdot 2 \end{cases}$$

$$z = 2 //$$

$$2y - 3 \cdot 2 = 0$$

$$2y = 6$$

$$y = 3 //$$

$$x - 2 \cdot 3 = 0$$

$$x = 6 //$$

Alternativa B.

$$x + 2y + 3z$$

$$6 + 2 \cdot 3 + 3 \cdot 2$$

$$6 + 6 + 6 = 18$$

$$3. \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6x + 3z = -12 \end{cases}$$

$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \begin{matrix} \\ \swarrow -2 \\ \end{matrix}$$

$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \begin{matrix} \\ \swarrow -2 \\ \end{matrix}$$

$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 0 & -5 & -10 \end{array} \right)$$

$$\div 5: z = -10$$

$$z = -10$$

$$-5$$

$$z = 2 //$$

Alternativa D)



$$4. \begin{cases} x = y + \frac{1}{3} \cdot z \\ 3y = z + \frac{1}{3} \cdot x \\ z + x + y = 68 \end{cases}$$

$$\begin{cases} x - y - \frac{1}{3}z = 0 \\ (-\frac{1}{3})x + 3y - z = 0 \\ x + y + z = 68 \end{cases}$$

$$\left( \begin{array}{ccc|c} 1 & (-1) & (-\frac{1}{3}) & 0 \\ (-\frac{1}{3}) & 3 & (-1) & 0 \\ 1 & 1 & 1 & 68 \end{array} \right) \begin{array}{l} \cdot \frac{1}{3} \\ \cdot (-1) \\ \cdot (-1) \end{array}$$

$$\left( \begin{array}{ccc|c} 1 & (-1) & (-\frac{1}{3}) & 0 \\ 0 & 14/3 & 4/5 & 0 \\ 0 & 2 & 6/5 & 68 \end{array} \right) \begin{array}{l} \cdot (-3/14) \\ \cdot (-5/4) \\ \cdot (-5/4) \end{array}$$

$$\left( \begin{array}{ccc|c} 1 & (-1) & (-1/9) & 0 \\ 0 & 14/5 & 4/5 & 0 \\ 0 & 0 & 68/35 & 68 \end{array} \right)$$

$$\frac{68}{35} \cdot z = 68 \quad z = 35$$

$$z = 35$$

$$2y + \frac{6}{5} \cdot 35 = 68$$

$$2y = 68 - 42$$

$$2y = 26$$

$$y = 26/2 \rightarrow 13,11$$

$$x + 13 + 35 = 68$$

$$x = 68 - 48$$

$$x = 20,11$$

$$x \rightarrow R\$ 20,00$$

$$y \rightarrow R\$ 13,00$$

$$z \rightarrow R\$ 25,00$$

Alternativa A

S.

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A \cdot x = \begin{bmatrix} 134 \\ 115 \\ 48 \end{bmatrix}$$

$$\begin{cases} 3 + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$\left( \begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \xrightarrow{-2} \left( \begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 1 & -10 & -162 \end{array} \right)$$

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$$\left( \begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 1 & -10 & -182 \end{array} \right) \cdot \frac{-1}{3}$$

$$\left( \begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 0 & -\frac{34}{3} & -\frac{680}{3} \end{array} \right) \cdot \frac{-1}{3}$$

$$\left( -\frac{34}{3} \right) \cdot z = \left( -\frac{680}{3} \right) \cdot \left( -\frac{3}{34} \right) \rightarrow$$

$$z = -\frac{680}{34} \cdot \left( -\frac{3}{34} \right)$$

$$z = \frac{680}{34} \rightarrow z = 20 //$$

$$\begin{array}{l|l} y - 10 \cdot 20 = (-182) & x + 5 \cdot 20 = 115 \\ y = (-182) + 200 & x = 115 - 100 \\ y = 18 // & x = 15 // \end{array}$$

$$x + y + z = ?$$

$$15 + 18 + 20 = 53 //$$

Alternativa (A)