09. Chi Square Test

Adapted From :

Probability & Statistics for Engineers & Scientists, 9th Ed.

Walpole/Myers/Myers/Ye (c)2010

Introduction to Business Statistics, 5e

Kvanli/Guynes/Pavur (c)2000

South-Western College Publishing

Statistics for Managers

Using Microsoft® Excel 4th Edition

χ^2 Goodness of Fit Test

- A goodness of fit test helps answer the questions: "Is a die fair?", "Is a population normally distributed", etc.
- For any situation comparing expected and observed frequencies in k different categories, if e_i is the expected, and o_i is the observed frequency in category i,

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$

is approximately χ^2 distributed with $\nu = k - 1$ degrees of freedom.

  - The expected frequency, e_i, in each category must be ≥ 5.
  - Categories with $e_i < 5$ may be combined, but without reference to the observed frequencies (don't cheat!).
  - If the frequencies don't match, the χ^2 statistic will be high, so we generally set up a 1-sided test in that direction.

Goodness of Fit Test Summary

- Steps for a χ^2 goodness of fit test of the hypothesis H_0 that data follows a given distribution:
  - Break the observed data up into a logical group of categories based on the range of the data. Do not base the categories in any way on the observed frequency values.
  - Determine the total number of observations, n, for the observed data.
  - For the given distribution, calculate the probability of of a randomly selected observation falling in each category. (These probabilities add to 1).
  - Multiply each probability by n to get the expected number of observations, e_i, in each category. (These and the o_i's add to n).
  - Determine the observed frequencies, o_i, in each category.
  - Combine categories if necessary to ensure that each $e_i \geq 5$. Do not use the o_i's when deciding which categories to combine.
  - Calculate χ^2 and reject H_0 if the χ^2 value is too high.

Degrees of Freedom

- The χ^2 goodness of fit test can apply to many situations. However, the number of degrees of freedom must be adjusted for parameters taken from the observed data and used to calculate the expected data.
    - For example, to test data for normality, if we estimate μ and σ using x_{bar} and s, we must subtract 2 <u>additional</u> degrees of freedom and use $\nu = k - 3$.
- Example of Goodness-of-Fit test : page371-373 (walpole)

Contingency Tables

Contingency Tables

- Useful in situations involving multiple population proportions
- Used to classify sample observations according to two or more characteristics
- Also called a cross-classification table.

Contingency Table Example

Left-Handed vs. Gender

Dominant Hand: Left vs. Right

Gender: Male vs. Female

- 2 categories for each variable, so called a 2 x 2 table
- Suppose we examine a sample of size 300

Contingency Table Example

(continued)

Sample results organized in a contingency table:

sample size = n = 300:

120 Females, 12 were left handed

180 Males, 24 were left handed

Gender	Hand Preference		
	Left	Right	
Female	12	108	120
Male	24	156	180
	36	264	300

χ^2 Test for the Difference Between Two Proportions

H_0: $p_1 = p_2$ (Proportion of females who are left handed is equal to the proportion of males who are left handed)

H_1: $p_1 \neq p_2$ (The two proportions are not the same – Hand preference is not independent of gender)

- If H_0 is true, then the proportion of left-handed females should be the same as the proportion of left-handed males
- The two proportions above should be the same as the proportion of left-handed people overall

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell

  f_e = expected frequency in a particular cell if H_0 is true

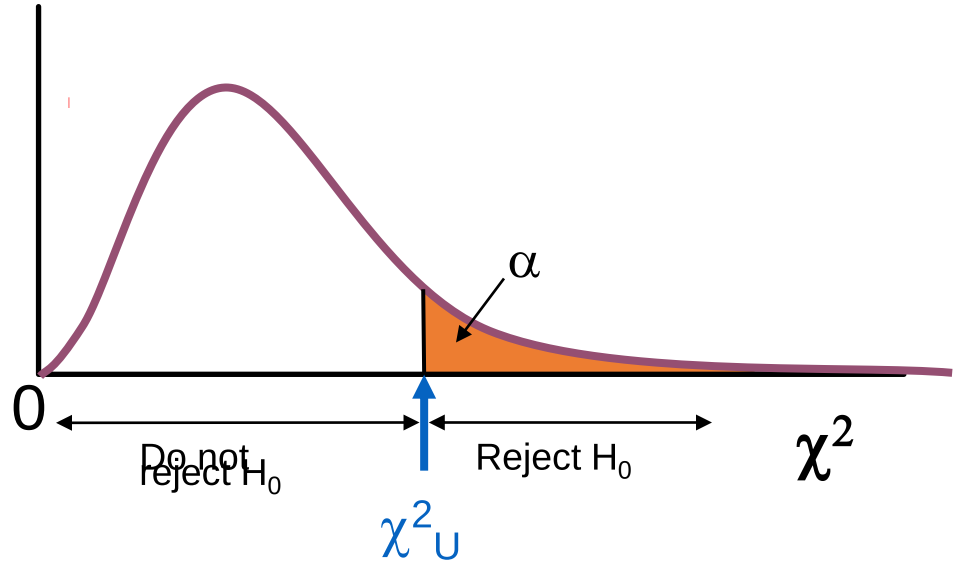
χ^2 for the 2 x 2 case has 1 degree of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 5)

Decision Rule

The χ^2 test statistic approximately follows a chi-squared distribution with one degree of freedom

Decision Rule:
If $\chi^2 > \chi^2_U$, reject H_0, otherwise, do not reject H_0



Computing the Average Proportion

The average proportion is:

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{X}{n}$$

120 Females, 12 were left handed

180 Males, 24 were left handed

Here:

$$\bar{p} = \frac{12 + 24}{120 + 180} = \frac{36}{300} = 0.12$$

i.e., the proportion of left handers overall is 12%

Finding Expected Frequencies

- To obtain the expected frequency for left handed females, multiply the average proportion left handed (p) by the total number of females
- To obtain the expected frequency for left handed males, multiply the average proportion left handed (p) by the total number of males

If the two proportions are equal, then

P(Left Handed | Female) = P(Left Handed | Male) = .12

i.e., we would expect (.12)(120) = 14.4 females to be left handed
(.12)(180) = 21.6 males to be left handed

Observed vs Expected Frequencies

Observed frequencies vs. expected frequencies:

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

Jika erc = ekspektasi baris ke r dan kolom ke c

Maka erc = (∑ Oc . ∑ Or) / n

The Chi-Square Test Statistic

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The test statistic is:

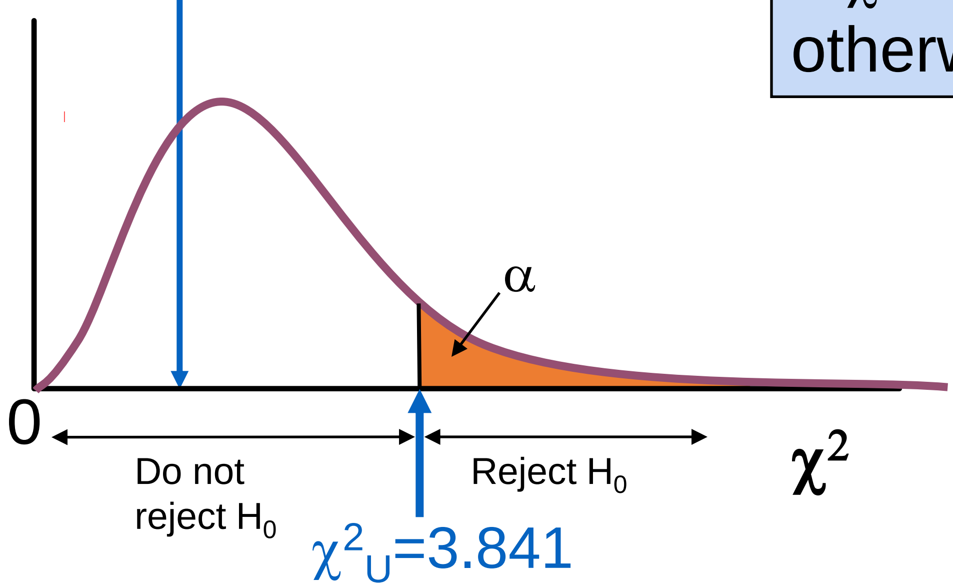
$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.6848$$

Decision Rule Example

The test statistic is $\chi^2 = \boxed{0.6848}$, χ^2_U with 1 d.f. $= 3.841$

Decision Rule:
If $\chi^2 > 3.841$, reject H_0, otherwise, do not reject H_0



Here,
$\chi^2 = 0.6848 < \chi^2_U = 3.841$,
so we do not reject H_0
and conclude that there is not sufficient evidence that the two proportions are different at $\alpha = .05$

χ^2 Test for the Difference Between More Than Two Proportions

- Extend the χ^2 test to the case with more than two independent populations:

H_0: $p_1 = p_2 = \ldots = p_c$

H_1: Not all of the p_j are equal ($j = 1, 2, \ldots, c$)

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell of the 2 x c table

  f_e = expected frequency in a particular cell if H_0 is true

χ^2 for the 2 x c case has (2-1)(c-1) = c - 1 degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

Computing the Overall Proportion

The overall proportion is: $\bar{p} = \frac{X_1 + X_2 + \cdots + X_c}{n_1 + n_2 + \cdots + n_c} = \frac{X}{n}$

- Expected cell frequencies for the c categories are calculated as in the 2 x 2 case, and the decision rule is the same:

Decision Rule:
If $\chi^2 > \chi^2_U$, reject H_0, otherwise, do not reject H_0

Where χ^2_U is from the chi-squared distribution with c – 1 degrees of freedom

χ^2 Test of Independence (Categorical Data)

- Similar to the χ^2 test for equality of more than two proportions, but extends the concept to contingency tables with r rows and c columns

H_0: The two categorical variables are independent

(i.e., there is no relationship between them)

H_1: The two categorical variables are dependent

(i.e., there is a relationship between them)

χ^2 Test of Independence

(continued)

The Chi-square test statistic is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell of the r x c table

  f_e = expected frequency in a particular cell if H_0 is true

χ^2 for the r x c case has (r-1)(c-1) degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

Expected Cell Frequencies

- Expected cell frequencies:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

Where:

row total = sum of all frequencies in the row

column total = sum of all frequencies in the column

n = overall sample size

Decision Rule

- The decision rule is

If $\chi^2 > \chi^2_U$, reject H_0,

otherwise, do not reject H_0

Where χ^2_U is from the chi-squared distribution with $(r - 1)(c - 1)$ degrees of freedom

Example

- The meal plan selected by 200 students is shown below:

Class Standing	Number of meals per week			Total
	20/week	10/week	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Example

(continued)

- The hypothesis to be tested is:

H_0: Meal plan and class standing are independent
(i.e., there is no relationship between them)
H_1: Meal plan and class standing are dependent
(i.e., there is a relationship between them)

Example: Expected Cell Frequencies *(continued)*

Observed:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Expected cell frequencies if H_0 is true:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24.5	30.8	14.7	70
Soph.	21.0	26.4	12.6	60
Junior	10.5	13.2	6.3	30
Senior	14.0	17.6	8.4	40
Total	70	88	42	200

Example for one cell:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

$$= \frac{30 \times 70}{200} = 10.5$$

Example: The Test Statistic

(continued)

- The test statistic value is:

$$\chi^2 = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

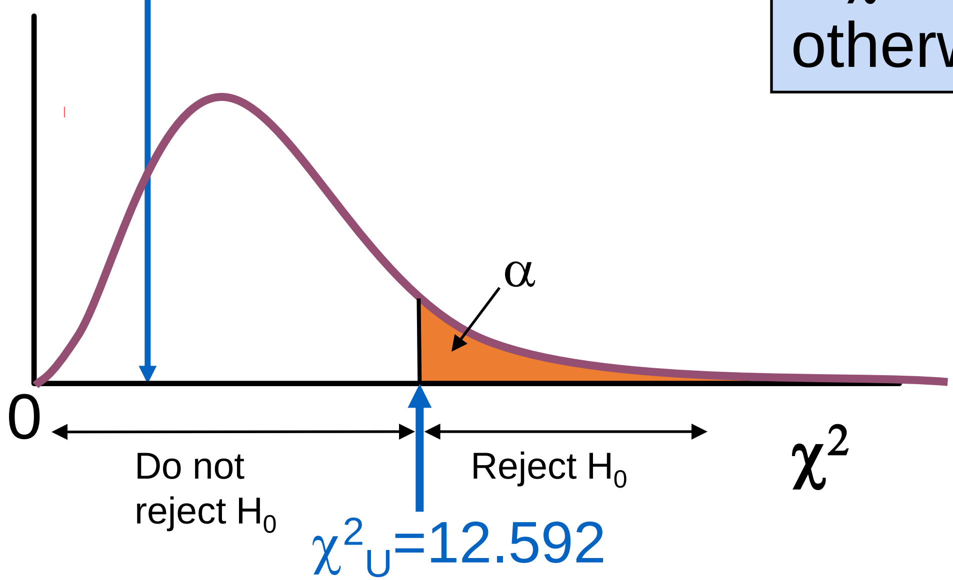
$$= \frac{(24-24.5)^2}{24.5} + \frac{(32-30.8)^2}{30.8} + \cdots + \frac{(10-8.4)^2}{8.4} = 0.709$$

$\chi^2_U = 12.592$ for $\alpha = .05$ from the chi-squared distribution with $(4 – 1)(3 – 1) = 6$ degrees of freedom

Example: Decision and Interpretation

(continued)

The test statistic is $\chi^2 = \boxed{0.709}$, χ^2_U with 6 d.f. $= 12.592$



Decision Rule:
If $\chi^2 > 12.592$, reject H_0, otherwise, do not reject H_0

Here,
$\chi^2 = 0.709 < \chi^2_U = 12.592$,
so do not reject H_0

Conclusion: there is not sufficient evidence that meal plan and class standing are related at $\alpha = .05$

Chapter Summary

- Developed and applied the χ^2 test for Goodness-of-Fit
- Developed and applied the χ^2 test for the difference between two proportions
- Developed and applied the χ^2 test for differences in more than two proportions
- Examined the χ^2 test for independence

Exercises

LATIHAN CHI-SQUARE AS A TEST OF VARIANCE

SOAL 1

Seorang manajer pabrik saat ini sedang mengevaluasi sebuah metode proses produksi baru. Pihak perusahaan sangat berhati-hati sekali tidak hanya terhadap rata-rata berat yang dihasilkan tetapi juga penyimpangan yang terjadi. Penyimpangan yang diijinkan relatif sangat kecil yaitu kurang dari 1 microgram bila metode proses produksi ingin diterapkan. Dari 30 sampel diperoleh standar deviasi 0,73 microgram Pada α 0,01 apakah metode proses produksi ini dapat diterima ?

SOAL 2

Standar deviasi kekuatan putus kabel produksi PT ABC adalah 240 kN. Setelah metode produksi diganti, kekuatan putus kabel dari 25 sebagai sample adalah 300 kN. Apakah terjadi peningkatan penyimpangan kekuatan putus kabel setelah proses produksi diganti pada α 0,05 ?

Exercises

LATIHAN CHI-SQUARE AS A TEST OF INDEPENDENCE

SOAL 1.

Sebuah LSM ingin menguji apakah ada hubungan antara latar belakang pendidikan dengan pemberian suara pada pemilihan seorang presiden. Dari 150 orang responden diperoleh data sebagai berikut :

	Suara	
Pendidikan	Setuju	Tidak Setuju
SD/SMP	10	20
SMU	30	40
Perguruan Tinggi	30	20

Ujilah pada α 0,01 !

Exercises

SOAL 2.

Sebuah perusahaan dagang secara rutin membeli komponen sejumlah 1000 unit. Perusahaan menetapkan untuk membeli dari 4 supplier A, B, C dan D dengan rasio masing-masing pembelian 2 : 2: 1 : 1 Sampel acak dari 240 unit ditelusuri berturut-turut berasal dari pembelian 130 unit supplier A, 40 unit supplier B, 40 unit supplier C dan 30 unit supplier D. Apakah sampel mendukung ketetapan perusahaan dalam rasio distrbusi supplier pada α 0,1 ?

SOAL 3

Bagian pemasaran sabun mandi melakukan penelitian untuk melihat apakah ada pengaruh warna sabun terhadap pembeli . Dari 200 pembeli diperoleh data sebagai berikut :

Warna Sabun	Pembeli
Merah	50
Putih	75
Biru	30
Hijau	45

Sabun-sabun tersebut sama kualitasnya hanya warnanya saja berbeda. Apakah memang ada pengaruh warna terhadap pembeli ? Ujilah pada α 0,01

Exercises

LATIHAN CHI SQUARE AS A TEST OF GOODNESS OF FIT

SOAL 1.

Sebuah layanan informasi via telepon melakukan penelitian jumlah telepon yang masuk per menit. Data yang diperoleh sebagai berikut :

Juml.telepon/menit	0	1	2	3	4	5	6	7≤
Frekuensi terjadinya	4	15	42	60	89	94	52	80

Pada α 0,05 ujilah apakah mengikuti pola distrbusi Poisson pada λ = 5 !

SOAL 2.

Seorang tenaga pemasaran sedang mengevaluasi hasil penjualannya. Setiap hari ia membawa 5 unit yang harus dijual dan data masa lalu sebagai berikut :

Jumlah terjual/hari	0	1	2	3	4	5
Frekuensi penjualan	10	41	60	20	6	3

Ia menduga bahwa pola penjualannya mengikuti distribusi binomial, jika peluang menjual setiap unit 0,4 Pada α 0,05 apakah benar dugaannya ?

Thank You

"We trust in GOD, all others must bring data"