

# 08. Hypothesis Testing: Two Samples

#### Adapted From:

Probability & Statistics for Engineers & Scientists, 9th Ed.

Walpole/Myers/Myers/Ye (c)2010

Introduction to Business Statistics, 5e

Kvanli/Guynes/Pavur (c)2000

South-Western College Publishing

Statistics for Managers

Using Microsoft® Excel 4th Edition



### Learning Objectives

#### After completing this session, you should be able to:

- Test hypotheses for the difference between two independent population means (standard deviations known or unknown)
- Test two means from related samples for the mean difference
- Complete a Z test for the difference between two proportions
- Use the F table to find critical F values
- Complete an F test for the difference between two variances



### Two Sample Tests

**Two Sample Tests** 

Population Means, Independent Samples

Population Means, Paired Observations

Population Proportions

Population Variances

**Examples:** 

Group 1 vs. independent Group 2

Same group before vs. after treatment

Proportion 1 vs. Proportion 2

Variance 1 vs. Variance 2



### Difference Between Two Means

Population means, independent samples



 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1 = \sigma_2$  but unknown

 $\sigma 1 \neq \sigma 2$  and unknown

Goal: Test hypotheses or form a confidence interval for the difference between two population means,  $\mu_1 - \mu_2$ 

The point estimate for the difference is

$$\overline{X}_1 - \overline{X}_2$$



### Independent Samples

# Population means, independent samples



 $\sigma_1$  and  $\sigma_2$  known

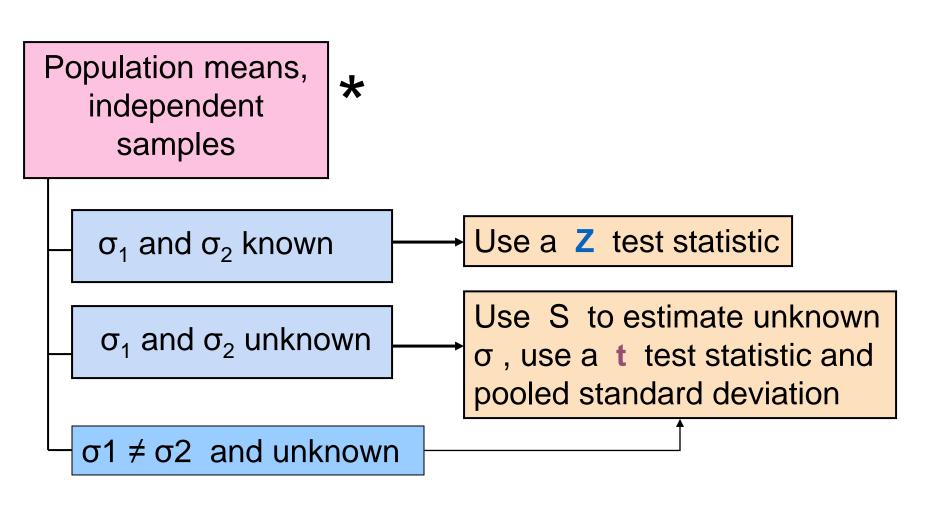
 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

- Different data sources
  - Unrelated
  - Independent
    - Sample selected from one population has no effect on the sample selected from the other population
- Use the difference between 2 sample means
- Use Z test or pooled variance t test



### Difference Between Two Means





Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

#### Assumptions:

- Samples are randomly and independently drawn
- population distributions are normal or both sample sizes are ≥ 30
- Population standard deviations are known



(continued)

Population means, independent samples

When  $\sigma_1$  and  $\sigma_2$  are known and both populations are normal or both sample sizes are at least 30, the test statistic is a Z-value...

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

...and the standard error of  $\overline{X}_1 - \overline{X}_2$  is

$$\sigma_{\overline{X}_{1}-\overline{X}_{2}} = \sqrt{\frac{\sigma_{1}^{2}}{n_{1}} + \frac{\sigma_{2}^{2}}{n_{2}}}$$



(continued)

Population means, independent samples

The test statistic for

$$\mu_1 - \mu_2$$
 is:

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

$$Z = \frac{\left(\overline{X}_{1} - \overline{X}_{2}\right) - \left(\mu_{1} - \mu_{2}\right)}{\sqrt{\frac{\sigma_{1}^{2}}{n_{1}} + \frac{\sigma_{2}^{2}}{n_{2}}}}$$



### Hypothesis Tests for Two Population Means

#### Two Population Means, Independent Samples

#### Lower tail test:

$$H_0$$
:  $\mu_1 = \mu_2$   
 $H_1$ :  $\mu_1 < \mu_2$   
i.e.,

$$H_0$$
:  $\mu_1 - \mu_2 = 0$   
 $H_1$ :  $\mu_1 - \mu_2 < 0$ 

#### Upper tail test:

$$H_0$$
:  $\mu_1 = \mu_2$   
 $H_1$ :  $\mu_1 > \mu_2$   
i.e.,

$$H_0$$
:  $\mu_1 - \mu_2 = 0$   
 $H_1$ :  $\mu_1 - \mu_2 > 0$ 

#### Two-tailed test:

$$H_0$$
:  $\mu_1 = \mu_2$   
 $H_1$ :  $\mu_1 \neq \mu_2$   
i.e.,

$$H_0$$
:  $\mu_1 - \mu_2 = 0$   
 $H_1$ :  $\mu_1 - \mu_2 \neq 0$ 



### Hypothesis tests for $\mu_1 - \mu_2$

Two Population Means, Independent Samples

#### Lower tail test:

$$H_0$$
:  $\mu_1 - \mu_2 = 0$ 

 $H_1$ :  $\mu_1 - \mu_2 < 0$ 

#### Upper tail test:

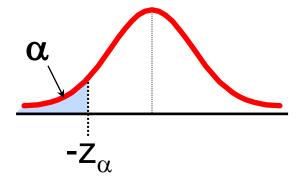
$$H_0$$
:  $\mu_1 - \mu_2 = 0$ 

 $H_1$ :  $\mu_1 - \mu_2 > 0$ 

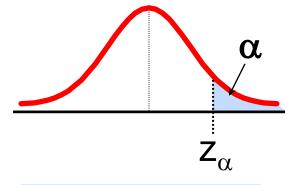
#### Two-tailed test:

$$H_0$$
:  $\mu_1 - \mu_2 = 0$ 

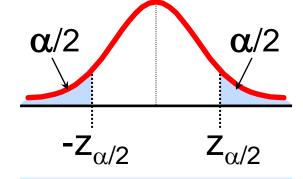
$$H_1$$
:  $\mu_1 - \mu_2 \neq 0$ 



Reject  $H_0$  if  $Z < -Z_{\alpha}$ 



Reject  $H_0$  if  $Z > Z_{\alpha}$ 



Reject  $H_0$  if  $Z < -Z_{\alpha/2}$ or  $Z > Z_{\alpha/2}$ 



Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

#### **Assumptions:**

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown but assumed equal



(continued)

Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

### Forming interval estimates:

- The population variances are assumed equal, so use the two sample standard deviations and pool them to estimate σ
- the test statistic is a t value with (n<sub>1</sub> + n<sub>2</sub> - 2) degrees of freedom



(continued)

Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

The pooled standard deviation is

$$S_{p} = \sqrt{\frac{(n_{1}-1)S_{1}^{2} + (n_{2}-1)S_{2}^{2}}{(n_{1}+n_{2}-2)}}$$



(continued)

Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

The test statistic for

$$\mu_1 - \mu_2$$
 is:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where t has  $(n_1 + n_2 - 2)$  d.f.,

and

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 1)}$$



### $\sigma_1 \neq \sigma_2$ and Unknown

Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

#### Assumptions:

\*

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown and unequal



### $\sigma_1 \neq \sigma_2$ and Unknown

(continued)

Population means, independent samples

 $\sigma_1$  and  $\sigma_2$  known

 $\sigma_1$  and  $\sigma_2$  unknown

 $\sigma 1 \neq \sigma 2$  and unknown

The statistics :

$$\mathbf{T'} = \frac{(\overline{\mathbf{X}}_1 - \overline{\mathbf{X}}_2) - (\mu_1 - \mu_2)}{\sqrt{\mathbf{S}_1^2 / n_1 + \mathbf{S}_2^2 / n_2}}$$

has an approximate t distribution with approximate degrees of fredom:

$$\upsilon = \frac{\left(S_{1}^{2} / n_{1} + S_{2}^{2} / n_{2}\right)^{2}}{\left(S_{1}^{2} / n_{1}\right)^{2} / (n_{1} - 1) + \left(S_{2}^{2} / n_{2}\right)^{2} / (n_{2} - 1)}$$



### Pooled S<sub>p</sub> t Test: Example

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

Number Sample mean Sample std dev

<b>NYSE</b>	<b>NASDAQ</b>	
21	25	
3.27	2.53	
1.30	1.16	

Assuming equal variances, is there a difference in average yield ( $\alpha = 0.05$ )?





### Calculating the Test Statistic

The test statistic is:

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(3.27 - 2.53\right) - 0}{\sqrt{1.5021 \left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(21 - 1)1.30^2 + (25 - 1)1.16^2}{(21 - 1) + (25 - 1)} = 1.5021$$



### Solution

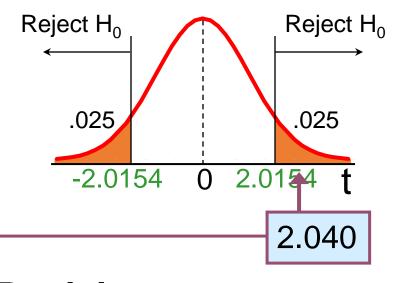
$$H_0$$
:  $\mu_1 - \mu_2 = 0$  i.e.  $(\mu_1 = \mu_2)$ 

$$H_1$$
:  $\mu_1 - \mu_2 \neq 0$  i.e.  $(\mu_1 \neq \mu_2)$ 

$$\alpha$$
 = 0.05

$$df = 21 + 25 - 2 = 44$$

Critical Values:  $t = \pm 2.0154$ 



#### **Test Statistic:**

$$t = \frac{3.27 - 2.53}{\sqrt{1.5021 \left(\frac{1}{21} + \frac{1}{25}\right)}}$$

#### **Decision:**

2.040

Reject  $H_0$  at  $\alpha = 0.05$ 

#### **Conclusion:**

There is evidence of a difference in means.



### Paired Observations

Related samples

Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$D = X_1 - X_2$$

- Eliminates Variation Among Subjects
- Assumptions:
  - Both Populations Are Normally Distributed
  - Or, if Not Normal, use large samples



### Mean Difference

Related samples

The i<sup>th</sup> paired difference is D<sub>i</sub>, where

$$D_i = X_{1i} - X_{2i}$$

The point estimate for the population mean\_paired difference is D:

$$\overline{D} = \frac{\sum_{i=1}^{n} D_{i}}{n}$$

n is the number of pairs in the paired sample



### Sample Standard Deviation

Related samples

We can estimate the unknown population standard deviation with a sample standard deviation:

The sample standard deviation is

$$S_{D} = \sqrt{\frac{\sum_{i=1}^{n} (D_{i} - \overline{D})^{2}}{n-1}}$$



### Mean Difference

(continued)

Paired samples

The test statistic for D is now a t statistic, with n-1 d.f.:

$$T = \frac{\overline{D} - \mu_D}{S_D / \sqrt{n}}$$

Where t has n - 1 d.f. and 
$$S_D$$
 is:  $S_D = \sqrt{\frac{\sum_{i=1}^{n} (D_i - \overline{D})^2}{n-1}}$ 



### Hypothesis Testing for Mean Difference

#### Paired Samples

#### Lower tail test:

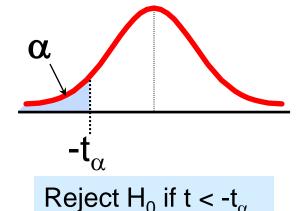
$$H_0$$
:  $\mu_D = 0$   
 $H_1$ :  $\mu_D < 0$ 

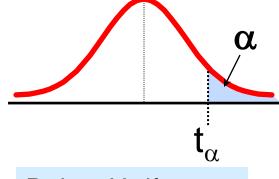
#### Upper tail test:

$$H_0$$
:  $\mu_D = 0$   
 $H_1$ :  $\mu_D > 0$ 

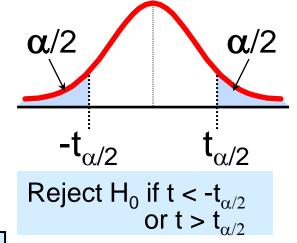


$$H_0$$
:  $\mu_D = 0$   
 $H_1$ :  $\mu_D \neq 0$ 





Reject 
$$H_0$$
 if  $t > t_{\alpha}$ 



Where t has n - 1 d.f.

### Paired Samples Example

 Assume you send your salespeople to a "customer service" training workshop. Is the training effective? You collect the following data:

	<b>Number of Complaints:</b>		(2) - (1)
Salesperson	Before (1)	After (2)	<u>Difference,</u> <u>D</u> <sub>i</sub>
C.B.	6	4	- 2
T.F.	20	6	-14
M.H.	3	2	- 1
R.K.	0	0	0
M.O.	4	0	<u>- 4</u>
			-21

$$\overline{D} = \frac{2D_i}{n}$$

$$= -4.2$$

$$S_D = \sqrt{\frac{\sum (D_i - \overline{D})^2}{n-1}}$$

$$= 5.67$$



### **Paired Samples: Solution**

 Has the training made a difference in the number of complaints (at the 0.01 level)?

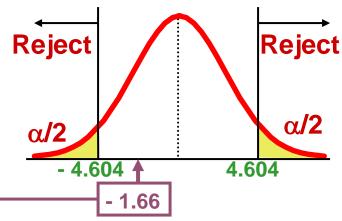
$$H_0: \mu_D = 0$$
  
 $H_1: \mu_D \neq 0$ 

$$\alpha = .01$$
  $\overline{D} = -4.2$ 

Critical Value = 
$$\pm 4.604$$
  
d.f. = n - 1 = 4

#### **Test Statistic:**

$$t = \frac{\overline{D} - \mu_{D}}{S_{D} / \sqrt{n}} = \frac{-4.2 - 0}{5.67 / \sqrt{5}} = \boxed{-1.66}$$



Decision: Do not reject  $H_0$  (t stat is not in the reject region)

Conclusion: There is not a significant change in the number of complaints.



### **Two Population Proportions**

Population proportions

Goal: test a hypothesis or form a confidence interval for the difference between two population proportions,

$$p_1 - p_2$$

#### Assumptions:

$$n_1p_1 \ge 5$$
 ,  $n_1(1-p_1) \ge 5$ 

$$n_2p_2 \ge 5$$
 ,  $n_2(1-p_2) \ge 5$ 

The point estimate for the difference is

$$\hat{p}_1 - \hat{p}_2$$



### **Two Population Proportions**

Population proportions

Since we begin by assuming the null hypothesis is true, we assume  $p_1 = p_2$  and the pooled estimate p of proportion p is

$$\hat{p} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{\mathbf{n}_1 + \mathbf{n}_2}$$

where  $X_1$  and  $X_2$  are the numbers from samples 1 and 2 with the characteristic of interest



### **Two Population Proportions**

(continued)

Population proportions

The test statistic for  $p_1 - p_2$  is a Z statistic:

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (p_1 - p_2)}{\sqrt{p_1 q_1 / n_1 + p_2 q_2 / n_2}}$$
$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{p_2 q_1 / n_1 + 1 / n_2}}$$

where

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$$
,  $\hat{p}_1 = \frac{X_1}{n_1}$ ,  $\hat{p}_2 = \frac{X_2}{n_2}$ 



### Hypothesis Tests for Two Population Proportions

#### Population proportions

#### Lower tail test:

$$H_0: p_1 \ge p_2$$
  
 $H_A: p_1 < p_2$   
i.e.,

$$H_0: p_1 - p_2 \ge 0$$
  
 $H_A: p_1 - p_2 < 0$ 

#### Upper tail test:

$$H_0: p_1 \le p_2$$
  
 $H_A: p_1 > p_2$   
i.e.,

$$H_0: p_1 - p_2 \le 0$$
  
 $H_A: p_1 - p_2 > 0$ 

#### Two-tailed test:

$$H_0$$
:  $p_1 = p_2$   
 $H_A$ :  $p_1 \neq p_2$   
i.e.,

$$H_0$$
:  $p_1 - p_2 = 0$   
 $H_A$ :  $p_1 - p_2 \neq 0$ 



### Hypothesis Tests for Two Population Proportions

(continued)

#### Population proportions

#### Lower tail test:

$$H_0$$
:  $p_1 - p_2 = 0$ 

$$H_A$$
:  $p_1 - p_2 < 0$ 

#### Upper tail test:

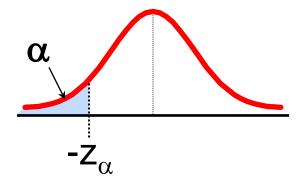
$$H_0$$
:  $p_1 - p_2 = 0$ 

$$H_A: p_1 - p_2 > 0$$

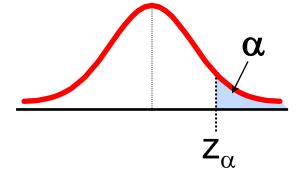
#### Two-tailed test:

$$H_0$$
:  $p_1 - p_2 = 0$ 

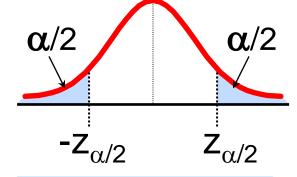
$$H_A: p_1 - p_2 \neq 0$$



Reject  $H_0$  if  $Z < -Z_\alpha$ 



Reject  $H_0$  if  $Z > Z_\alpha$ 



Reject  $H_0$  if  $Z < -Z_{\alpha/2}$  or  $Z > Z_{\alpha/2}$ 



# Example: Two population Proportions

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition A?



- In a random sample, 36 of 72 men and 31 of 50 women indicated they would vote Yes
- Test at the .05 level of significance

### **Example:** Two population Proportions (continued)

The hypothesis test is:

 $H_0$ :  $p_1 - p_2 = 0$  (the two proportions are equal)

 $H_A$ :  $p_1 - p_2 \neq 0$  (there is a significant difference between proportions)

The sample proportions are:

• Men: 
$$p_{s1} = 36/72 = .50$$

• Women:  $p_{s2} = 31/50 = .62$ 

The pooled estimate for the overall proportion is:

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{36 + 31}{72 + 50} = \frac{67}{122} = .549$$



## Example: Two population Proportions

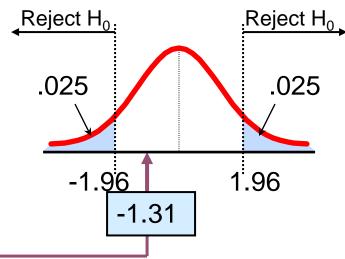
(continued)

The test statistic for  $p_1 - p_2$  is:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(.50 - .62) - (0)}{\sqrt{.549(1 - .549)\left(\frac{1}{72} + \frac{1}{50}\right)}} = \boxed{-1.31}$$

Critical Values =  $\pm 1.96$ For  $\alpha = .05$ 



Decision: Do not reject H<sub>0</sub>

Conclusion: There is not significant evidence of a difference in proportions who will vote yes between men and women.



### Hypothesis Tests for Variances

Tests for Two
Population
Variances

F test statistic

$$H_0$$
:  $\sigma_1^2 = \sigma_2^2$   
 $H_1$ :  $\sigma_1^2 \neq \sigma_2^2$ 

$$H_0$$
:  $\sigma_1^2 = \sigma_2^2$   
 $H_1$ :  $\sigma_1^2 < \sigma_2^2$ 

$$H_0$$
:  $\sigma_1^2 = \sigma_2^2$   
 $H_1$ :  $\sigma_1^2 > \sigma_2^2$ 

Two tailed test

Lower tail test

Upper tail test



### Hypothesis Tests for Variances

(continued)

Tests for Two Population Variances

F test statistic

#### The F test statistic is:

$$F = \frac{S_1^2}{S_2^2}$$

$$S_1^2$$
 = Variance of Sample 1  
 $n_1$  - 1 = numerator degrees of freedom

$$S_2^2$$
 = Variance of Sample 2  
 $n_2$  - 1 = denominator degrees of freedom

#### The F Distribution

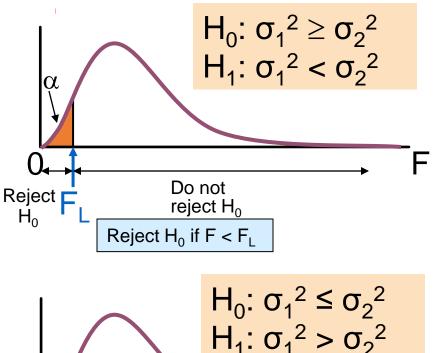
- The F critical value is found from the F table
- The are two appropriate degrees of freedom: numerator and denominator

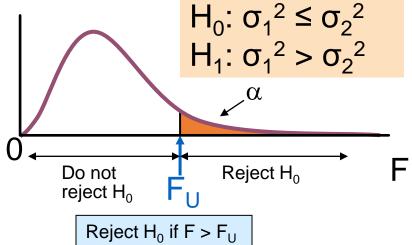
$$F = \frac{S_1^2}{S_2^2}$$
 where  $df_1 = n_1 - 1$ ;  $df_2 = n_2 - 1$ 

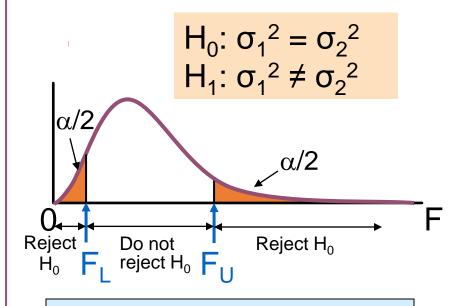
- In the F table,
  - numerator degrees of freedom determine the column
  - denominator degrees of freedom determine the row



## Finding the Rejection Region







rejection region for a two-tailed test is:  $F = \frac{S_1^2}{S_2^2} > F_U$   $F = \frac{S_1^2}{S_2^2} < F_L$ 

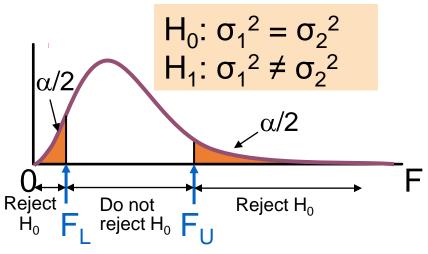


## Finding the Rejection Region

(continued)

#### To find the critical F values:

1. Find  $F_U$  from the F table for  $n_1 - 1$  numerator and  $n_2 - 1$  denominator degrees of freedom



**2**. Find  $F_L$  using the formula:

$$F_L = \frac{1}{F_{U^*}}$$

Where  $F_{U^*}$  is from the F table with  $n_2 - 1$  numerator and  $n_1 - 1$  denominator degrees of freedom (i.e., switch the d.f. from  $F_U$ )



You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	<u>NASDAQ</u>
Number	21	25
Mean	3.27	2.53
Std dev	1.30	1.16

Is there a difference in the variances between the NYSE & NASDAQ at the  $\alpha$  = 0.1 level?





$$H_0$$
:  $\sigma^2_1 - \sigma^2_2 = 0$ 

$$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$$

 $H_0: \sigma_1^2 - \sigma_2^2 = 0$  (there is no difference between variances)

 $H_1: \sigma^2_1 - \sigma^2_2 \neq 0$  (there is a difference between variances)

■ The test statistic is: 
$$F = \frac{S_1^2}{S_2^2} = \frac{1.30^2}{1.16^2} = 1.256$$

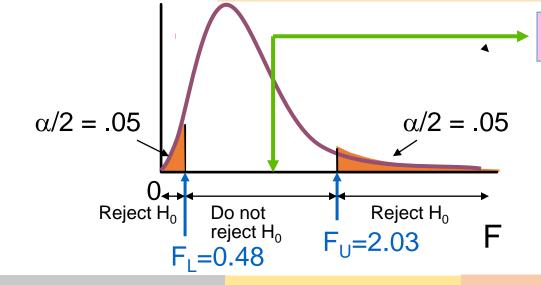
$$F_U = F_{\alpha/2, n, d}$$
  
=  $F_{.05, 20, 24}$   
= 2.03

$$F_L = F_{(1-\alpha/2), n, d} =$$

$$= 1/F_{\alpha/2, d, n} = 1/F_{.05, 24, 20}$$

$$= 1/2.08 = .48$$

1.256





#### F Test: Example Solution

(continued)

 $H_0$ :  $\sigma_1^2 = \sigma_2^2$ 

The test statistic is:

$$F = \frac{S_1^2}{S_2^2} = \frac{1.30^2}{1.16^2} = \frac{1.256}{1.16^2}$$

$$\alpha/2 = .05$$

$$Reject H_0 Do not reject H_0 Fu=2.03$$

 $F_1 = 0.48$ 

- F = 1.256 is not in the rejection region, so we do not reject H<sub>0</sub>
- Conclusion: There is not sufficient evidence of a difference in variances at  $\alpha = 0.1$

$$H_0$$
:  $\sigma_2^2 - \sigma_1^2 = 0$ 

$$H_1: \sigma_2^2 - \sigma_1^2 \neq 0$$

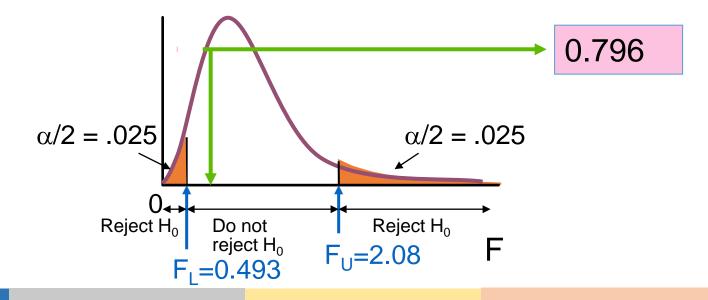
 $H_0: \sigma_2^2 - \sigma_1^2 = 0$  (there is no difference between variances)

 $H_1: \sigma_2^2 - \sigma_1^2 \neq 0$  (there is a difference between variances)

• The test statistic is:  $F = \frac{S_2^2}{S_2^2} = \frac{1.16^2}{1.30^2} = 0.796$ 

$$F_U = F_{\alpha/2, n, d}$$
  
=  $F_{.05, 24, 20}$   
= 2.08

$$F_L = F_{(1-\alpha/2), n, d}$$
  
=1/ $F_{\alpha/2, d, n} = 1/F_{.05, 20.24}$   
= 1/2.03 = .493





### F Test: Example Solution

(continued)

• The test statistic is:  $H_0: \sigma_2^2 = \sigma_1^2$   $H_1: \sigma_2^2 \neq \sigma_1^2$  A/2 = .05• F = 0.796 is not in the rejection region, so we do not reject  $H_0$ 

• Conclusion: There is not sufficient evidence of a difference in variances at  $\alpha = 0.1$ 



#### ANALYSIS OF VARIANCE UJI 1 ARAH

Seorang supervisor pengendalian mutu perusahaan otomotif sangat memperhatikan jumlah kerusakan yang terjadi pada setiap perakitan. Jika sebuah perakitan mempunyai varian kerusakan yang tinggi, maka perbaikan harus segera dilakukan. Supervisor tersebut telah mengumpulkan data dari 2 perakitan sebagai berikut :

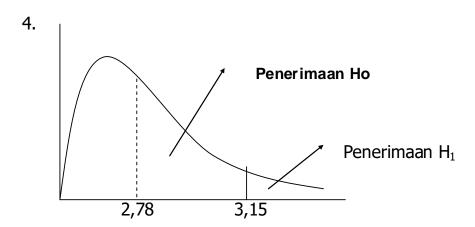
	JUMLAH KERUSAKAN			
	Perakitan A	Perakitan B		
Rata-rata	10	11		
Varian	9	25		
Ukuran sampel	20	16		

Ujilah pada  $\alpha$  0,01 apakah varian perakitan B lebih besar daripada A ?

- 1. Menentukan Hipotesis:
  - Ho :  $s_A^2 = s_B^2$ ; varian kerusakan B sama dgn varian A
  - $H_1: s_B^2 > s_A^2$ ; varian kerusakan B > varian A
- 2. Critical Value uji 1 arah pada  $\alpha$  0,01 dengan numerator  $n=n_B-1=16-1=15$ ;  $d=n_A-1=20-1=19$  adalah  $F_{\alpha,n,d}=3,15$
- 3. Perhitungan F<sub>s</sub>:

$$F_s = s_B^2 / s_A^2 = 25 / 9 = 2,78$$





5. Karena F<sub>s</sub> berada didalam penerimaan Ho artinya varian kerusakan yang terjadi pada perakitan B tidak lebih besar dari perakitan A

#### Alternatif Lain:

- 1. Menentukan Hipotesis:
  - Ho :  $s_A^2 = s_B^2$ ; varian kerusakan B tidak > varian A
  - $H_1: s_A^2 < s_B^2$ ; varian kerusakan B > varian A
- 2. Critical Value uji 1 arah pada  $\alpha$  0,01 dengan numerator n = n<sub>A</sub> -1 = 20 -1 = 19 ; d = n<sub>B</sub> -1 = 16 -1 = 15 adalah :

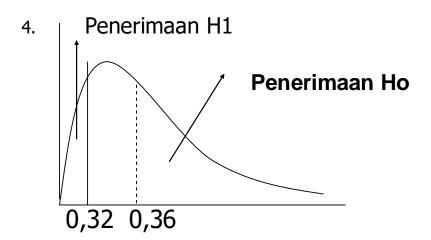
$$F_{1-\alpha, n, d}$$
 (lower tail) =  $\frac{1}{F_{\alpha, d, n}}$  =  $\frac{1}{3,15}$  = 0,32

3. Perhitungan F<sub>s</sub>:

$$F_s = s_A^2 / s_B^2 = 9 / 25 = 0.36$$



#### Alternatif Lain:



5. Karena F<sub>s</sub> berada didalam penerimaan Ho artinya varian kerusakan yang terjadi pada perakitan B tidak lebih besar dari perakitan A



## Exercises

#### PROBLEM 1

In a recent survey, college students were asked the amount of time they spend watching television and surfing on the Internet. The researchers were interested in determining whether the time spent on both activities was equal. They collected the following data:

Person #	1	2	3	4	5	6	7	8
Internet	2	7	3	8	9	15	7	2
TV	4	15	5	3	4	4	4	8

Test the hypothesis at  $\alpha = 0.05$ !

#### PROBLEM 2

National Park rangers were surveyed as to whether they endorsed the idea of carrying firearms. Of the 260 ranges polled west of the Mississippi, 78% endorsed the idea. Of the 184 rangers polled east of the Mississippi, 64% endorsed the idea. Is there evidence that the level of support for carrying firearms is BIGGER in the West than it is in the East?

#### PROBLEM 3

You are comparing the precision of two brands of stamping machines. From a random sample of 12 units of output from Brand A machine, you find that it produces with a standard deviation of 15.2. For the Brand B machine, in a sample of 20 units of output, you find a standard deviation of 10.1. Assume that the output of both machines follows a normal distribution, and the population variances are equal. Evaluate the null hypothesis of equal variances against the alternative hypothesis that Brand B machines produce with lower variance  $\alpha$  0.10.



# Thank You

"We trust in GOD, all others must bring data"