Anomalies and divergences in supergravity amplitudes

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Two main goals:

- Explain relation between anomalies and divergences in (super)gravity

- Show evidence for $\mathcal{N}=4$ supergravity without anomalous amplitudes

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- Explain relation between anomalies and divergences in (super)gravity

- Show evidence for $\mathcal{N}=4$ supergravity without anomalous amplitudes

Secondary goal:

- Show how one can calculate n -point multi-loop amplitudes in supergravity

Gravity as an EFT

- Non-renormalizable but pretty fine EFT

$$\mathcal{L} = M_{pl}^2 R + c_2 R^2 + \cdots + c_{n,k} D^{2k} R^n \qquad c_{n,k} \sim 1/\Lambda^{2k+n-4}$$

- EFT totalitarian principle:

"everything that is allowed is compulsory"

- Appearance of counterterms dictated by divergences

- $S \lesssim 1$ vs. $S > 1$ Q: Are the rules the same?

Progress over last decade

Explicit calculations enabled by double-copy / unitarity

N \ L	1	2	3	4	5
0	$0/-$	∞/R^3	$-$	$-$	$-$
4	$0/-$	$0^{D=5}/R^4$	$0/R^4$	∞/D^2R^4	$-$
5	$0/-$	$0/-$	$0/-$	$0/D^2R^4$	$-$
8	$0/-$	$0/-$	$0/-$	$0/-$	$0/-$

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$\mathcal{N}$ \ L	1	2	3	4	5
0	0 / –	∞ / R^3	–	–	–
4	0 / –	$0^{D=5}$ / R^4	0 / R^4	∞ / D^2R^4	–
5	0 / –	0 / –	0 / –	0 / D^2R^4	–
8	0 / –	0 / –	0 / –	0 / –	0 / –

⟶ Counterterm available respecting known symmetries

Defying EFT lore? Unknown symmetry?

Progress over last decade

Explicit calculations enabled by double-copy / unitarity

N \ L	1	2	3	4	5
0	$0 / -$	∞ / R^3	$-$	$-$	$-$
4	$0 / -$	$0^{D=5} / R^4$	$0 / R^4$	$\infty / D^2 R^4$	$-$
5	$0 / -$	$0 / -$	$0 / -$	$0 / D^2 R^4$	$-$
8	$0 / -$	$0 / -$	$0 / -$	$0 / -$	$0 / -$

→ Counterterm available respecting known symmetries

→ Only divergences in four dimensions

Will focus on divergent cases

I. Pure gravity at two loops

One-loop finiteness of gravity

- Evanescent counterterm

in $D=4$

$$E_4 = (\mathrm{Riem})^2 - 4(\mathrm{Ric})^2 + R^2 = d\Omega$$

- Divergence not numerically zero

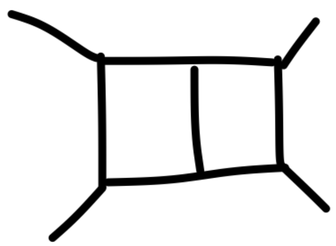
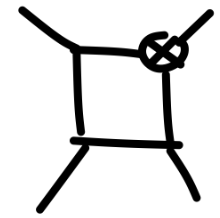
$$\mathcal{M}^{\text{1-loop}}\Big|_{\text{div}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\frac{a+c}{2}\mathcal{M}_{R^2}$$

- Related to "trace anomaly"

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{1}{360}\left(a E_4 + c W^2\right) + \cdots$$

$E_4 = W^2$ on-shell

Contamination at two loops

 $\sim (\mu_R^2)^{2\epsilon}\,\frac{c_1}{\epsilon}$  $\sim (\mu_R^2)^{\epsilon}\,\frac{c_2}{\epsilon}$

$$\mathcal{M}^{\text{2-loop}} = (c_1+c_2)\frac{1}{\epsilon} + (2c_1+c_2)\log\mu_R^2 + \cdots$$

- Divergence and scale dependence disconnected

$$\mathcal{M}^{\text{2-loop}}_{\text{grav}} = \left(\frac{1}{\epsilon}\,\frac{209}{24} - \frac{1}{4}\log\mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$

$$\mathcal{M}^{\text{2-loop}}_{\mathcal{N}=1} = \left(\frac{1}{\epsilon}\,\frac{341}{32} - \underline{\underline{0}}\,\log\mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$

- Simple scale dependence $\sim \dfrac{N_B - N_F}{8}$!

II. $\mathcal{N}=4$ sugra at four loops

$\mathcal{N}=4$ supergravity

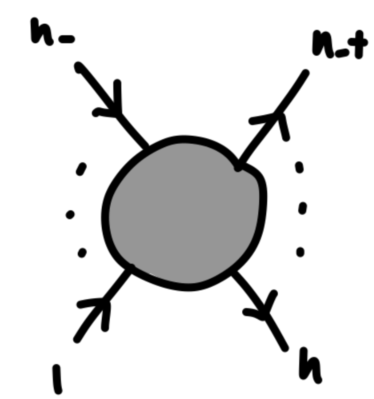
- Multiplets

$$\phi^+ = h^{++} + \psi^+_A \eta^A + A^+_{AB}\,\frac{1}{2!}\,\eta^A\eta^B + \chi^+_{ABC}\,\frac{1}{3!}\,\eta^A\eta^B\eta^C + \bar{t}\,\eta^1\eta^2\eta^3\eta^4$$

$$\phi^- = t + \chi^-_A \eta^A + A^-_{AB}\,\frac{1}{2!}\,\eta^A\eta^B + \psi^-_{ABC}\,\frac{1}{3!}\,\eta^A\eta^B\eta^C + h^{--}\eta^1\eta^2\eta^3\eta^4$$

- Amplitudes

$$M_{n,k}^{(n_-,n_+)} = M_{n,k}\big(\underbrace{\phi^-_1\,\phi^-_2 \cdots \phi^-_{n_-}\,\phi^+_{n_-+1}\cdots \phi^+_n}_{\text{choice of multiplet}}\big) =$$



η-counting
SYM

pure YM

Duality

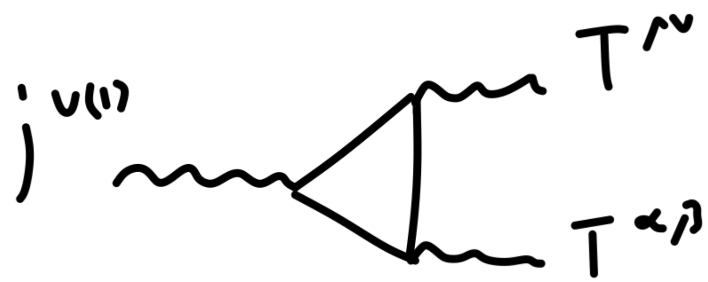
- Scalar t parameterizes coset $\underbrace{SU(1,1)}_{\text{non-linearly realized}}/U(1)$

- Covariant formulation
  - Scalars T^α $SU(1,1)$ doublet $\longrightarrow$ $\underbrace{U^\alpha{}_\beta}_{SU(1,1)} T^\beta e^{i\theta(x)}$ gauged $U(1)$
  - Vectors $\begin{pmatrix} F \\ *F \end{pmatrix} \to U \begin{pmatrix} F \\ *F \end{pmatrix}$

- Amplitudes perspective
  - t Goldstone boson $\longrightarrow$ Vanishing soft limit
  - $U(1)$ Selection rule $\longrightarrow$ $\boxed{K = n_- - 2}$

Duality anomaly

Deviation from standard transformation rule of Effective action

""

$$\Gamma \sim \int \frac{\partial \cdot A^{U(1)}}{\Box} R \wedge R$$

Amplitudes violate selection rule beyond tree level

e.g.

$$M^{\text{1-loop}}(h_1^{--}\, h_2^{--}\, t_3 \cdots t_n) = i \left(\frac{\kappa}{2}\right)^n (n-3)!\, [12]^4$$

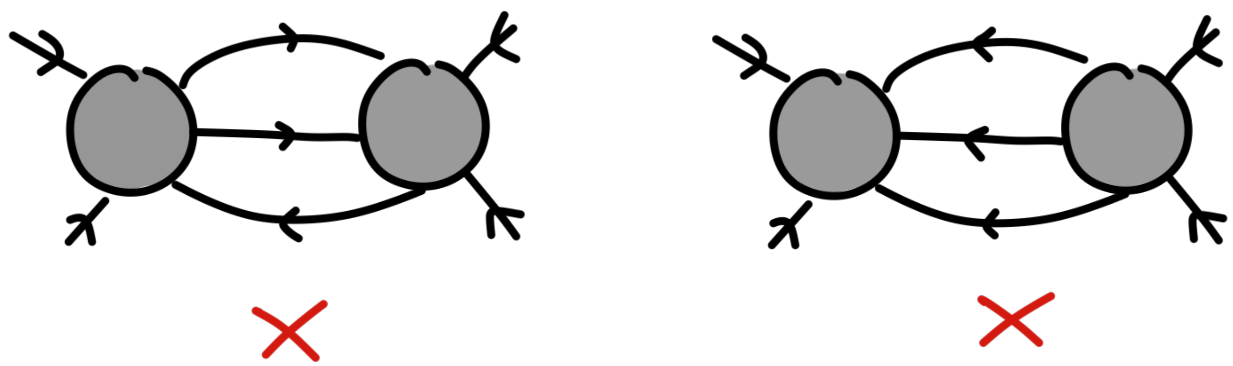
contained in $\quad M^{(n,0)}_{n,0} = i \left(\frac{\kappa}{2}\right)^n (n-3)!\, \delta^{(8)}(Q)$

others non-local

Structure of anomalous amplitudes

- Unitarity cuts with trees vanish in four dimensions

e.g



- Simple argument to all loops: charge conservation
- Anomalous amplitudes suppressed by $\epsilon = \frac{D-4}{2}$

tree = 0 one-loop ~ tree ...

- Transcendental properties delayed

OK, BUT WHAT ABOUT THE DIVERGENCE ?!

Four-loop divergence

Mixed transc.

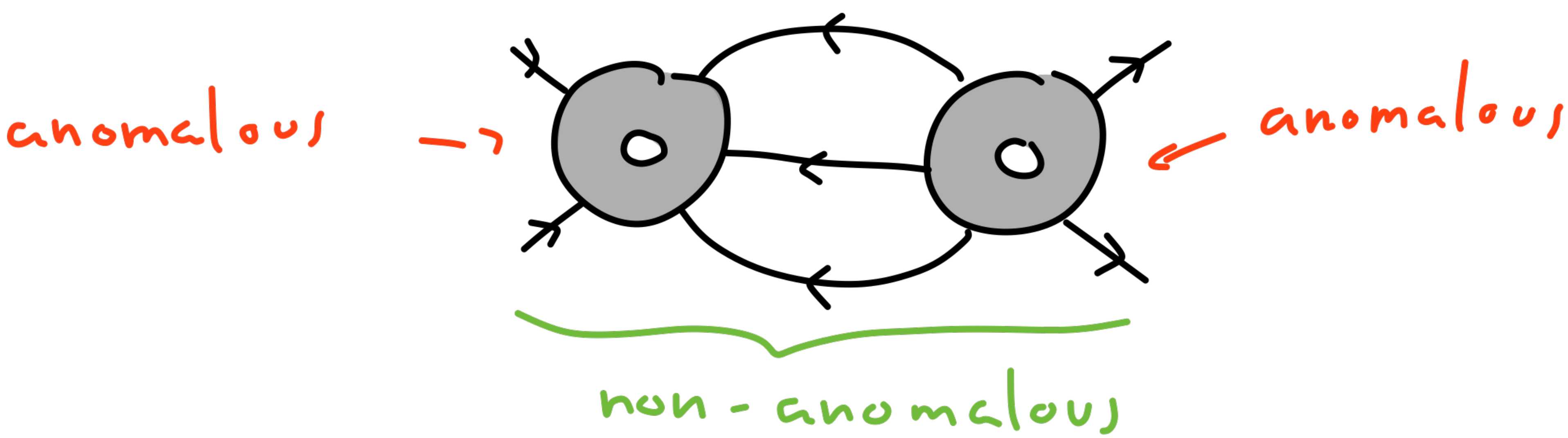
$$\mathcal{M}_4^{(4)}\Big|_{\text{div}} = \frac{1}{\epsilon}\,\frac{1-264\,\zeta_3}{144}\, s t \, A_4^{\text{tree}} \times$$

$$\underbrace{\left(\mathcal{O}^{(2,2)} + \mathcal{O}^{(3,1)} + \mathcal{O}^{(4,0)}\right)}_{\text{Same structure!}}$$

Anomalous

Q: Is the anomaly related to the divergence?

1st piece of evidence: anomalous amplitudes contribute in cuts



More modest Q:

Can the anomalous amplitudes be removed by adding a finite local counterterm?

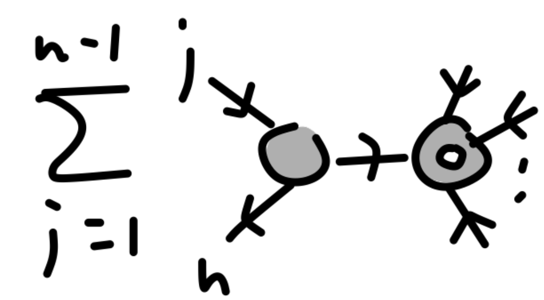
OR

Is there a scheme in which $\mathcal{N}=4$ supergravity does not have anomalous amplitudes?

Structure of anomalous amplitudes - one loop

- Inverse soft recursion

$$\mathcal{M}^{(n-1,1)}_{0,n} = i\left(\frac{\kappa}{2}\right)^{n} \delta^{8}(Q) \underbrace{\sum_{j=3}^{n-1} \frac{[jn]\langle 1j\rangle\langle 2j\rangle}{\langle jn\rangle\langle 1n\rangle\langle 2n\rangle}}_{\text{Graviton soft factor!}} \sim \sum_{j=1}^{n-1}$$



- General formula

inverse soft limit $\quad \mathcal{M}^{(n_-,n_+)}_{0,n} = S[M]\, \mathcal{M}^{(n_-,0)}_{0,n}$

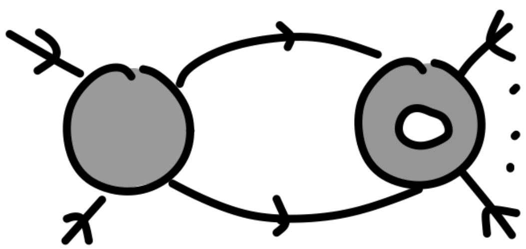
$\sim \phi^-$ states

$S[M] = |H|^{m_1 \ldots m_{n_-}}_{m_1 \ldots m_{n_-}}$ $\quad$ minor of Hodges matrix

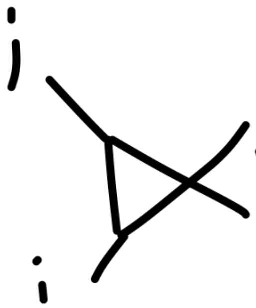
$$H_i^{\,j} = \frac{[ij]}{\langle ij\rangle} \,, \qquad H_i^{\,i} = \sum_{j\neq i} \frac{[ij]\langle jx\rangle\langle jy\rangle}{\langle ij\rangle\langle ix\rangle\langle iy\rangle}$$

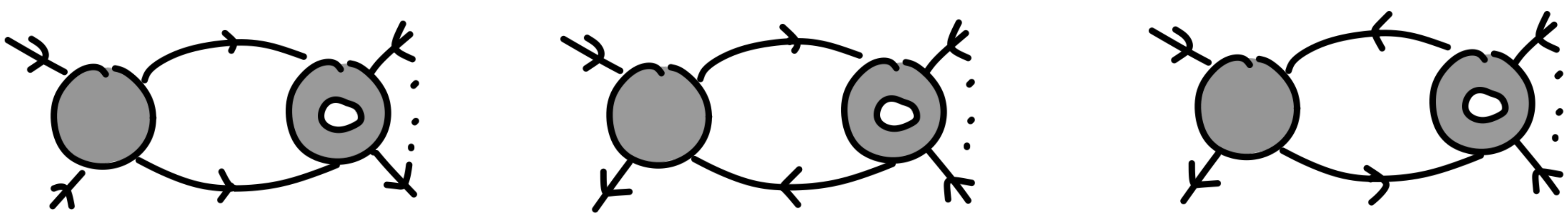
Structure of anomalous amplitudes - two loops

- Few unitarity cuts contribute (recall all trees vanish!)



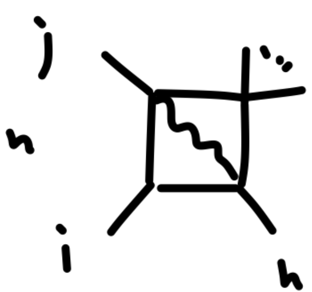
$$\longrightarrow \mathcal{M}_n^{\text{2-loop}\,(n,0)} = \mathcal{M}_n^{\text{1-loop}}(\epsilon) \sum_{i\neq j}^{n} s_{ij}^2$$



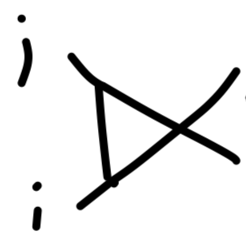


one-loop seed

$$\longrightarrow \mathcal{M}_n^{\text{2-loop}\,(n-1,n)} = \sum_{i\neq j}^{n-1} \frac{\langle ij\rangle^2}{\langle in\rangle^2\langle jn\rangle^2}\, s_{ij}\, s_{jn}$$



$$+\, \mathcal{M}_n^{\text{1-loop}}(\epsilon) \sum_{i\neq j}^{n-1} s_{ij}^2$$



Seed of all anomalous amplitudes is local

$$\mathcal{M}^{(n,0)}_{n,0} = i\left(\frac{\kappa}{2}\right)^{n} (n-3)!\, \delta^{(8)}(Q)$$

Can be removed by the following finite counterterm

$$i - i \ln(1-t)\, R^{+} \wedge R^{+} + c.c + susy$$

So the answer to our second question seems to be YES!

Checks - double-copy

- Double copy of spectrum $(\phi^+, \phi^-) = \phi_{\mathcal{N}=4} \otimes (g^+, g^-)$

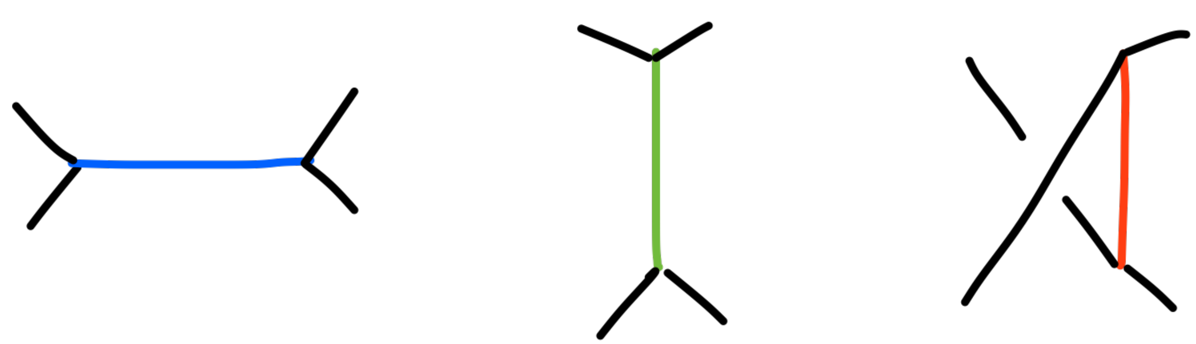
$\rightarrow$ selection rule : MHV deggrees aligned

- Double copy of amplitudes

$$\sum_i \frac{n_i c_i}{P_i} \longrightarrow \sum_i \frac{n_i \tilde{n}_i}{P_i}$$

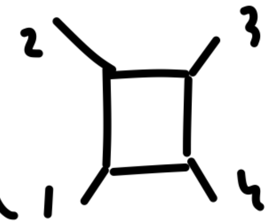
$$c_s + c_t + c_u = 0$$

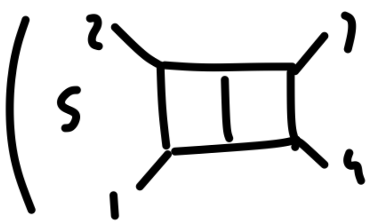
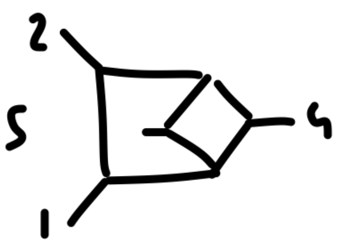
$$n_s + n_t + n_u = 0$$



Checks - double-copy

- One and two-loop amplitudes

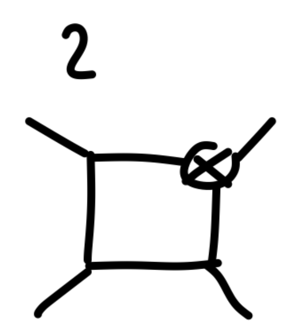
$\mathcal{M}^{\text{1-loop}} = s\, t\, A^{\text{tree}}_{\mathcal{N}=4} \Big($  $+ \text{ perms} \Big)$

$\mathcal{M}^{\text{2-loop}} = s\, t\, A^{\text{tree}}_{\mathcal{N}=4} \Big(s$  $+\ s$  $+ \text{ perms} \Big)$

- Counterterm insertion - double-copy for higher dim. operators

$$\mathcal{M}^{\text{tree}}_{t^n R^2} = A^{\text{tree}}_{\mathcal{N}=4} \otimes A^{\text{tree}}_{F^3} \qquad (\text{KLT})$$

$$\mathcal{M}^{\text{1-loop}}_{t^n R^2} = \underbrace{s\, t\, A^{\text{tree}}_{\mathcal{N}=4}}_{n_{\mathcal{N}=4}} \left(A^{\text{1-loop}}_{F^3}(1,2,3,4) + \text{perms} \right)$$



In all cases we could check double copy reproduces results from soft/collinear & cut analysis

Double copy of F^3 confirms cancellation of anomalous amplitudes

Q: Is the anomaly related to the divergence?

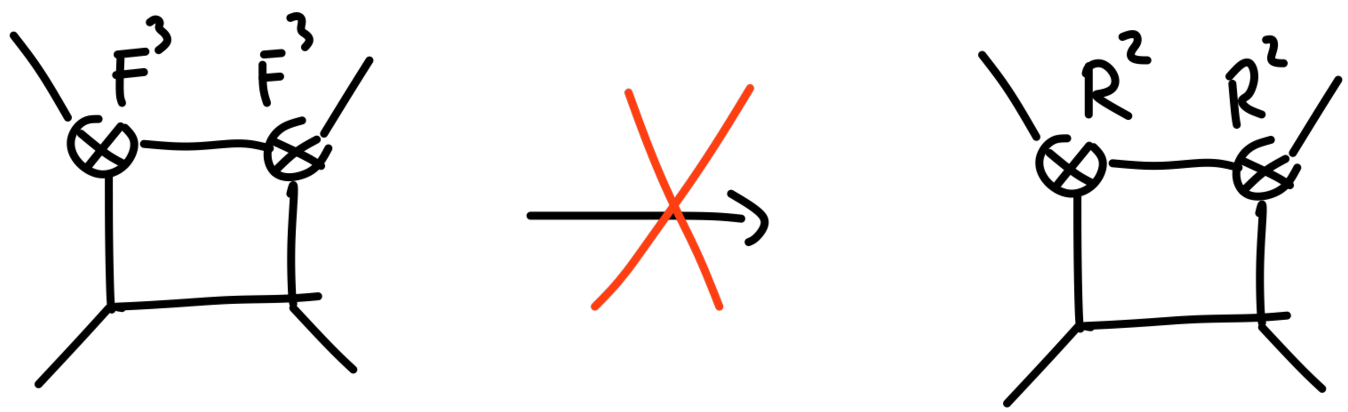
2nd piece of evidence: evanescent contribution in non-anomalous amplitude

$$\mathcal{M}^{(2,2)\ \text{1-loop}} = \mathcal{M}_{R^2} + \cdots$$

Cancelled by same counterterm, including feed-up to higher loops

Path forward

- Three loop calculations beyond current (integral) technology
- Double copy subtle for multiple insertions



WE NEED A DIFFERENT APPROACH!

If cancellations persist...

$$\mathcal{M}_4^{(4)}\Big|_{div} = \frac{1}{\epsilon}\,\frac{1-264\,\zeta_3}{144}\, s\,t\,A_4^{tree} \times \left(\mathcal{O}^{(2,2)} + \cancel{\mathcal{O}^{(3,1)}} + \cancel{\mathcal{O}^{(4,0)}} \right)$$

+ anomalous & evanescent contributions to $\mathcal{M}^{(2,2)}$ ✗

Four loop divergence should be reanalyzed!

Summary

- Anomalies make analysis of gravity divergences non-trivial

- In $\mathcal{N}=4$ sugra duality anomaly is suspect for divergence

- One and two-loop anomalous amplitudes surprisingly simple
  (even grad student can calculate)

- Anomalous and evanescent amplitudes can be removed by adding a finite local counterterm

- Fate of the four-loop diverge ?!