UCLA

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Gravity Amplitudes from the Ultraviolet

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Main goals of the talk:

- Explain first hints (and challenges) in understanding geometry behind gravity amplitudes

- Convince you that the UV properties of gravity are “good”

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- Explain first hints (and challenges) in understanding geometry behind gravity amplitudes

- Convince you that the UV properties of gravity are “good”

More modest goal:

- At least convince you that the UV properties of gravity are not well understood and deserve more attention

Outline

- Puzzles about gravity in the UV
- Hidden symmetry in SYM and the Amplituhedron
- Explorations of gravity towards the UV
- Gravity amplitudes from the UV

Puzzles about gravity in the UV

Gravity as an EFT

- Non-renormalizable but very fine EFT

$$\mathcal{L} = M_{pl}^2 R + c_2 R^2 + \cdots + c_{n,k} D^{2k} R^n \qquad c_{n,k} \sim 1/\Lambda^{2k+n-4}$$

- EFT totalitarian principle:

"everything that is allowed is compulsory"

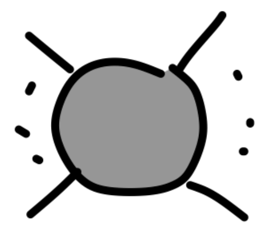
- Counterterms tied to divergences

- Check by explicit calculation

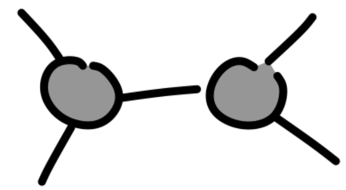
On-shell methods

- Factorization and unitarity as basic principles

$p_i^2 \to 0$

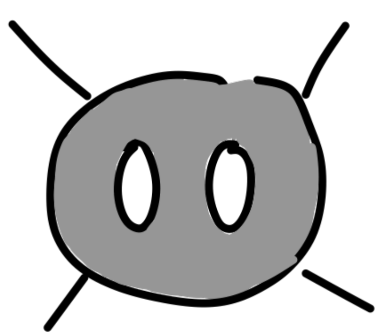


$\to$

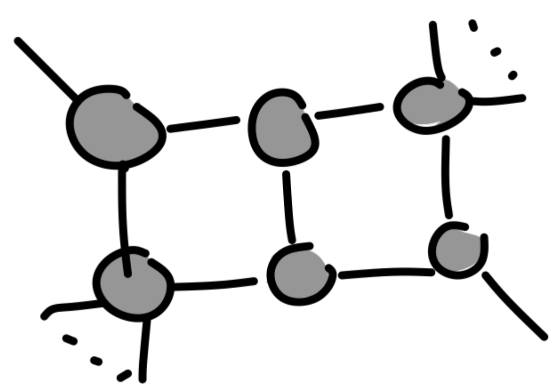


$\Rightarrow A^{\text{tree}}$

$\{(\ell_i - q_j)^2 = 0\}_{i,j}$ going on-shell



$\to$



$\Rightarrow A^{\text{loop}}$

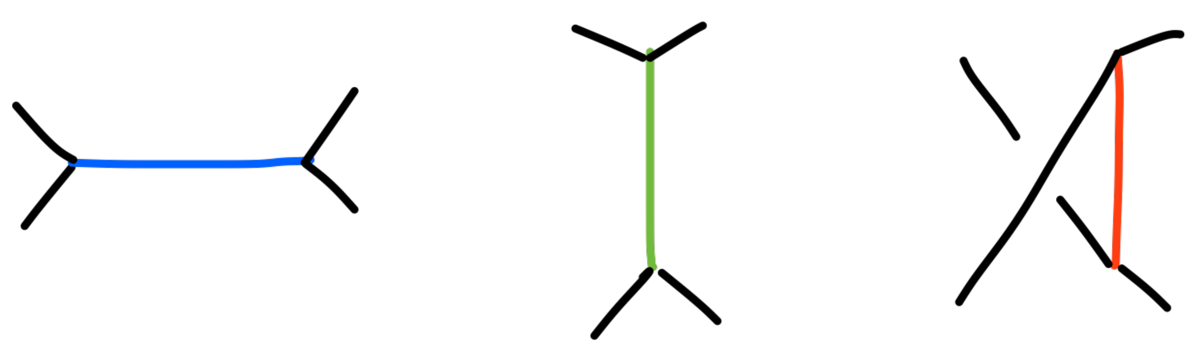
Double copy

- Gravity = Yang-Mills x Yang Mills
- Double copy of amplitudes

$$\sum_i \frac{n_i c_i}{P_i} \longrightarrow \sum_i \frac{n_i \tilde{n}_i}{P_i}$$

$$c_s + c_t + c_u = 0$$

$$n_s + n_t + n_u = 0$$



Progress over last decade

N \ L	1	2	3	4	5
0	$0/-$	∞/R^3	$-$	$-$	$-$
4	$0/-$	$0^{D=5}/R^4$	$0/R^4$	∞/D^2R^4	$-$
5	$0/-$	$0/-$	$0/-$	$0/D^2R^4$	$-$
8	$0/-$	$0/-$	$0/-$	$0/-$	$0/-$

- Counterterm available respecting known symmetries

Progress over last decade

N \ L	1	2	3	4	5
0	0 / –	∞ / R^3	–	–	–
4	0 / –	$0^{D=5}$ / R^4	0 / R^4	∞ / D^2R^4	–
5	0 / –	0 / –	0 / –	0 / D^2R^4	–
8	0 / –	0 / –	0 / –	0 / –	0 / –

- Counterterm available respecting known symmetries
- Only divergences in four dimensions

One loop finiteness of gravity

- Evanescent counterterm

in $D=4$

$$E_4 = (\mathrm{Riem})^2 - 4(\mathrm{Ric})^2 + R^2 = d\Omega$$

- Divergence not numerically zero

$$\mathcal{M}^{\text{1-loop}}\Big|_{\text{div}} = -\frac{1}{(4\pi)^2}\frac{1}{360\,\epsilon}\frac{a+c}{2}\mathcal{M}_{R^2}$$

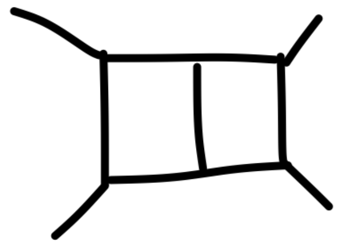
- Related to “trace anomaly”

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{1}{360}\left(a E_4 + c W^2\right) + \cdots$$

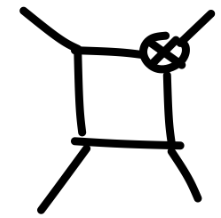
$E_4 = W^2$ on-shell

11

- Contamination at two loops



$$\sim (\mu_R^2)^{2\epsilon}\,\frac{c_1}{\epsilon}$$



$$\sim (\mu_R^2)^{\epsilon}\,\frac{c_2}{\epsilon}$$

$$\mathcal{M}^{\text{2-loop}} = (c_1 + c_2)\,\frac{1}{\epsilon} + (2c_1 + c_2)\log \mu_R^2 + \cdots$$

- Divergence and scale dependence disconnected

$$\mathcal{M}^{\text{2-loop}}_{\text{grav}} = \left(\frac{1}{\epsilon}\,\frac{209}{24} - \frac{1}{4}\log \mu_R^2 \right) \mathcal{M}_{R^3} + \cdots$$

$$\mathcal{M}^{\text{2-loop}}_{\mathcal{N}=1} = \left(\frac{1}{\epsilon}\,\frac{341}{32} - \underline{\underline{0}}\,\log \mu_R^2 \right) \mathcal{M}_{R^3} + \cdots$$

- Simple scale dependence $\sim \dfrac{N_B - N_F}{8}$!

Simplest example

- Gravity minimally coupled to a Weyl fermion is finite at two loops!

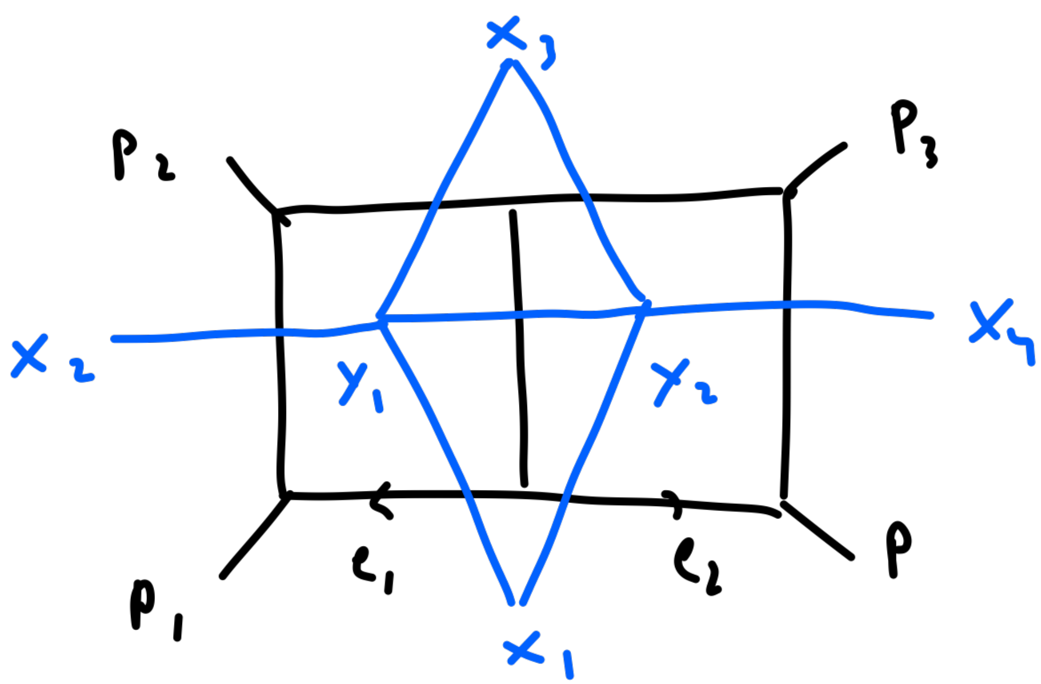
- Despite existing counterterm compatible with all <u>known</u> symmetries...

- If EFT lore applies, there must be a hidden symmetry. And it should be quite generic.

Hidden symmetry in SYM and the Amplituhedron

Hidden symmetry in SYM

- Dual Conformal Invariance (DCI)

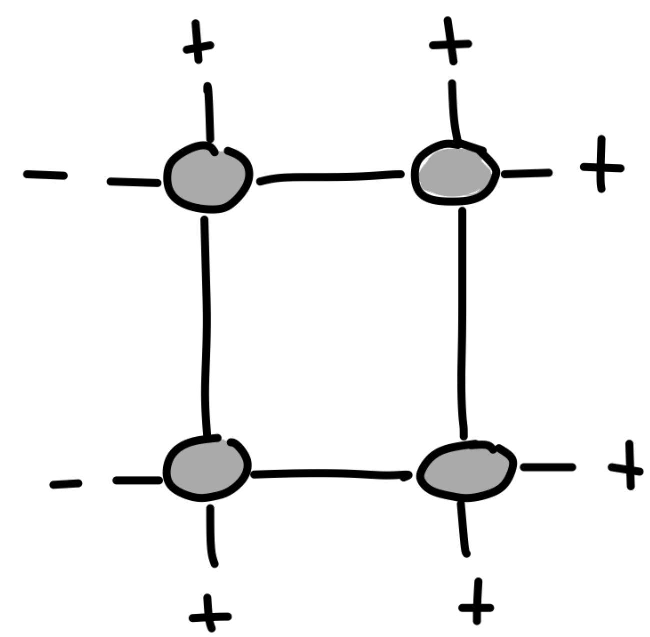


$$p_i = x_{i+1} - x_i$$

- Hidden symmetry of $\mathcal{N}=4$ planar SYM integrands

$$\delta x_i^\mu = \frac{1}{2} x_i^2 b^\mu - (x_i \cdot b) x_i^\mu$$

- DCI ⟷ analytic structure of the integrand
  - Logarithmic singularities $\cancel{\frac{dx}{x^2}}$
  - No poles at infinity $\cancel{\frac{dz}{z}}$ $\ell \propto z$
- Homogeneous cuts
  - Spurious singularities
  - Wrong helicity assignment



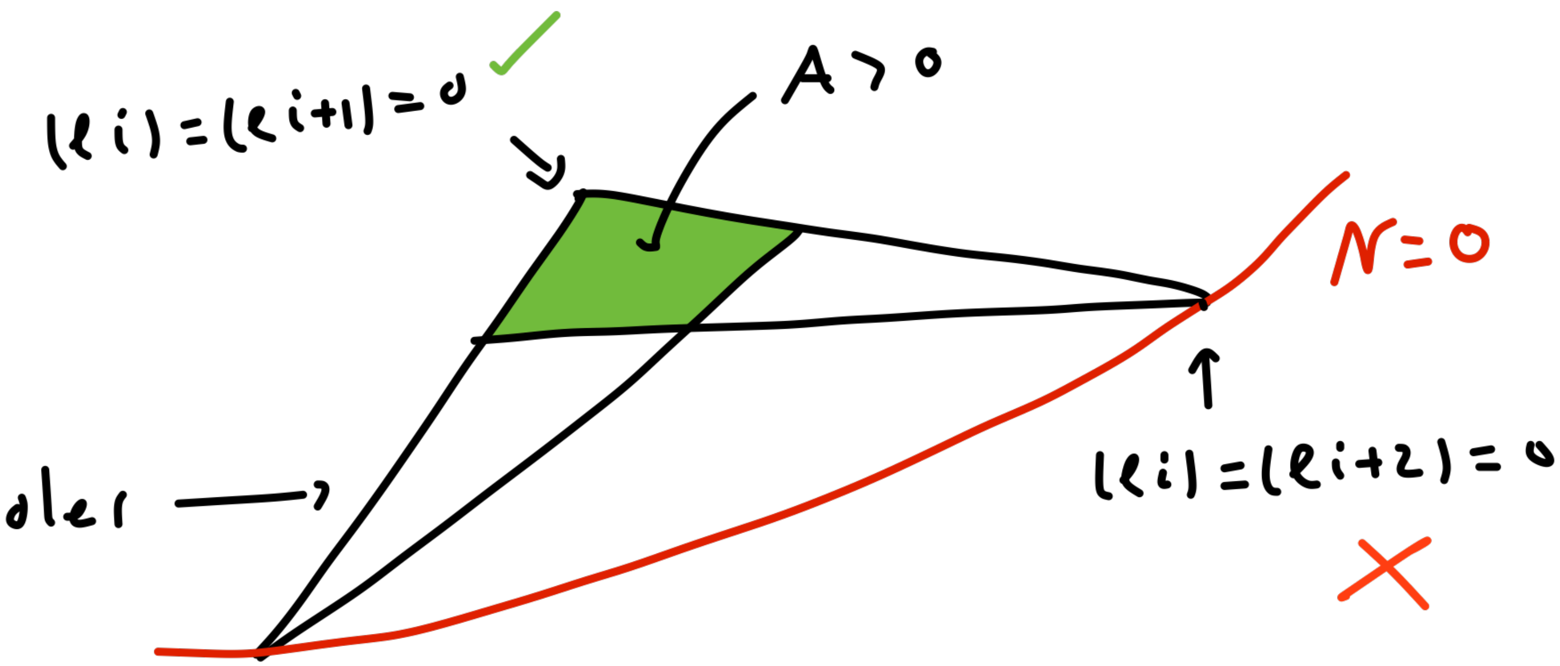
- The two together fix the integrand completely

Dual Amplituhedron

- Amplitude = Volume of “positive space”

$$A = \frac{\mathcal{N}}{(\ell \cdot i)\cdots(\ell \cdot n)}$$

local poles



- Zero conditions ensure poles at right boundaries

Homogeneous cuts $\rightarrow$ Fix $\mathcal{N}$

Non-planar $\mathcal{N}=4$ SYM

- Non-planar integrand not well-defined (labels)
- Can still define amplitude as a sum over diagrams

$$A = \sum_i a_i \, c_i \, LS_i \int I$$

color

rational

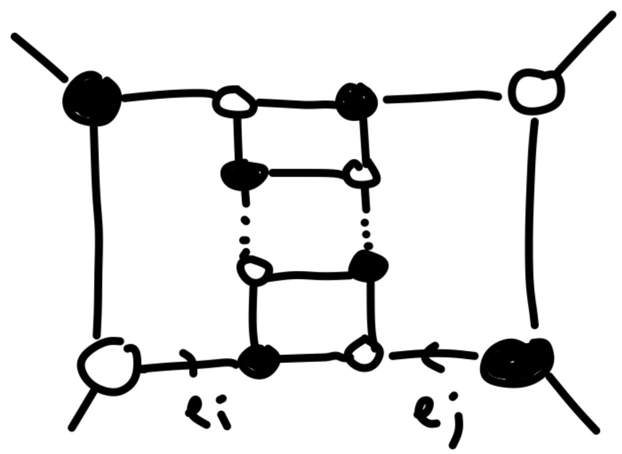
log sings
no pole at ∞

- Homogeneous cuts fix all coefficients a_i
- Considered evidence for non-planar Amplituhedron

Explorations of gravity towards the UV

Gravity analog ?

- Gravity amplitudes certainly have poles at infinity
- In fact they can have multiple poles at infinity



$$\sim (\ell_i \cdot \ell_j)^{2(L-3)}$$

- Homogeneous cuts do not contain enough info

Q: What else is special about gravity?

0. Tree level

- Large z behavior under BCFW deformation

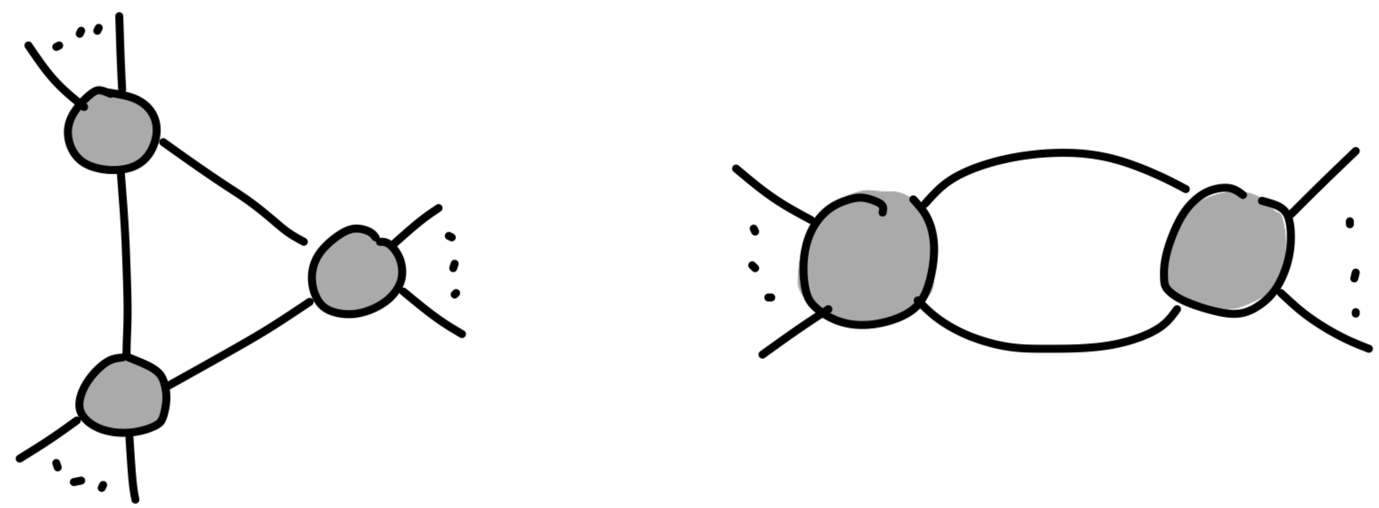
$$\lambda_i \to \lambda_i - z\lambda_j \qquad \tilde{\lambda}_j \to \tilde{\lambda}_j + z\tilde{\lambda}_i$$

$$A_n \overset{z\to\infty}{\sim} \frac{1}{z} \qquad M_n \overset{z\to\infty}{\sim} \frac{1}{z^2}$$

- z^{-1} scaling required for recursion
- However z^{-2} required for factorization
- Does not depend on number of legs

1. One loop

- Tree level behavior feeds into 1 loop through cuts



- "No triangle" in $\mathcal{N}=8$ supergravity
- Bubble cuts directly related to divergence
- Role of SUSY is minor, cancellations generic

2. Two loops

- Maximal vs. minimal cuts

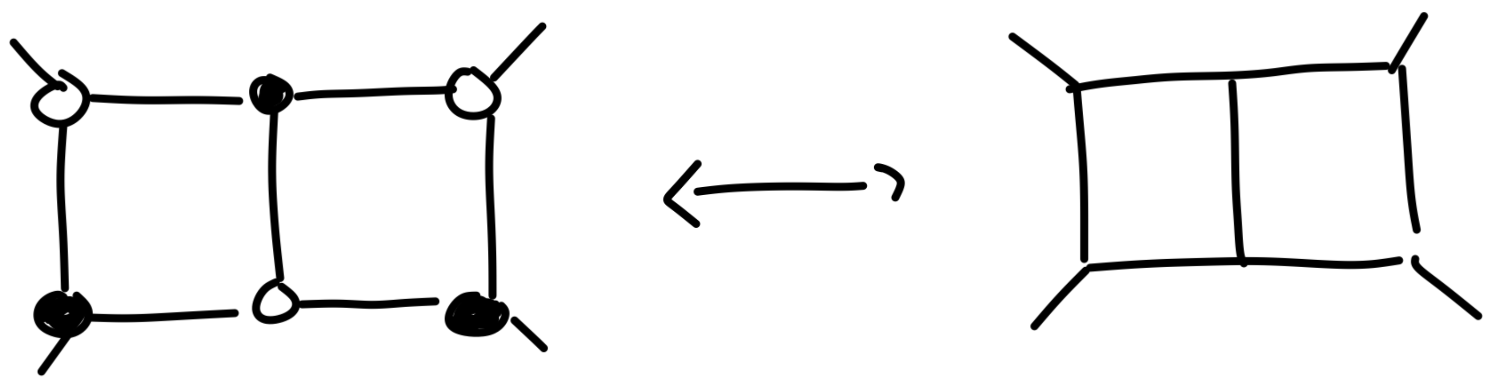
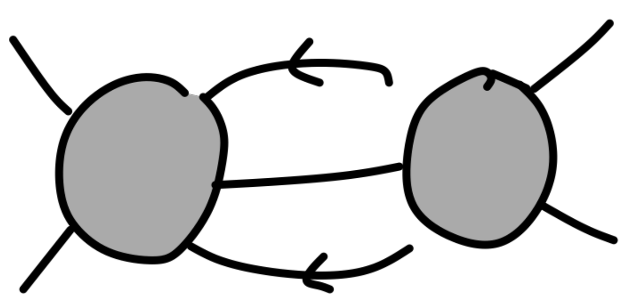


diagram p.c.
cancellations unlikely

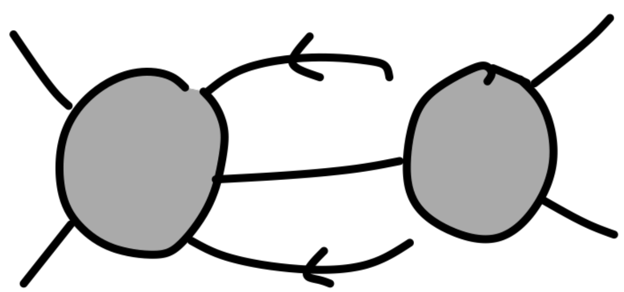


$$= \sum \text{diag}$$

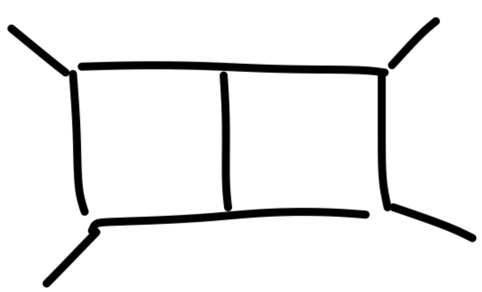
global info.
cancellations likely

- 2-loop d-dimensions (2017) - no cancellations

$$\ell_i^{cut} \to \ell_i^{cut} + z\, q_i \,, \quad q_i^2 = 0 \quad q_i \cdot \ell_i = 0$$



$$\sim \quad \frac{1}{z^4} \,, \quad z^0$$

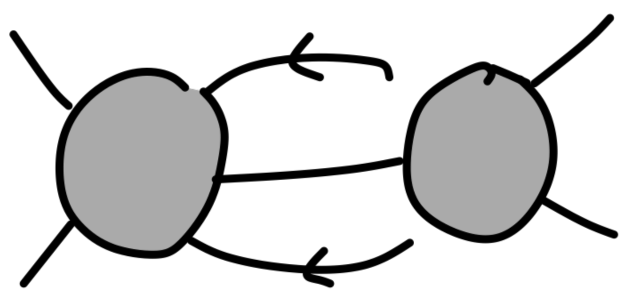


$$\sim \quad \frac{1}{z^4} \,, \quad z^0$$

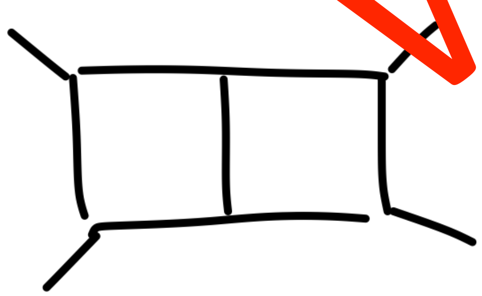
max SUGRA	half-max SUGRA

- 2-loop d-dimensions (2017) - no cancellations

$$\ell_i^{cut} \to \ell_i^{cut} + z\, q_i \,, \quad q_i^2 = 0 \quad q_i \cdot \ell_i = 0$$



$$\sim \frac{1}{z^4} \,, \qquad z^0$$



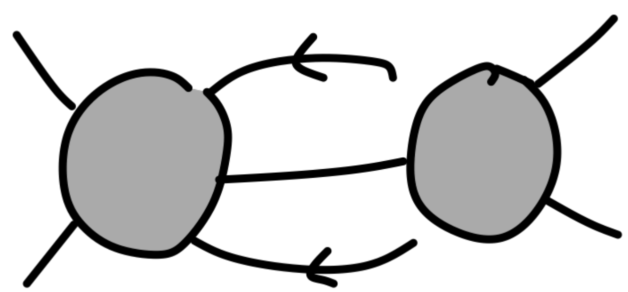
$$\sim \frac{1}{z^4} \,, \qquad z^0$$

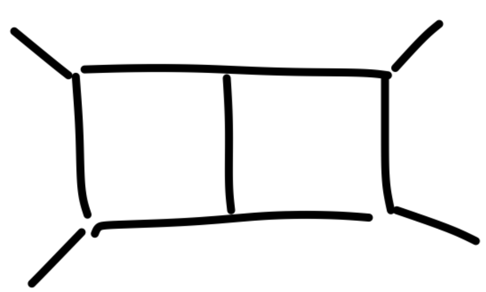
max SUGRA	half-max SUGRA

- 2-loop 4-dimensions (2018) - yes cancellations!!

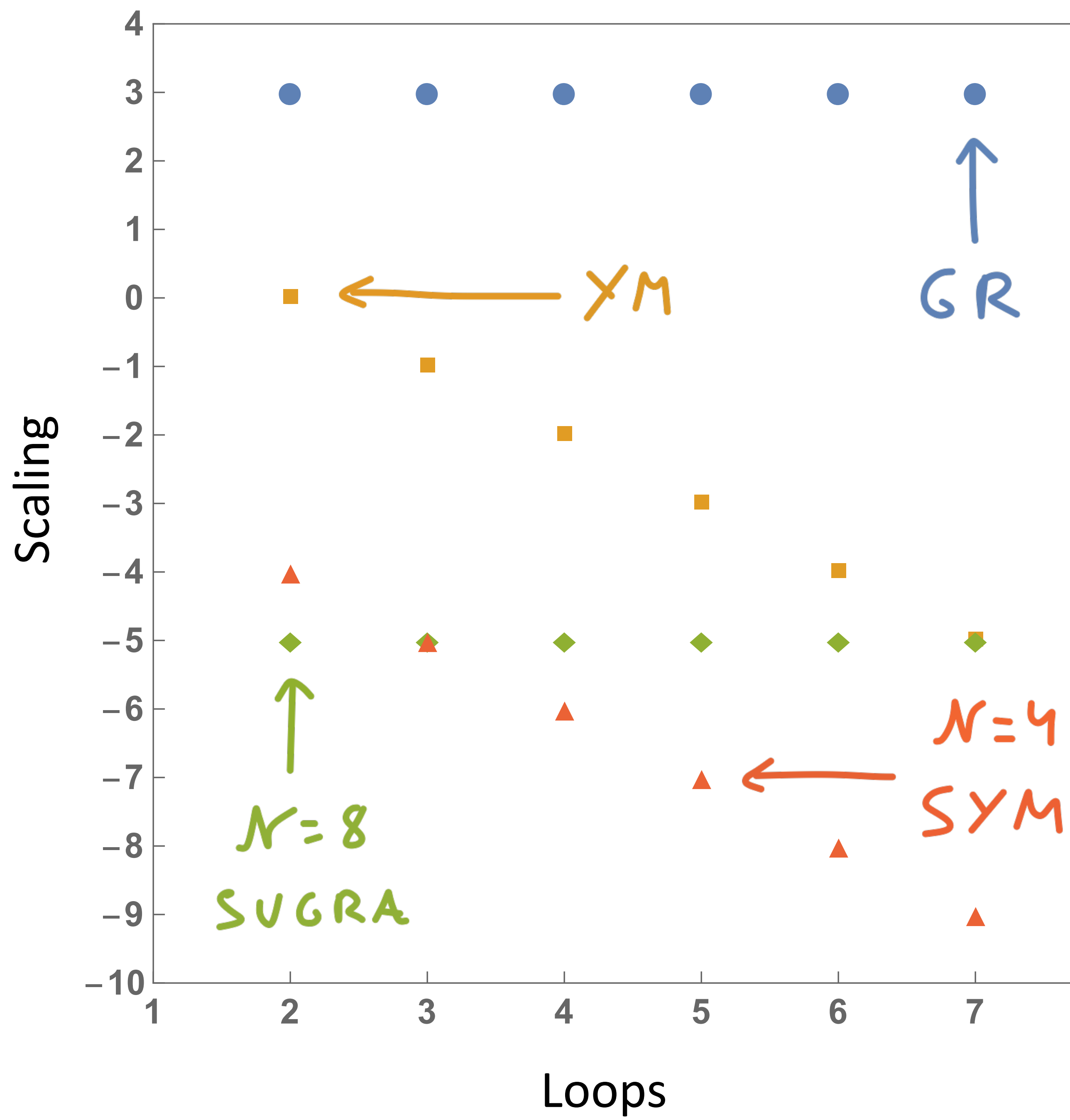
$$\ell_i = \lambda_{\ell_i}\left(\bar{\eta} + c_i z \bar{\chi}\right) \qquad i = 1, \dots, L$$

$$\ell_{L+1} = \xi \, \frac{-p^2}{2\xi \cdot p} \qquad \xi = \lambda_{L+1}\left(\bar{\eta} + c_{L+1} z \bar{\chi}\right)$$

 $\sim \dfrac{1}{z^5}$

 $\sim \dfrac{1}{z^4}$

7. Higher loops

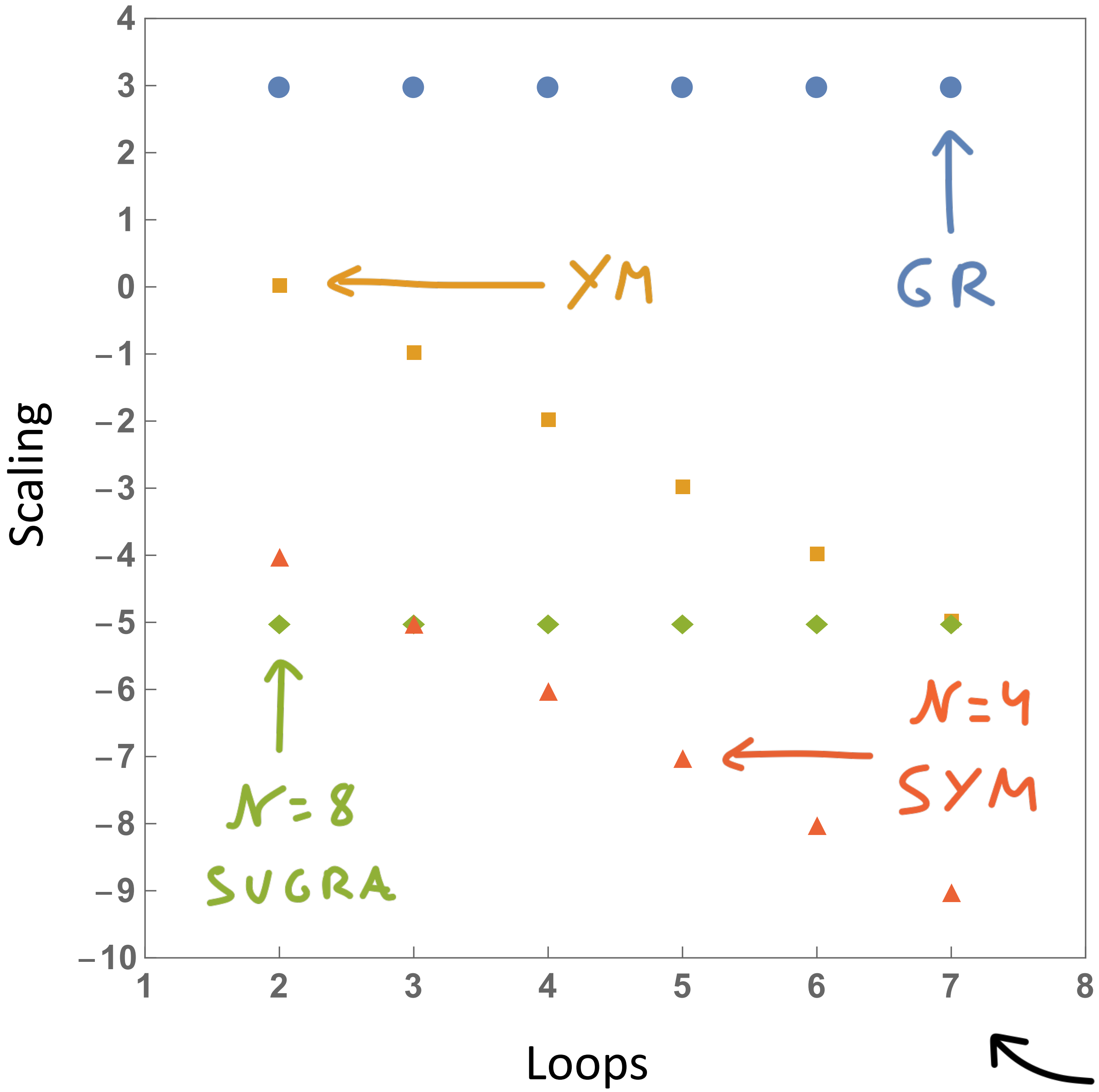


Yang-Mills scaling = diagram p.c. dominated by planar (WYSIWYG)

Nontrivial cancellation in gravity
Compare to worst diagram!

$$z^{2(L+1)-\mathcal{N}}$$

7. Higher loops



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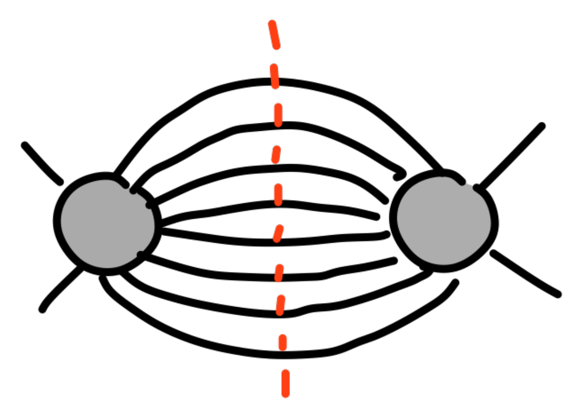


$L = 7$

GR !

Tree origin

- Good UV behavior still comes from trees
- No cancellations between different cont. to cut
- Cut scaling   multi-line shift

$$\ell_i = \lambda_{\ell_i}\left(\bar{\eta} + c_i z \bar{\chi}\right) \qquad i = 1, \dots, L$$

$$\ell_{L+1} = \xi \frac{-P^2}{2\xi\cdot P} \qquad \xi = \lambda_{L+1}\left(\bar{\eta} + c_{L+1} z \bar{\chi}\right)$$

More comments later…

A: Improved UV behavior in cuts is special in gravity

Analog of logarithmic singularities and no poles at ∞ in $\mathcal{N}=4$?

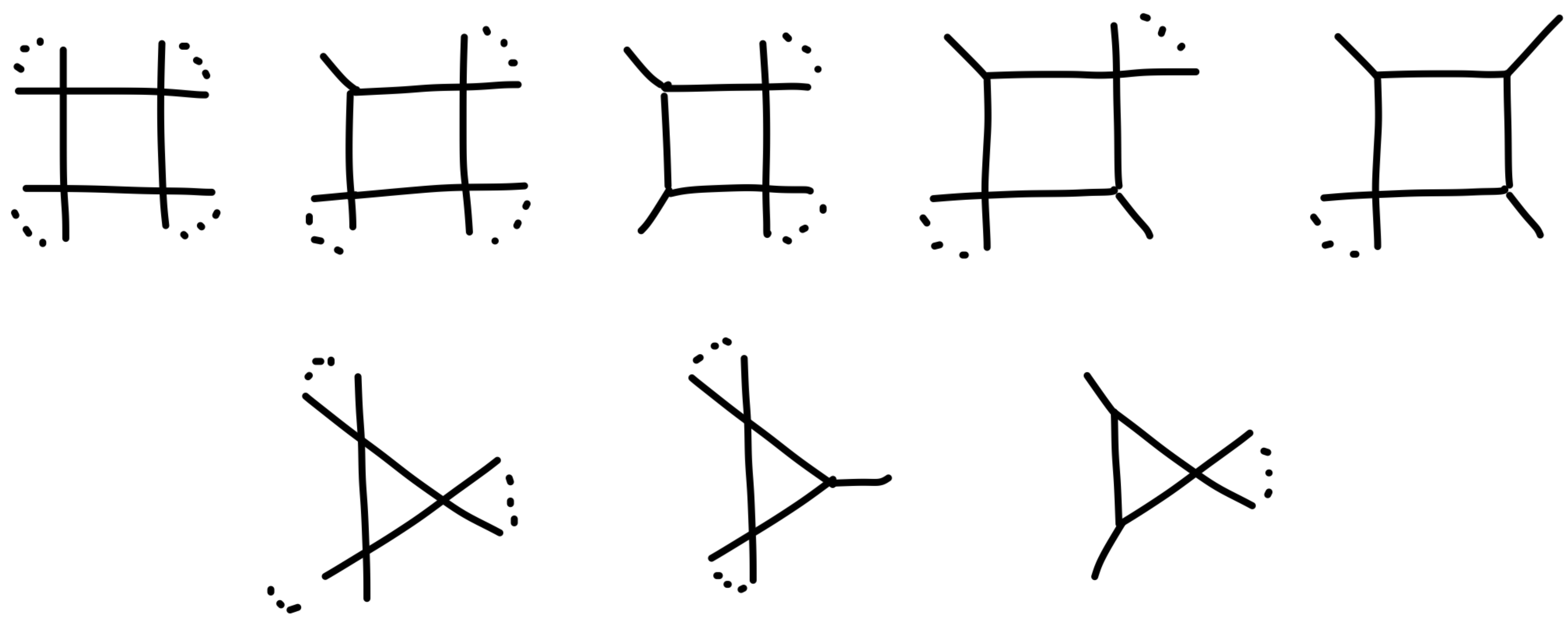
Gravity amplitudes from the UV

Tentative Program

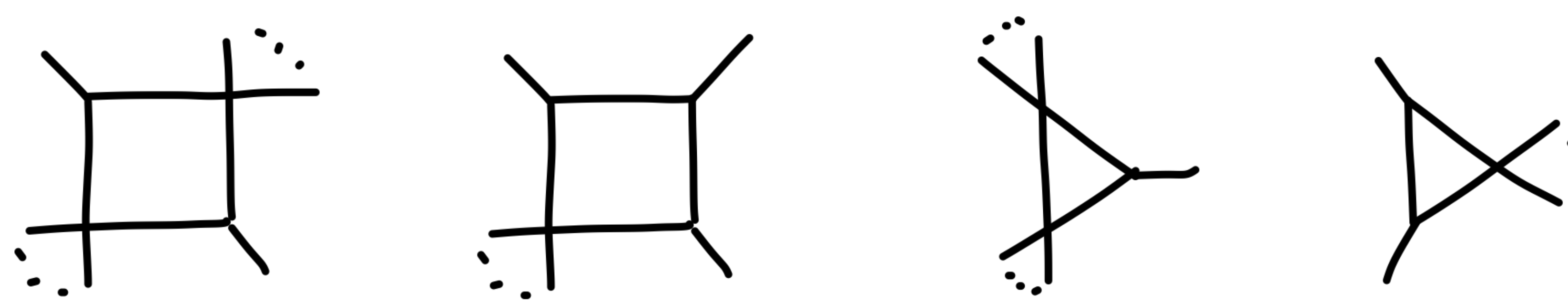
- Start with an ansatz
- Constrain using forbidden cuts
- Constrain using improved UV behaviour
- Is the amplitude uniquely fixed?

One-loop n-pt MHV

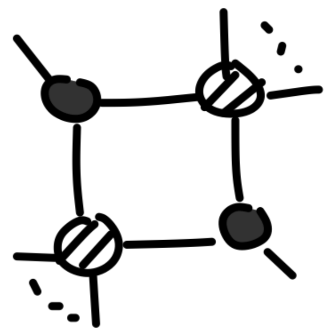
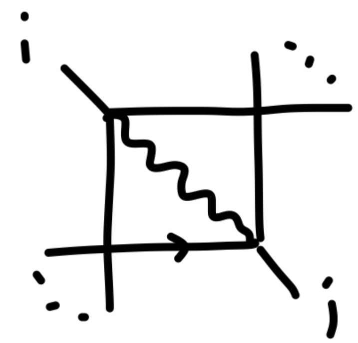
- Initial ansatz: triangle p.c $\mathcal{N} = 8$



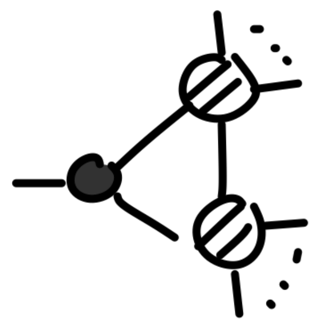
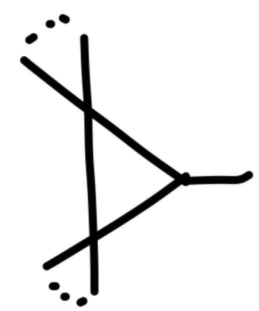
- Forbidden cuts: first pass



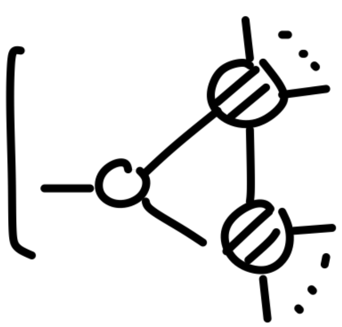
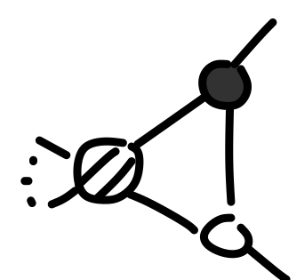
- Forbidden cuts: second pass

$0 =$  $\longrightarrow$ 

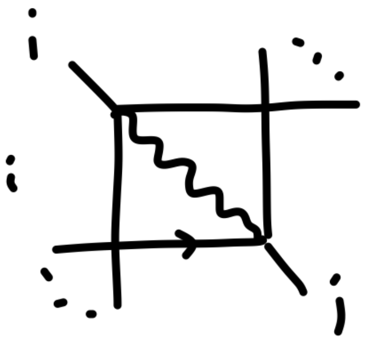
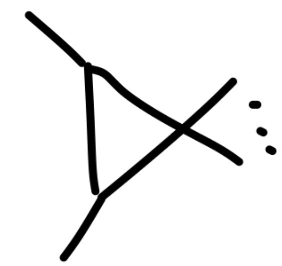
$s_{ij}\,(\ell-\ell_*)^2$

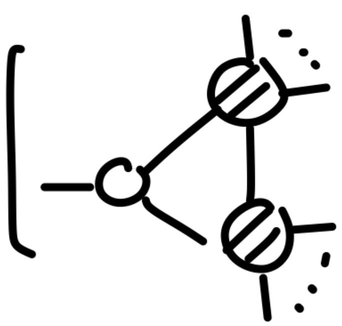
$0 =$  $\longrightarrow$  $\longrightarrow 0$

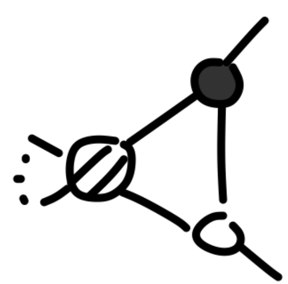
- Improved UV behavior

$0 = \mathrm{Res}\Big[$  $, \infty\Big] = \mathrm{Res}\Big[$  $, \infty\Big]$

- Left-over ansatz like in SYM

$A = \sum_i q_i \, LS_i$  $+ \sum_j c_j^{tri}$ 

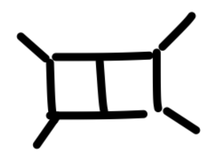
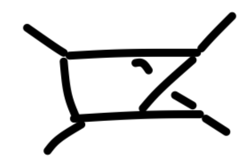
$0 = \mathrm{Res}\Big[$  $, \infty\Big] \to \sum q_i \, LS_i = 0$ GRT: 2 choices

$0 = \mathrm{Res}\Big[$  $, \infty\Big] \to c_j^{tri} = \sum \pm LS_i$

- Amplitude fixed up to overall scale!

Two-loop four point

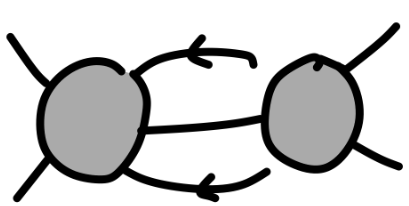
- Initial ansatz

$A = s t u \, M^{tree} \Big[\sum_i$  N +  $\tilde{N} \Big]$

- Usual power counting: $2x$ powers for $x+4$ -gon

$$N = a s + b t \, , \quad \bar{N} = \sum c_i \{ s, t, \ell_i \cdot k_j, \ell_i^2 \}$$

- UV scaling alone fixes the whole amplitude

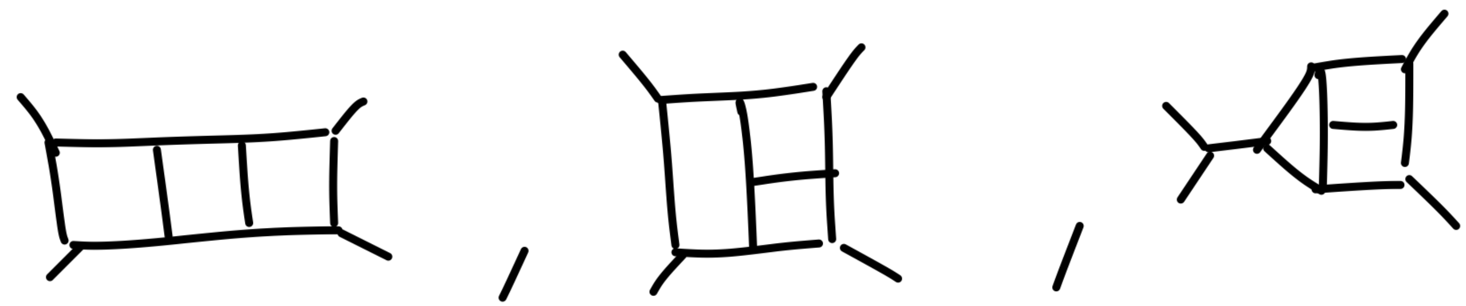
 $\sim \frac{1}{z^5}$

$$A_{ns} \sim \frac{a}{z^2} + \frac{b}{z^3} + \frac{c}{z^4} + \cdots$$

$$a = b = c = 0$$

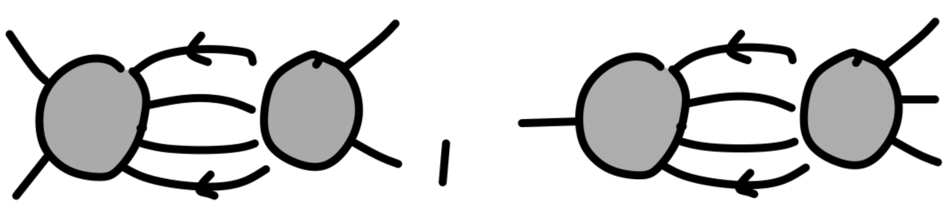
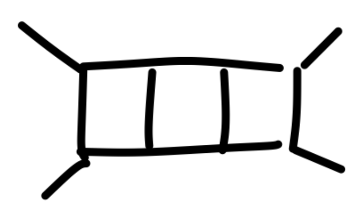
Three-loop four point

- Initial ansatz



$+$ non-planar

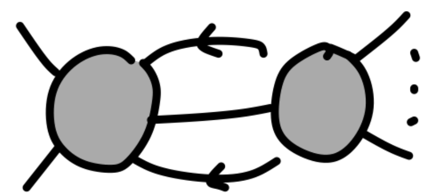
- UV scaling fixes everything but ladders

 $\sim \frac{1}{z^5}$  $\sim \frac{1}{z^6}$

- Working on using forbidden cuts to fix this

Challenges ahead

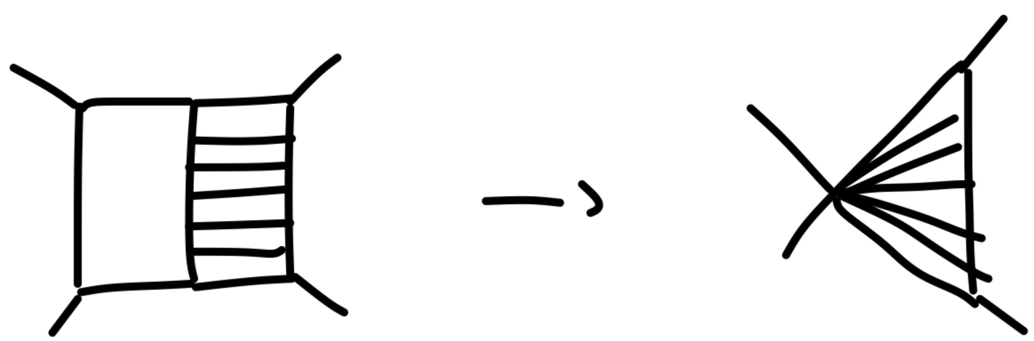
- Order of poles at infinity that grows with n



$\sim \quad z^{n-9}$

(But no divergence)

- Polynomial/rational terms unfixed by 4d cuts



- Still have not defined an integrand...

Unanswered questions

- Can trees be fixed by requiring large z scaling?

- Relation to UV divergences after integration

- Geometric structure

$\mathcal{N}=4$ SYM

Amplituhedron $\longrightarrow$ DCI $\longrightarrow$ Log sing. No poles at

$\mathcal{N}=8$ SUGRA

???? $\longleftarrow$ Symmetry? $\longleftarrow$ Improved UV scaling

Summary

- UV behavior of gravity is surprising
- Hints of an unknown symmetry
- UV behavior strongly constrains (fixes!) amplitudes
- Still a lot to be learned from gravity in the UV