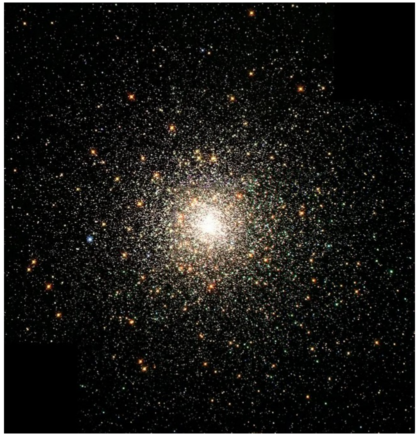
N-body project

Crédit : NASA

The N-body problem : what is it ?

- Dynamical interaction of N bodies along the time
- Interaction are a set of forces, bodies can be more or less complex
- Follow the trajectories, rotation and deformation of the bodies

- Molecular dynamics
- Astronomy (cosmology, planets, galaxy, stellar cluster…)
- Plasmas
- ...



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The N-body problem : how to solve numerically ?

- Euler method
- **Leapfrog method**
- Runge-Kutta (RK1, RK2, RK4)
- Fast-Multiple method, Ahmad-Cohen, Barnes-Hut ….

The N-body problem : Leapfrog integrator

- Start from initial positions $\mathbf{r_0}$ and speeds $\mathbf{v_0}$ at time t_0
- For t_{i+1}

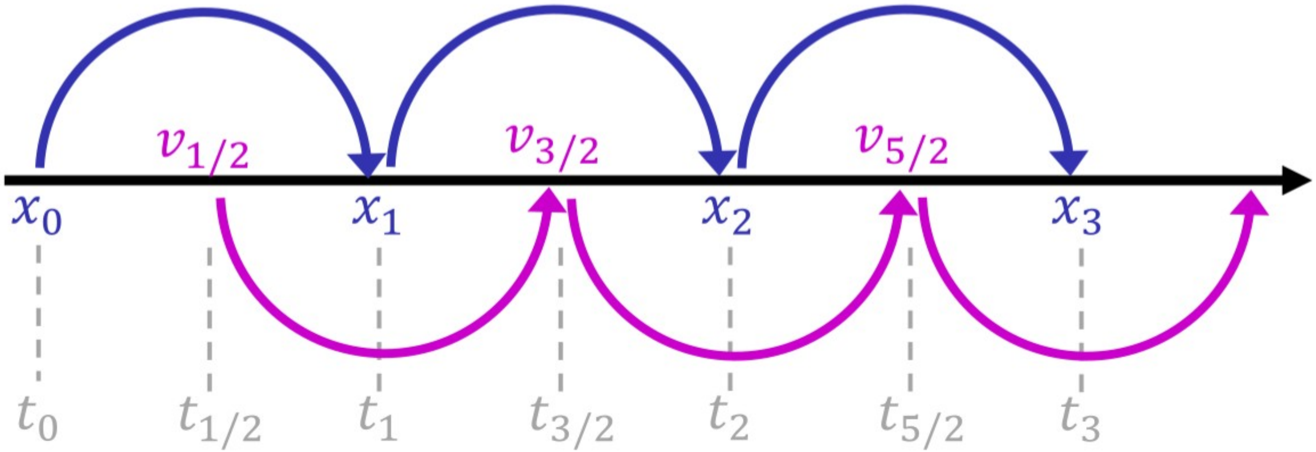
Leapfrog

$$a_i = A(r_i)$$

Integer steps version

$$v_{i+0.5} = v_{i-0.5} + a_i \Delta t$$
$$r_{i+1} = r_i + v_{i+0.5} \Delta t$$

$$r_{i+1} = r_i + v_i \Delta t + 0.5 a_i \Delta t^2$$
$$v_{i+1} = v_i + 0.5(a_i + a_{i+1}) \Delta t$$



The N-body problem for M2 CompuPhys

- The universal law of gravitation

$$m_i \vec{a}_i = \sum \vec{F}_i \qquad\qquad F_{ij} = G m_i \sum_{j \neq i} m_j \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^3}$$

- For simplicity, let's set G = 1 and $m_i = 1/N_p$
- Initial positions inside a sphere of radius < 1
- Initial speeds as a rigid rotation along the z-axis $\vec{v}_i = \vec{w} \otimes \vec{r}_i = (-y; x; 0)$

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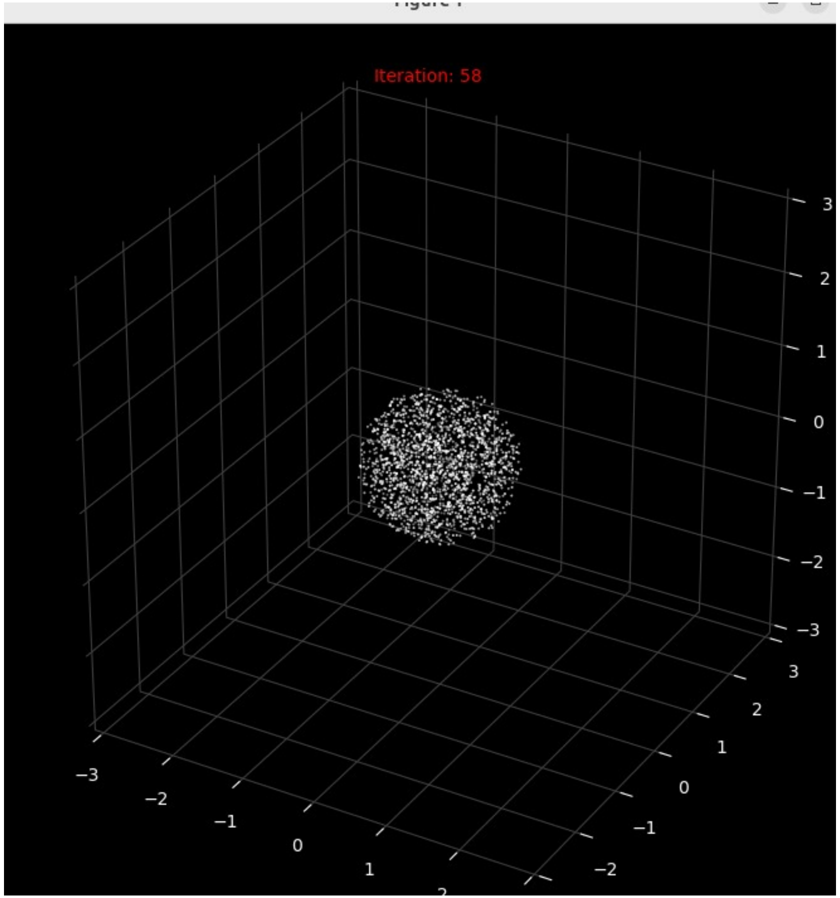
- **CAUTION** when $\vec{r}_i \approx \vec{r}_j \Rightarrow \vec{a}_i \rightarrow \infty$
- (Quadratically) Add a smoothing length to the denominator $\varepsilon \ll 1$
- Start with N_p = 1000, N_t = 1000, dt = 0.005, ε = 0.005

The N-body problem for M2 CompuPhys

- Write first a sequential code !
  - Save the position and speeds for each iterations
- Parallelize the code, try different places/strategies
- Study the performances as a function of various parallelization technics

The N-body problem for M2 CompuPhys : few tips

- Optimize the sequential code
  - Be careful on loops order
  - try compiler optimization
  - …..
- Check conservation of fundamental quantities
  - Total mecanic energy: E_p+E_c = cte
  - Total angular momentum: $\vec{L}=cte$
  - Center of mass position: $\overrightarrow{COM}=cte$
- Compare parallel code versus sequential results
- Check effects of Np, Nt, dt etc...
- You can use CompuPhys server !



The N-body problem for M2 CompuPhys : presentation

- Give a general introduction to the N body problem
- Present the leapfrog integrator and its implmentation in the sequential code
- Present the parallelization choice you made, you **have to present the speed-up curve and Amdahl’s law**
- Present the different tests you did
- Anything you find relevent/challenging
- You **need** to gives all of your codes (Github access works)
- 15 min for talk+few questions