2013年3月7日
13:33

extension $\{1, 9, 25, 49, 81\}$

intension $\{n^2 \mid 0 < n < 10,\ n \text{ odd}\}$

```
[sqr n | n <- [1..9], odd n]
do  n <- [1..9]
    odd n
    return (sqr n)
```

$$[\,e \mid \varepsilon\,] = [e]$$

$$[\,e \mid b\,] = \text{if } b \text{ \underline{then} } [e] \text{ \underline{else} } [\,]$$

$$[\,e \mid a \leftarrow x\,] = \text{map } (\lambda a \to e)\ x$$

$$[\,e \mid q, q'\,] = \text{concat } [\,[e \mid q'] \mid q\,]$$

$$[\,e \mid p \leftarrow x\,] = \text{concat } (\text{map } h\ x) \text{ \underline{where}}$$

$$h\ p = [e]$$

$$h\ \_ = [\,]$$

<u>class</u> Monad m => MonadZero m <u>where</u>
    mzero :: m a

$$[\,e \mid a \leftarrow \text{mzero}, q\,] = \text{mzero}$$

join mzero = mzero

---

class Monad m => MonadPlus m where

class Monad m ⇒ MonadPlus m where
    mplus :: m a → m a → m a

[e | a ← x `mplus` y, q]
    = [e | a ← x, q] `mplus` [e | a ← y, q]

join (x `mplus` y) = join x `mplus` join y

mzero ↦ $\emptyset$ mplus ↦ $\uplus$

"ringads"

{} ⦃⦄ [] ?   Boom Hierarchy

ACI AC A .

"(a,k) is an algebra for monad m"

$k :: m\ a \to a$

$k \cdot return = id$

$k \cdot join = k \cdot map\ k$

eg (m a, join)

algebra for ringad if also

$k\ \emptyset = i_\oplus$

$k\ (x \uplus y) = k\,x \oplus k\,y$

— ie $\oplus$/

$$[e \mid q]^{\oplus} = \oplus / [e \mid q]$$

eg $[e \mid \varepsilon]^{\oplus}$

$= \oplus / [e \mid \varepsilon]$

$= \oplus / (\text{return } e)$

$= e$

$$[e \mid \varepsilon]^{\oplus} = e$$

$$[e \mid b]^{\oplus} = \underline{\text{if}}\ b\ \underline{\text{then}}\ e\ \underline{\text{else}}\ i_{\oplus}$$

$$[e \mid p \leftarrow x]^{\oplus} = \oplus / (\text{map } h\ x)\ \underline{\text{where}}$$

$$h\ p = e$$

$$h\ \_ = i_{\oplus}$$

$$[e \mid q, q']^{\oplus} = [[e \mid q']^{\oplus} \mid q]^{\oplus}$$

"$\varphi :: m\,a \to n\,a$ is a monad morphism"

$$\varphi \cdot \text{return}_m = \text{return}_n$$

$$\varphi \cdot \text{join}_m = \text{join}_n \cdot \varphi \cdot \text{map}\ \varphi$$

"ringad morphism" if also

$$\varphi\ \emptyset_m = \emptyset_n$$

$$\varphi\ (x \uplus_m y) = \varphi\, x \uplus_n \varphi\, y$$

x a bag $\Rightarrow$

$$\text{bag2set}\left([\, a^2 \mid a \leftarrow x,\ \text{odd } a \,]_{\text{Bag}}\right)$$

$$= [\, a^2 \mid a \leftarrow \text{bag2set } x,\ \text{odd } a \,]_{\text{set}}$$

write $[\, a^2 \mid a \leftarrow x,\ \text{odd } a \,]_{\text{set}}$

$$[\, a+b \mid a \leftarrow [1,2,3],\ b \leftarrow \{4,4,5\} \,]_{\text{set}}$$

$$= \{5,6,7,8\}$$

$$[\, \text{makeInvoice } c\ o \mid c \leftarrow \text{customers},\ o \leftarrow \text{orders},\ c.\text{id} == o.\text{cid} \,]$$