Institut for Matematik og Datalogi
Syddansk Universitet, Odense

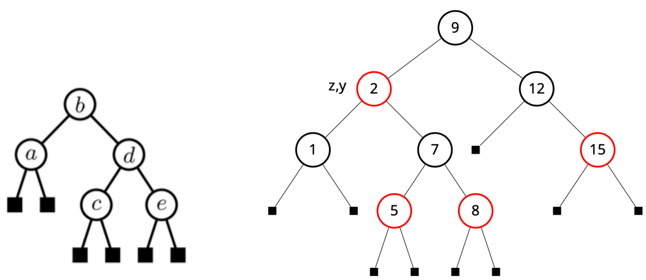
27. March 2025

Exercise Sheet Week 7

DSK814

A. Solve in the practice sessions

1. Specify a coloring of the nodes in the left tree below that makes it a red-black tree.



2. Describe what happens to the right tree above when key 2 is deleted.

3. Consider the hash function $h(x) = (7x + 4) \bmod 11$ and the hash table below. Whenever a collision occurs, use linear hashing. Describe what happens if you insert the keys $18, 26$.

0	1	2	3	4	5	6	7	8	9	10
67	20	17		33		16	2			15

4. Cormen et al., 4. Edition: Exercise 11.2-2 (page 281) [Cormen et al., 3. Edition: Exercise 11.2-2 (page 261)].

   Consider the hash function $h(k) = k \bmod 9$ and a hash table with 9 slots. Demonstrate what happens upon inserting the keys $5, 28, 19, 15, 20, 33, 12, 17, 10$ with collisions resolved by chaining.

5. Cormen et al., 4. Edition: Exercise 11.4-1 (page 300) [Cormen et al., 3. Edition: Exercise 11.4-1 (page 277)].

   Consider inserting the keys $10, 22, 31, 4, 15$ into a hash table of length $m = 11$ using open addressing. Illustrate the result of inserting these keys using linear probing with $h(i, k) = (k+i) \bmod m$ and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \bmod (m-1))$.

6. Consider the hash function $h(x) = x \bmod 16$ and $c_1 = c_2 = 1/2$. Whenever a collision occurs, use quadratic hashing. Describe what happens when 71 is inserted in the hash table below.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
32				20			23	7		39					

7. Consider the hash function $h(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod 11$ where $h_1(k) = k \bmod 11$ and $h_2(k) = 1 + (k \bmod 10)$. Describe what happens when 18 is inserted with open addressing and double hashing in the hash table below.

			14				3		41	
0	1	2	3	4	5	6	7	8	9	10

B. Solve at home before tutorial in week 6

1. Cormen et al., 4. Edition: Exercise 13.4-4 (page 354) [Cormen et al., 3. Edition: Exercise 13.4-3 (page 330)].

   In Cormen et al., 4. Edition: Exercise 13.3-2 (page 364) [Cormen et al., 3. Edition: Exercise 13.3-2 (page 322)], you found the red-black tree that results from successively inserting the keys $41, 38, 31, 12, 19, 8$ into an initially empty tree. Now show the red-black trees that result from the successive deletion of the keys in the order $8, 12, 19, 31, 38, 41$.