

Calculation of Satellite Position from Ephemeris Data

Table A3-1. Representation of GPS Broadcast Ephemeris

Time Parameters	
t_{0e}	Reference time, ephemeris parameters (s)
t_{0c}	Reference time, clock parameters (s)
a_0, a_1, a_2	Polynomial coefficients for clock correction (bias (s), drift (s/s), drift rate (aging) (s/s ²))
Keplerian Parameters	
\sqrt{A}	Square root of the semi-major axis (m ^{1/2})
e	Eccentricity (dimensionless)
i_0	Inclination angle at reference time (semicircles)
Ω_0	Longitude of ascending node at reference time (semicircles)
ω	Argument of perigee (semicircles)
\bar{M}_0	Mean anomaly at reference time (semicircles)
Perturbation Parameters	
Δn	Mean motion difference from computed value (semicircles/s)
$\dot{\Omega}$	Rate of change of right ascension (semicircles/s)
\dot{i}	Rate of change of inclination (semicircles/s)
C_{us}	Amplitude of the sine harmonic correction term to the argument of latitude (rad)
C_{uc}	Amplitude of the cosine harmonic correction term to the argument of latitude (rad)
C_{is}	Amplitude of the sine harmonic correction term to the angle of inclination (rad)
C_{ic}	Amplitude of the cosine harmonic correction term to the angle of inclination (rad)
C_{rs}	Amplitude of the sine harmonic correction term to the orbit radius (m)
C_{rc}	Amplitude of the cosine harmonic correction term to the orbit radius (m)

Source: Seeber (2003).

The individual satellite time, t_{SV} , is corrected to GPS system time, t , using:

$$t = t_{SV} - \Delta t_{SV}$$

in which

$$\Delta t_{SV} = a_0 + a_1(t - t_{0c}) + a_2(t - t_{0c})^2 \quad (\text{A3-1})$$

Differentiating Eq. A3-1 with respect to time yields satellite clock drift.

The satellite coordinates in the WGS-84 Cartesian system are computed for a given epoch, t . The time, t_k , elapsed since the reference epoch, t_{0e} , is $t_k = t - \Delta t_{0e}$.

Table A3-2. Calculating Satellite Coordinates from GPS Broadcast Ephemeris

Constants	
$GM = 3.986005 \cdot 10^{14} \text{ m}^3/\text{s}^2$	WGS-84 value for the product of gravitational constant G and the mass of the Earth M
$\omega_e = 7.292115 \cdot 10^{-5} \text{ rad/s}$	WGS-84 value of the Earth's rotation rate
$\pi = 3.1415926535898 \text{ (exactly)}$	
$T = 2\pi / \sqrt{GM/A^3}$	Keplerian Parameters to ECEF Coordinates Satellite orbital period
$n_0 = \sqrt{\frac{GM}{A^3}}$	Computed mean motion
$n = n_0 + \Delta n$	Corrected mean motion
$\bar{M}_k = \bar{M}_0 + nt_k$	Mean anomaly
$E_k = \bar{M}_k + e \sin E_k$	Kepler's equation of eccentric anomaly is solved by iteration. Because of the small eccentricity of GPS orbits ($e < 0.001$), two steps are usually sufficient: $E_0 = \bar{M}$, $E_i = \bar{M} + e \sin E_{i-1}$, $i = 1, 2, 3, \dots$
$\cos v_k = \frac{\cos E_k - e}{1 - e \cos E_k}$	True anomaly
$\sin v_k = \frac{\sqrt{1 - e^2} \sin E_k}{1 - e \cos E_k}$	True anomaly
$\Phi_k = v_k + \omega$	Argument of latitude
$\delta u_k = C_{uc} \cos 2\Phi_k + C_{us} \sin 2\Phi_k$	Argument of latitude correction
$\delta r_k = C_{rc} \cos 2\Phi_k + C_{rs} \sin 2\Phi_k$	Radius correction
$\delta i_k = C_{ic} \cos 2\Phi_k + C_{is} \sin 2\Phi_k$	Inclination correction
$u_k = \Phi_k + \delta u_k$	Corrected argument of latitude
$r_k = A(1 - e \cos E_k) + \delta r_k$	Corrected radius
$i_k = i_0 + \delta i_k$	Corrected inclination
$X'_k = r_k \cos u_k$	Position in the orbital plane
$Y'_k = r_k \sin u_k$	Position in the orbital plane
$\Omega_k = \Omega_0 + (\Omega - \omega_e)t_k - \omega_e t_{0e}$	Corrected longitude of ascending node
$X_k = X'_k \cos \Omega_k - Y'_k \sin \Omega_k \cos i_k$	Earth-fixed geocentric satellite coordinate
$Y_k = X'_k \sin \Omega_k + Y'_k \cos \Omega_k \cos i_k$	Earth-fixed geocentric satellite coordinate
$Z_k = Y'_k \sin i_k$	Earth-fixed geocentric satellite coordinate

ECEF, Earth-centered, Earth-fixed.

Source: Seeber (2003).