# General Observations

Newly added text is highlighted in green in the corrected manuscript.

# Editor comments (XY adapt if necessary)

# Reviewer #1

1. This study developed a robust multi-stage MPC framework to optimize substrate feedings with uncertain influent macronutrients to an agricultural AD plant, and the results were good. It should be interesting to the readership of Bioresource technology, I think the manuscript can be considered for publication after major revision.

Thank you for positively evaluating our research. We are happy that our efforts to progress in the field of biogas technology are recognized.

1. To highlight the innovation of this paper, the authors should clarify the differences and advancements between the model in this paper and previous models in the Introduction section.

Thank you for your remark. We highlighted the differences and advancements of the model used in this study and previously proposed models more clearly in the introduction, cf. lines XY-XY.

1. Lines 163-166: Crop straws are typical agricultural AD substrates and should be considered in this paper.

Thank you for your comment. We recognize that crop straw is an abundant and valuable agricultural residue. Therefore, we considered it as an additional disturbance substrate in case study 1, cf. lines XY-XY, Fig. 4 and Fig. 3.

1. Food waste is also a commonly used raw material in AD plants. Is it applicable to the model established in this paper?

Thank you for your thoughtful inquiry. In principle, the ADM1-R3 is capable to describe anaerobic digestion of multiple anaerobically degradable substrates, including foodwaste. However, due to the usually high lipid content of food waste, the substrate characterization method used in this study is not ideally suitable for foodwaste. Therefore, we decided against considering foodwaste as a substrate.

1. Some words in Figure 1 (e) are not clear. Please revise them for easier reading.

Thank you for your remark. The bottom subfigure 1 (e) is indeed not well visible. Therefore, we increased the figure resolution and font size of the respective figure text.

1. Line 177: "where crude CH include lignin" - what does CH refer to?

Not applicable. CH abbreviates carbohydrates, as introduced in line 130

# Reviewer #2

1. In this paper, the authors address the robust control design of an anaerobic digester (AD) aimed at maximizing biogas production in response to demand from a combined heat and power unit (CHP) and gas storage volumes (GS). The robust control approach is based on a multi-stage nonlinear model predictive control (MS-NMPC), which is applied to a simplified ADM1 model suitable for precise prediction of gas production rates enhanced by slow- and fast-degradable fractions of carbohydrates.

In recent years, several MPC and NMPC designs have been applied to anaerobic digesters (AD), aimed at both single and co-substrate degradation and biogas production. In this paper, the authors applied MS-NMPC aimed at biogas production using multi-substrate feeds, considering uncertain influent macronutrients.

I have several comments that I believe are necessary to address to strengthen the paper's scope and contribution. These comments are as follows:

Thank you for positively evaluating our research. We are happy that our efforts to progress in the field of biogas technology are recognized, and we acknowledge that modifying the manuscript according to your comments improves its quality.

1. Introduction: I believe that some relevant contributions related to the focus of the paper have been overlooked, for instance:
   1. Mendiola-Rodriguez, T. A., & Ricardez-Sandoval, L. A. (2022). Robust control for anaerobic digestion systems of Tequila vinasses under uncertainty: A Deep Deterministic Policy Gradient Algorithm. Digital Chemical Engineering, 3, 100023.
   2. Tawai, A., & Sriariyanun, M. (2022). Nonlinear Optimization‐Based Robust Control Approach for a Two‐Stage Anaerobic Digestion Process. Journal of Chemistry, 2022(1), 8966350.
   3. Yoshida, K., & Shimizu, N. (2020). Biogas production management systems with model predictive control of anaerobic digestion processes. Bioprocess and biosystems engineering, 43(12), 2189-2200.

Thank you for your constructive suggestions of missing relevant research in the field. We added the paper of Mendiola-Rodriguez et al. (2022) in the introduction for their contribution with a model-free robust MPC with parametric uncertainty.

1. MS-NMPC design using a medium-size phenomenological model: In my opinion, the paper requires a depth thorough discussion and justification of the practical implementation of the proposed MS-NMPC in two aspects:
   1. (i) Despite the ADM1-R3 model providing a robust prediction of biogas production, I believe that a sensitivity analysis of the model parameters is required to include some parameters in the uncertain scenarios for the robust NMPC design, and not only the uncertain substrate characterization.

Thank you for your valuable remark. We acknowledge that parameters cannot be identified uniquely, and are thus uncertain. Their uncertainty strongly depends on experimental conditions, the quality of the measurement data, and the method of parameter identification. In this study, uncertainty of influent concentrations was rigorously derived from underlying measurement uncertainty. However, appropriately quantifying the parameter uncertainty bounds even based on a sensitivity analysis is subject to various assumptions and difficult to define uniquely.

For this reason and given the space limitation of the manuscript, we do not believe that adding further uncertain model parameters adds clarity to the layout of the manuscript. Instead, we aim to address the aspect of parameter uncertainty in a separate publication which also considers results of a currently investigated sensitivity analysis of the ADM1-R3 model parameters.

* 1. (ii) In the numerical simulations and evaluation of the MS-NMPC, I believe that both additional uncertain model parameters and the introduction of a state estimator for the used model will lead to significant computational cost and optimization convergence problems. These are essential aspects that should be addressed to provide a complete evaluation of the robust NMPC approach.

Thank you for your valuable comment. We are aware that adding further uncertain parameters and including a state observer will increase the computational complexity. Therefore, we further stressed these aspects in the results and discussion section, cf. lines XY-XY for the aspect of additional uncertain parameters, and lines XY-XY for a state observer.

1. Supplementary material: To be reproducible, the numerical simulation results, it is suggested that the authors provide in supplementary material model equations, parameters, and initial conditions.

We appreciate your comment. As referenced in line 129, the model equations and parameters of the ADM1-R3 with a single carbohydrate fraction are published in Hellmann et al. (2023). The assumed nominal influent concentrations of considered substrates as well as initial conditions are summarized in the SI, Tab. B.1. The kinetic constants used in this study are further summarized in Tab. 1.

To further clarify the small modifications compared with Hellmann et al. (2023) which were introduced by the second carbohydrate fraction, the model equations of both carbohydrate states were added to the SI, cf. lines XY-XY. The simulation results should now be reproducible by the readers.