JMO Spectrum Library Documentation

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Rationale

We deal routinely with spectra in illumination optics. We need to analyze LED spectra, compute color coordinates and color rendering values from spectra, integrate them, add spectra, multiply spectra with scalar weights and with other spectra (like transmission spectra), interpolate a given measured spectrum with non-equidistant wavelength values to a regular 1 nm array, and so on.

In practice, this is tedious: spectra come in various formats, and the problem of dealing with two spectra als tabulated values which have two different sets of wavelengths is annoying.

This open source Matlab library is designed to make these engineering tasks easy and transparent. It is compatible with GNU Octave, and with Matlab for Mac and Linux.

License -- public domain

I release this software into the public domain under CC0

Getting started

To define a spectrum and compute its CIE xy color coordinates, simply say

```
clear s;
s.lam = [360 830];
s.val = [1 1];
% this is a flat spectrum, known as CIE standard illuminant E.
% By definition, it should have color coordinates x == y == 1/3-
s.XYZ = CIE1931_XYZ(s);
% This is a common pattern: Compute information from a spectrum, and then
% add this information to the same spectrum as an additional field
s
```

```
s = struct with fields:
    lam: [360 830]
    val: [1 1]
    XYZ: [1×1 struct]
```

s.XYZ

```
ans = struct with fields:

X: 106.8654

Y: 106.8569

Z: 106.8919

x: 0.3333

y: 0.3333

z: 0.3334
```

Design decisions

Spectrum

In this library, a spectrum is a struct with at least two fields named lam and val, which meet the following requirements:

- 1. 1am is a 1D vector of numeric values which are not complex, > 0, and strictly ascending
- 2. val is a 1D vector of numeric values which are not complex.
- 3. lam and val are of same length and have at least two elements

It is desirable for lam and val to be column vectors.

There is a function rv = SpectrumSanityCheck(rhs), which tests all these requirements and, if met, returns the same struct except that lam and val are converted to column vectors if necessary.

A *spectrum* models the function S(lambda) which represents a physical scalar function of wavelength. Like spectral radiant flux, spectral irradiance, spectral radiant intensity, spectral radiance, spectral transmission, spectral absorption, spectral efficiency. The tabulated values in val are linearly interpolated. Outside the range given by lam, S(lambda) == 0.

lam is considered to have units of nanometers in all library functions that make use of this unit, e.g. color calculations.

Why structs and not classes? Classes are nice to guarantee that properties like 1am and val always are present, and would allow methods which operate directly on spectra. However, structs are simpler and more versatile. I, as the library designer, cannot know which additional information a user (I myself, for example), wants to attach to a given spectrum. Name, date, name of LED, color coordinates and more. To make a spectrum s, I can simply say

```
clear s
nfig = 0; %figure number to be used in examples
s.lam = [360 830];
s.val = [1 1];
s.name = "CIE standard illuminant E";
s.hopp = "topp";
s

s = struct with fields:
    lam: [360 830]
    val: [1 1]
    name: "CIE standard illuminant E"
```

to give it an appropriate name and an arbitrary value in an arbitrary additional field. Of course, a user can always create a subclass to add properties to a given class, but it is just so much simpler to assign to an additional struct field in a single line. In my experience, these conventions for spectra are simple and few enough to be easily remembered and adhered to.

Conventions

hopp: "topp"

In my code, I like to have long unabbreviated variable and function names. Except for a few standards:

rv is my name for the return value of a function

spec is my name for a single spectrum argument of a function.

1hs and rhs are short for 'left hand side' and 'right hand side', the arguments of a binary function.

Proper library function names start with capital letters, e.g. MakeSpectrum. Internal helper functions start with a small "i", e.g. iLinInterpolProto.

A spectrum struct is valid if it has fields lam and val which fulfill the requirements above. In addition, I assume a spectrum may have the following fields:

name: A short character string with a name.

description: A longer character string with a description.

XYZ: A struct with tristimulus and color coordinate fields X, Y, Z, x, y, z, typically created by code like s.XYZ = $CIE1931_XYZ(s)$

Function reference

Alphabetic list

AddSpectra adds two spectra

AddWeightedSpectra adds multiple spectra with weights

CCT_from_xy computes correlated color temperature and uv-distance to Planck locus from spectrum

CIE1931_lam_x_y_z.mat contains CIE 1931 data for color matching functions, monochromatic border and Planck locus

CIE1931_XYZ computes CIE 1931 XYZ color coordinates and tristimulus values from spectrum

CODATA2018 returns a struct with CODATA 2018 values for relevant physical constants (speed of light, Boltzmann constant etc.)

GaussSpectrum creates a Gaussian spectrum for given mean and standard deviation

IsOctave determines if running on GNU Octave or Matlab

LinInterpol computes linear interpolation like interp1, but about five times faster on Matlab on Windows

LinInterpolAdd4Async computes linear interpolation of the sum of four functions

MakeSpectrum creates a valid spectrum from wavelength and value data

MultiplySpectra multiplies two spectra, e.g. an LED spectrum with a transmission spectrum

PlanckLocus returns a wealth of information about the Planck locus, including interpolation function objects and Judd lines of equal CCT

PlanckSpectrum creates a blackbody spectrum with various normalizations to choose from

SpectrumSanityCheck checks if a spectrum fulfills the requirements

TestLinInterpol tests the LinInterpol function

AddSpectra

Adds two spectra without weights. Convenience function with simpler interface than AddWeightedSpectra

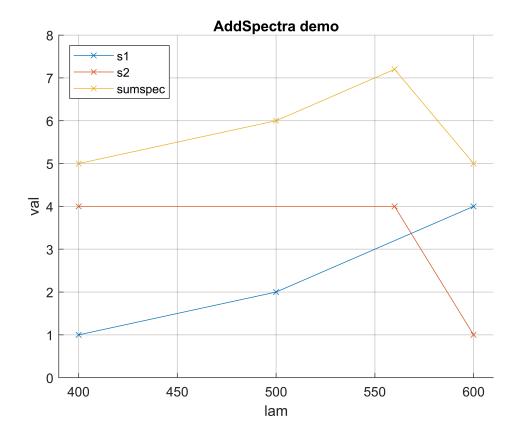
% function rv = AddSpectra(lhs, rhs)

Input: 1hs, rhs: Both must be valid spectra.

Output: rv: Sum of rhs + 1hs. When both spectra do not overlap, the wavelength ranges are concatenated, and the range in between is padded with zero. If they do overlap, then rv.1am contains all values from both input spectra, with duplicate values removed, and what is added are the linearly interpolated values from both input spectra. Thus, the sum spectrum is a perfect model of the underlying continuous function which is the sum of the continuous, linearly interpolated input spectra.

Additional fields present in 1hs or rhs will be stripped, rv will have only fields 1am and val.

```
clear s1 s2 sumspec
s1 = MakeSpectrum([400,500,600],[1, 2, 4]);
s2 = MakeSpectrum([400,560,600],[4, 4, 1]);
sumspec = AddSpectra(s1,s2);
nfig = nfig + 1;
figure(nfig);
clf;
hold on;
plot(s1.lam, s1.val, 'Marker', 'x');
plot(s2.lam, s2.val, 'Marker', 'x');
plot(sumspec.lam, sumspec.val, 'Marker', 'x');
legend({'s1','s2','sumspec'},'Location','NorthWest');
axis([390 610 0 8]);
grid on;
xlabel('lam');
ylabel('val');
title('AddSpectra demo');
```



AddWeightedSpectra

Computes weighted sum of several spectra.

```
% function rv = AddWeightedSpectra(spectra,weights)
```

Input:

spectra: Nonempty cell array of spectra

weights: Vector of numeric non-complex values, same length as spectra.

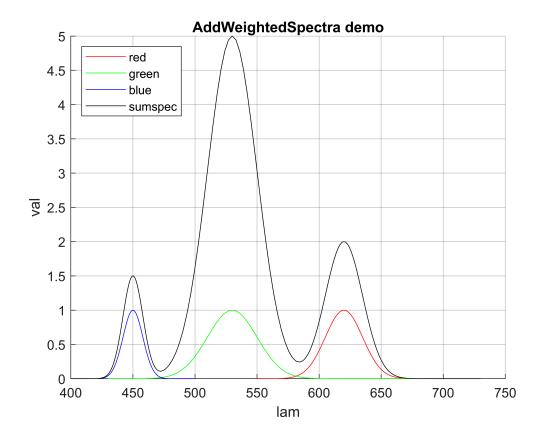
Output: rv: Weighted sum of input spectra. When spectra do not overlap, the wavelength ranges are concatenated, and the range in between is padded with zero. If they do overlap, then rv.lam contains all values from all input spectra, with duplicate values removed, and sorted, and what is added are the linearly interpolated values from all input spectra. Thus, the sum spectrum is a perfect model of the function which is the sum of the continuous, linearly interpolated input spectra.

Additional fields present in spectra will be stripped, rv will have only fields lam and val.

Usage Example:

Add a red, a green, and a blue spectrum created by GaussSpectrum

```
clear red green blue sumspec
red = GaussSpectrum(linspace(550,700),620,15);
green = GaussSpectrum(linspace(430,730),530,20);
blue = GaussSpectrum(linspace(400,500),450,8);
sumspec = AddWeightedSpectra({red, green, blue},[2, 5, 1.5]);
nfig = nfig + 1;
figure(nfig);
clf;
hold on;
plot(red.lam, red.val, 'r');
plot(green.lam, green.val, 'g');
plot(blue.lam, blue.val, 'b');
plot(sumspec.lam, sumspec.val, 'k');
legend({'red', 'green', 'blue', 'sumspec'}, 'Location', 'NorthWest');
grid on;
xlabel('lam');
ylabel('val');
title('AddWeightedSpectra demo');
```



CCT_from_xy

Compute correlated color temperature of an xy color point. Uses parabolic interpolation between nearest Planck locus points

```
% function [CCT, dist_uv] = CCT_from_xy(x,y)
```

Input: x, y scalar numbers, x/y color coordinates

Output:

CCT: Correlated color temperature in Kelvin

dist_uv: Distance to Planck locus in u/v color space. When dist_uv>0.05, a warning is issued, when dist_uv > 0.09, an error. Positive when x/y is above Planck locus (on the "green" side), negative when below.

```
clear T pl jl uv den x y CCT dist_uv
T = 3456;
pl = PlanckLocus;
jl = pl.JuddLine_func(T);
% compute uv coordinates for color point with CCT == T,
% 0.04 away from Planck
uv = [jl.u, jl.v] + 0.04 * [jl.du, jl.dv];
% compute xy coordinates by standard transformation
den = 2*uv(1) - 8*uv(2) + 4;
x = 3*uv(1)/den;
```

```
y = 2 * uv(2) / den;
% compute CCT
[CCT, dist_uv] = CCT_from_xy(x,y);
CCT - T % should be zero, is about 1.3 mK
```

```
ans = 0.0013
```

```
dist_uv - 0.04 % should be zero
```

```
ans = -4.3846e-07
```

CIE1931 lam x y z.mat

A .mat file which contains a struct named 'CIE1931XYZ' with CIE 1931 data: x/y/z color matching functions, monochromatic border x/y coordinates with corresponding wavelengths, and coordinates of the Planck locus with corresponding absolute temperatures.

Usage Example:

```
clear CIE1931XYZ;
load('CIE1931_lam_x_y_z.mat','CIE1931XYZ')
CIE1931XYZ

CIE1931XYZ = struct with fields:
    lam: [471×1 double]
        x: [471×1 double]
        y: [471×1 double]
        z: [471×1 double]
        xBorder: [471×1 double]
        yBorder: [471×1 double]
        yBorder: [1001×1 double]
        PlanckT: [1001×1 double]
        Plancky: [1001×1 double]
        Plancky: [1001×1 double]
```

CIE1931_XYZ

Computes CIE 1931 color coordinates.

```
% function rv = CIE1931_XYZ(spec)
```

Input: spec is a *spectrum* struct, see above for requirements.

Output: A struct with fields X Y Z x y z.

Capital X Y Z are the CIE tristimulus values, i.e. the result of integrating spec with the CIE 1931 standard x y z color matching functions.

x y z are the corresponding color coordinates, x = X / (X + Y + Z) etc.

```
clear s;
s.lam = [360 830];
s.val = [1 1];
% this is a flat spectrum, known as CIE standard illuminant E.
% By definition, it should have color coordinates x == y == 1/3-
s.XYZ = CIE1931_XYZ(s);
```

```
% This is a common pattern: Compute information from a spectrum, and then
  % add this information to the same spectrum as an additional field
  s
  s = struct with fields:
      lam: [360 830]
      val: [1 1]
     XYZ: [1×1 struct]
  s.XYZ
  ans = struct with fields:
     X: 106.8654
     Y: 106.8569
     Z: 106.8919
     x: 0.3333
      y: 0.3333
      z: 0.3334
CODATA2018
A struct with the most relevant physical constants for optics, as defined by CODATA
  % function cd = CODATA2018()
Input: None
Output: Struct with fields:
b: Wien's wavelength displacement law constant
bprime: Wien frequency displacement law constant
c: speed of light in vacuum
e: elementary charge
h: Planck constant
k: Boltzmann constant
me: electron mass
mn: neutron mass
mp: proton mass
NA: Avogadro constant
R: molar gas constant
Vm: molar volume of ideal gas
sigma: Stefan-Boltzmann constant
c1: first radiation constant (2 pi h c^2)
```

c1L: first radiation constant for spectral radiance (2 h c^2)

c2: second radiation constant (h c / k)

Each field is a struct with fields name, value, reluncertainty, absuncertainty, unit. Most uncertainties are zero, since 2018.

Usage Example:

```
clear cd lambda freq pe c
cd = CODATA2018();
lambda = 500e-9; % in meters
c = cd.c.value % speed of light

c = 299792458

freq = c/lambda% frequency of 500 nm light about 600 THz

freq = 5.9958e+14

pe = freq * cd.h.value % energy of a 500 nm photon, in Joule

pe = 3.9729e-19
```

GaussSpectrum

Creates a normalized Gaussian spectrum with given mean and standard deviation

```
% function rv = GaussSpectrum(lam_vec,mean,sdev,varargin)
```

Input:

lam vec: A vector of positive reals, strictly ascending

mean: Scalar positive number. Mean value of the distribution. May or may not be in the lam_vec range.

sdev: Scalar positive number. Standard deviation

varargin: Optional string argument 'val only'.

Output:

rv: Spectrum struct, with additional name field. Except if optional argument 'val_only' is present: Then, rv is a column vector of the values.

Usage Example:

see also example for AddWeightedSpectra.

```
GaussSpectrum(400:500,450,10)
```

```
ans = struct with fields:
    val: [101×1 double]
    lam: [101×1 double]
    name: 'Gauss spectrum for mean 450 nm and sdev 10 nm, normalized to peak = 1'
```

IsOctave

Determines if code is running under GNU Octave (or Matlab)

```
% function rv = IsOctave()
```

Output: Returns logical 1 when running under GNU Octave, else returns logical 0

Usage Example:

```
IsOctave()
ans = logical
0
```

LinInterpol

Computes linearly interpolated values of scalar tabulated function. Very similar to built in interp1, but uses faster C++ DLL under Matlab (on my machine, a factor of five faster).

Used internally as a helper function for higher level library functions, but exposed as a proper library function because it may well be useful to speed up computation in another context.

```
% function yq = LinInterpol(xx,yy,xq)
```

Input:

xx is a 1D vector of numeric values which are not complex, and strictly ascending

yy is a 1D vector of numeric values which are not complex.

xx and yy are of same length and have at least two elements

xq is a 1D vector of numeric values which are not complex, and strictly ascending (the latter is NOT a requirement for interp1)

These preconditions are not checked.

Output:

yq is a vector of same length as xq, with linearly interpolated values. Zero if outside the xx range.

Usage Example:

```
LinInterpol([1 2],[3 4],[-100,1,2,1.7])

ans = 1×4

0  3.0000  4.0000  3.7000
```

LinInterpolAdd4Async

Computes linearly interpolated values of the sum of four input functions. Uses C++ DLL with multithreading under Matlab. On my machine, a factor of five faster than summing the result of interp1 calls, but not faster than summing the results of four LinInterpol calls. I still leave the function in place, there may be a difference for other input values and/or on another machine.

```
% function yq = LinInterpolAdd4Async(xx0,yy0,xx1,yy1,xx2,yy2,xx3,yy3,xq)
```

Input:

xx0 is a 1D vector of numeric values which are not complex, and strictly ascending

yy0 is a 1D vector of numeric values which are not complex.

xx0 and yy0 are of same length and have at least two elements.

Same applies to the xx1/yy1, xx2/yy2, xx3/yy3 pairs, but the xx arrays may all be different.

xq is a 1D vector of numeric values which are not complex, and strictly ascending (the latter is NOT a requirement for interp1!!)

Output: yq is a vector of same length as xq, with the sum of the four linearly interpolated values.

Usage Example:

```
LinInterpolAdd4Async([0 1],[1 1.1], [0 1],[2 2], [0 1],[3 3], [0 2],[4 4], [0, 1, 0.5])

ans = 1×3
10.0000 10.1000 10.0500
```

MakeSpectrum

Creates a spectrum struct out of arrays lam and val and checks if they meet the requirements

```
% function rv = MakeSpectrum(lam, val)
```

Input:

lam is a 1D vector of numeric values which are not complex, > 0, and strictly ascending

val is a 1D vector of numeric values which are not complex.

lam and val are of same length and have at least two elements.

The requirements are checked, an error thrown if violated

Output:

rv is a spectrum struct with column vector fields lam and val.

Usage Example:

```
clear s
s = MakeSpectrum([400 700], [1 1])
s = struct with fields:
    lam: [2×1 double]
```

MultiplySpectra

val: [2×1 double]

Multiply two spectra, e.g. an LED spectrum with a transmission spectrum

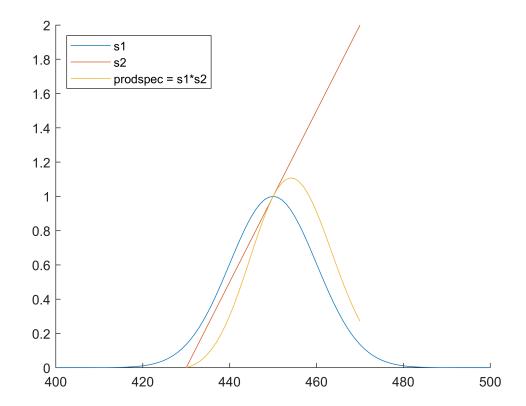
```
% function rv = MultiplySpectra(lhs, rhs)
```

Input: 1hs, rhs: Valid spectrum structs. May or may not overlap.

Output: rv: Spectrum struct, modeling the product of *lhs(lambda)* * *rhs(lambda)*. The rv.lam field covers the overlap region, if any, where it contains all wavelengths from both inputs. The rv.val field contains the product of the respective interpolated values. There is no need to have zero values outside the overlap range, as zero is always assumed outside anyway. When there is no overlap, rv contains a single interval as the overall min/max wavelength range, with zero value.

Usage Example:

```
clear s1 s2 prodspec;
s1 = GaussSpectrum(400:500, 450,10);
s2 = MakeSpectrum([430 470],[0 2]);
prodspec = MultiplySpectra(s1, s2);
nfig = nfig + 1;
figure(nfig);
clf;
hold on;
plot(s1.lam, s1.val);
plot(s2.lam, s2.val);
plot(prodspec.lam, prodspec.val);
legend({'s1','s2','prodspec = s1*s2'},'Location','northwest');
```



PlanckLocus

Compute the x/y, u/v, u/v' coordinates of the Planck locus, as well as interpolation function objects for x/y, u/v, and the Judd lines (lines of equal correlated color temperature) in u/v.

```
% function rv = PlanckLocus()
```

Input: None

Output: Struct with fields:

nT: Number of temperature points (1001)

invT: Inverse absolute temperature values, from near zero (1e-11) to 0.002 (1/K), in equidistant steps. (The Planck locus points are approximately equidistant in 1/T, and not at all equidistant in T)

T: Absolute temperature values, from 1e11 K down to 500 K

x: CIE 1931 x color coordinate values of the Planck locus curve

y: CIE 1931 y color coordinate values of the Planck locus curve

u: CIE u color coordinate values of the Planck locus curve (to be used only for CCT computation purposes)

v: CIE v color coordinate values of the Planck locus curve (to be used only for CCT computation purposes)

up: CIE u' (u_prime) color coordinate values of the Planck locus curve

vp: CIE v' (v_prime) color coordinate values of the Planck locus curve

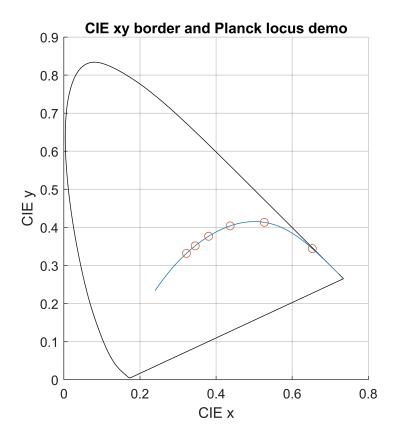
 xy_{func} : A function object to compute interpolated values of the x/y color coordinates of the Planck locus. To be called like $xy = rv.xy_{func}(T)$ where T is a scalar of vector of absolute temperatures, and returns an array of size [2, length(T)] with the interpolated x values as first column and the interpolated y values as second column. Returns NaN when T is out of range 500 .. 1e11

uv func: Same as xy func, except it returns u/v values.

JuddLine_func: A function object to compute interpolated values of the Judd line parameters. To be called like j1 = rv.JuddLine_func(T) where T is a scalar, absolute temperature. Returns a struct with fields u, v, du, dv where [u,v] are the u/v coordinates of the Planck locus point for T, and [du,dv] is the Judd line direction, normalized to length 1. By definition, the Judd line is perpendicular to the Planck locus in u/v color space, and all color points on a Judd line are deemed to have same CCT (correlated color temperature). Note that the "allowed" meaningful range is +/- 0.05 u/v length units away from the Planck locus.

```
clear pl xb yb T xy
pl = PlanckLocus();
load('CIE1931_lam_x_y_z.mat','CIE1931XYZ');
nfig = nfig + 1;
figure(nfig);
clf;
hold on;
xb = CIE1931XYZ.xBorder;
yb = CIE1931XYZ.yBorder;
xb(end+1)=xb(1); % close loop
```

```
yb(end+1)=yb(1);
plot(xb,yb,'k');
plot(pl.x,pl.y);
T = [1000 2000 3000 4000 5000 6000];
xy = pl.xy_func(T);
scatter(xy(:,1),xy(:,2));
axis equal;
grid on;
axis([0 0.8 0 0.9]);
xlabel('CIE x');
ylabel('CIE y');
title('CIE xy border and Planck locus demo')
```



PlanckSpectrum

Generate blackbody spectrum for some absolute temperature.

```
% function rv = PlanckSpectrum(lam_vec, T, varargin)
```

Input:

lam_vec: A wavelength vector, numeric, non-complex, positive, strictly ascending

T: Scalar real number, absolute temperature in K. May be inf, then returns lam_vec.^(-4), scaled to localpeak1 (this is the high temperature limit of the shape of the long wavelength tail).

varargin: Name/Value pairs.

 $1e-9:1am_vec$ given in nm, returned spectrum is W / (nm m^2 sr) or W / (nm m^2) for radiance/exitance scaling. rv.XYZ will be CIE XYZ values X, Y, Z, x, y

Output: Spectrum struct with fields

lam: Same as lam_vec, but column vector.

val: The spectral values, column vector of same length, normalized according to 'normalize' (default: global peak 1.0).

name: string, an appropriate name

```
clear bb T
nfig = nfig + 1;
figure(nfig);
clf;
hold on;
for T = [3000 4000 5000 6000]
         bb = PlanckSpectrum(200:2000,T,'normalize','radiance');
         plot(bb.lam,bb.val);
end
xlabel('\lambda (nm)');
ylabel('spectral radiance L_\lambda (W/(m² nm sr)')
legend({'3000K','4000K','5000K','6000K'})
title('Blackbody spectral radiance')
```

^{&#}x27;normalize' -> string, default 'globalpeak1'. Allowed values:

^{&#}x27;globalpeak1': scaled such that global peak would be 1.0 even if outside lambda range

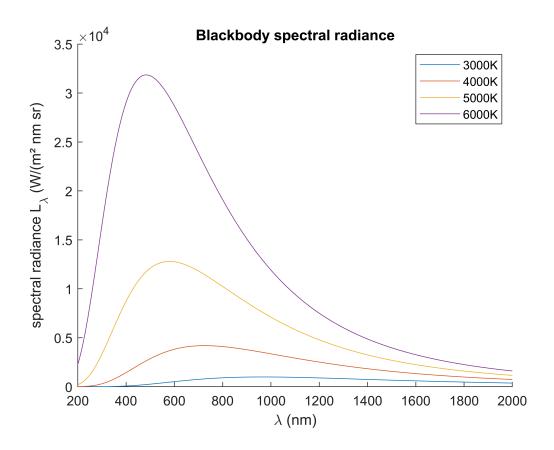
^{&#}x27;localpeak1': scaled such that the peak value is 1.0 for the given lam_vec. Not exactly identical if global peak is in range, due to discretization

^{&#}x27;localflux1': scaled such that integral over lam_vec is 1.0

^{&#}x27;radiance': scaled such rv is blackbody spectral radiance, W/(wlu m²sr). wlu is wavelengthUnit, see below

^{&#}x27;exitance': scaled such that rv is blackbody spectral exitance, W/(wlu m²). wlu is wavelengthUnit, see belo

^{&#}x27;wavelengthUnit' -> positive real, wavelength unit in meters, default 1e-9 (nanometers).



SpectrumSanityCheck

Checks requirements for a spectrum variable, and returns a sanitized version of the variable, with column vectors lam and val.

```
% function [ok, msg, rv] = SpectrumSanityCheck(spec, varargin)
```

Input:

spec is a spectrum which should adhere to the requirements above.

varargin are optional name/value pairs:

'doThrow', logical, default: true. If there is an error: If flag is true, throws error exception, else returns false in ok and an error message in *msg*.

'doStrip', logical, default: false: If true returns rv with only the lam and val fields, else returns rv with all fields of input spec

Output:

ok: Logical. True if there are no requirement violations, else false

msg: Character string. Empty string if there are no requirement violations, else diagnostic message.

rv: A spectrum struct where lam and val are same as in spec except that both are always column vectors. When the doStrip flag is false, also has all other fields of spec. At requirement violation, is empty array.

Usage Example:

```
clear good bad ok1 msg1 rv1 ok2 msg2 rv2
good = MakeSpectrum([400 700], [1 1]);
[ok1, msg1, rv1] = SpectrumSanityCheck(good)
ok1 = logical
msg1 =
    []
rv1 = struct with fields:
   lam: [2×1 double]
   val: [2×1 double]
bad = struct('lam', [1, 2, 1], 'val', [0 0 0]);
[ok2, msg2, rv2] = SpectrumSanityCheck(bad, 'doThrow', false)
ok2 = logical
  0
msg2 =
'lam must be strictly ascending'
rv2 =
    []
```

TestLinInterpol

A script to test the LinInterpol and LinInterpolAdd4Async functions, also measuring run times.

Input: none

Output: diagnostic text

Usage Example:

TestLinInterpol

```
LinInterpol elapsed time: 3.8385e-05
interp1 elapsed time: 0.00021416, diffnorm = 0
LinInterpolAdd4Async elapsed time: 0.00011751
4 x LinInterpol elapsed time: 0.00011388, diffnorm = 0
4 x interp1 elapsed time: 0.00053468, diffnorm = 0
```

template for copy paste

descr

%

Input: x

Output: x