1. To maintain efficiency, ensure all operations (insert, delete, search) run in O(log n) time.
2. Having a balance factor outside (-1, 0, 1) indicates the tree is unbalanced and requires rotations.
3. Trees with bigger height may slow down operations; AVL trees keep height minimal.
4. Inserting products with increasing IDs in a normal BST would result in a right-skewed tree, degrading performance to O(n).
5. Flight reservation systems - The balanced nature ensures that even during peak booking times with thousands of seats across multiple flights, operations remain fast.

OUTPUT:

```
Initial catalog:
[(101, 'Mouse', 15.5), (120, 'USB Cable', 5.5), (150, 'Webcam', 35.0), (205, 'Keyboard', 25.0), (250, '
Monitor', 90.0), (300, 'Headset', 55.0)]
Root = 150 | Height = 3
------------------------------------------------
After updates:
[(101, 'Mouse', 15.5), (120, 'USB Cable', 5.5), (150, 'HD Webcam', 40.0), (250, 'Monitor', 90.0), (300,
 'Headset', 55.0)]
Root = 150 | Height = 3
Next product after $20.0: (150, 'HD Webcam', 40.0)
Next product after $90.0: None
```

BONUS OUTPUT:

```
Initial catalog:
[(101, 'Mouse', 15.5), (102, 'Wireless Mouse', 25.0), (103, 'Gaming Mouse', 45.0), (104, 'Ergonomic Mou
se', 35.0), (105, 'Trackball Mouse', 40.0), (201, 'Keyboard', 30.0), (202, 'Mechanical Keyboard', 85.0)
, (203, 'Wireless Keyboard', 50.0), (204, 'Gaming Keyboard', 120.0), (205, 'Compact Keyboard', 45.0), (
301, 'Monitor', 180.0), (302, '4K Monitor', 350.0), (303, 'Ultrawide Monitor', 450.0), (304, 'Gaming Mo
nitor', 280.0), (305, 'Portable Monitor', 150.0), (401, 'Webcam', 35.0), (402, 'HD Webcam', 55.0), (403
, '4K Webcam', 95.0), (404, 'Streaming Webcam', 120.0), (405, 'Conference Webcam', 200.0), (501, 'Heads
et', 40.0), (502, 'Gaming Headset', 80.0), (503, 'Wireless Headset', 90.0), (504, 'Noise-Canceling Head
set', 150.0), (505, 'USB Headset', 35.0), (601, 'USB Cable', 5.5), (602, 'USB-C Cable', 8.0), (603, 'HD
MI Cable', 12.0), (604, 'DisplayPort Cable', 15.0), (605, 'Ethernet Cable', 10.0), (701, 'External Hard
 Drive 1TB', 60.0), (702, 'External Hard Drive 2TB', 85.0), (703, 'SSD 500GB', 70.0), (704, 'SSD 1TB',
120.0), (705, 'USB Flash Drive 32GB', 12.0), (801, 'Laptop Stand', 25.0), (802, 'Monitor Stand', 35.0),
 (803, 'Adjustable Desk', 250.0), (804, 'Laptop Cooling Pad', 30.0), (805, 'Cable Management Box', 15.0
), (901, 'Router', 65.0), (902, 'Gaming Router', 180.0), (903, 'Mesh WiFi System', 220.0), (904, 'Range
 Extender', 35.0), (905, 'Modem', 55.0), (1001, 'Printer', 120.0), (1002, 'Laser Printer', 280.0), (100
3, '3D Printer', 450.0), (1004, 'Photo Printer', 95.0), (1005, 'Label Printer', 85.0), (1101, 'Speaker'
, 45.0), (1102, 'Soundbar', 120.0), (1103, 'Studio Monitors', 280.0), (1104, 'Bluetooth Speaker', 55.0)
, (1105, 'Desktop Speakers', 35.0), (1201, 'Microphone', 40.0), (1202, 'USB Microphone', 65.0), (1203,
'Studio Microphone', 150.0), (1204, 'Lavalier Microphone', 25.0), (1205, 'Shotgun Microphone', 180.0),
(1301, 'Graphics Tablet', 85.0), (1302, 'Drawing Tablet', 250.0), (1303, 'Pen Display', 450.0), (1304,
'Stylus Pen', 35.0), (1305, 'Digital Pen', 55.0), (1401, 'Power Strip', 18.0), (1402, 'Surge Protector'
, 25.0), (1403, 'UPS Battery Backup', 120.0), (1404, 'USB Power Adapter', 15.0), (1405, 'Wireless Charg
er', 28.0), (1501, 'Desk Lamp', 35.0), (1502, 'LED Strip Lights', 22.0), (1503, 'Ring Light', 45.0), (1
504, 'Monitor Light Bar', 55.0), (1505, 'Smart Bulb', 18.0), (1601, 'Docking Station', 120.0), (1602, '
USB Hub', 25.0), (1603, 'Card Reader', 15.0), (1604, 'KVM Switch', 55.0), (1605, 'USB Switch', 20.0), (
1701, 'Mouse Pad', 12.0), (1702, 'Extended Mouse Pad', 25.0), (1703, 'RGB Mouse Pad', 35.0), (1704, 'Wr
```

```
USB Hub', 25.0), (1603, 'Card Reader', 15.0), (1604, 'KVM Switch', 55.0), (1605, 'USB Switch', 20.0), (
1701, 'Mouse Pad', 12.0), (1702, 'Extended Mouse Pad', 25.0), (1703, 'RGB Mouse Pad', 35.0), (1704, 'Wr
ist Rest', 18.0), (1705, 'Desk Mat', 30.0), (1801, 'Monitor Arm', 85.0), (1802, 'Dual Monitor Arm', 150
.0), (1803, 'TV Wall Mount', 45.0), (1804, 'Laptop Arm', 95.0), (1805, 'Tablet Stand', 28.0), (1901, 'C
leaning Kit', 15.0), (1902, 'Screen Cleaner', 8.0), (1903, 'Compressed Air', 10.0), (1904, 'Microfiber
Cloth', 5.0), (1905, 'Keyboard Brush', 7.0), (2001, 'Privacy Screen', 35.0), (2002, 'Screen Protector',
 12.0), (2003, 'Blue Light Filter', 25.0)]
Root = 1304 | Height = 7
--------------------------------------------------

📦 OPERATION 1: Holiday Sale Price Updates
Applied discounts to 3 items
Root = 1304 | Height = 7

🗑️OPERATION 2: Removing Discontinued Items
Removed 5 discontinued products
Root = 1304 | Height = 7
--------------------------------------------------

Final catalog size: 93
Next product after $50.0: None
Next product after $200.0: None
```

CONCLUSION:

After inserting 100 products into the AVL tree, the height stabilized at approximately 7-8 nodes, closely matching the theoretical optimal of $\log_2(100) \approx 6.64$. This demonstrates that AVL self-balancing successfully maintains logarithmic height regardless of insertion order. In contrast, an unbalanced binary search tree could degenerate to a height of 100 in the worst case, requiring 100 comparisons per search instead of just 7-8 a 93% performance improvement.

The key finding is that AVL trees guarantee O(log n) operations through automatic rebalancing via rotations. During our tests with price updates and product deletions, the tree consistently maintained its optimal height without manual intervention. This logarithmic scaling means that even with 10,000 products, search operations would require only ~14 comparisons, and with 100,000 products, only ~17 comparisons. The small overhead of performing rotations during insertions and deletions is far outweighed by the dramatic search performance gains, making AVL trees ideal for dynamic datasets requiring frequent lookups, such as product catalogs, flight reservation systems, and database indexes.