

OG-ZAF: Current Calibration

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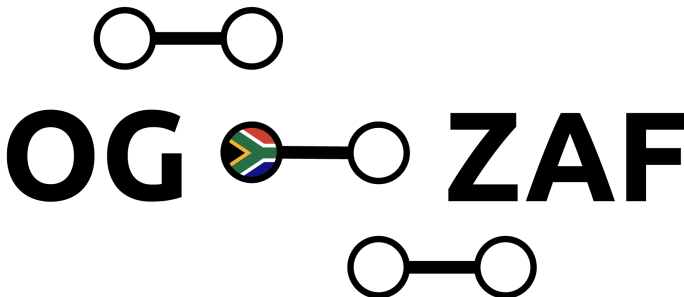
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OG-ZAF



- OG-ZAF is a calibration of the OG-Core model.

OG-ZAF: A Calibration

What it means for OG-ZAF to be a “calibration” of OG-Core...

- OG-Core has all the underlying theory (e.g., different households, an arbitrary number of production sectors)
- But the parameters of the model are set to represent the South African economy
- E.g.,
 - The model matches South Africa’s gini coefficient for the distribution of income
 - The size of production sectors represent the relative size in the South African economy
 - Policy parameters are set to represent South Africa’s tax system

NOTE: Any of these parameters can be changed to represent a counterfactual South Africa.

Model Dimensions

- OG-Core allows for arbitrary sizes in the various model dimensions. In OG-ZAF, the defaults are:
 - Number of labor skill groups, J : 7
 - Representing the 0-25%, 25-50%, 50-70%, 70-80%, 80-90%, 90-99%, Top 1%
 - Age at which economically active, E : 20
 - Number of periods representing ages E to 100, S : 80
 - With this, a model period represents one year
 - Number of model periods until assume reach steady-state, T : $4 \times S = 320$
 - Note, it's helpful to make T a bit larger than you think it needs to be since you need the economy to settle down before this point, but only computational cost if set it too high
 - Number of consumption goods, I : 1 (5)
 - Number of production industries, M : 1 (7)

OG-ZAF Calibration

- The following parameters have been calibrated specifically to South Africa:
 - Demographics
 - Lifetime earnings processes for heterogeneous households
 - Firm production functions (labor share of output)
 - Household consumption parameters (expenditure shares on differentiated goods)
 - Input-output mapping between firm output and consumption categories
 - Macro parameters (long run growth rate, gov't interest rate haircut)
 - Remittances

OG-ZAF Calibration

- Other parameters such as:
 - Household preference parameters
 - Elasticity of substitution between capital and labor
- Are set to standard values used in the literature, due to a lack of research (to our knowledge) on these parameters in the South African context

Full documentation of the OG-ZAF calibration:

<https://EAPD-DRB.github.io/OG-ZAF/>

OG-ZAF Demographics

- Demographic parameters include:
 - Fertility rates by age
 - Mortality rates age
 - Age distribution of the population in an initial period
 - *Not* immigration (to ensure things add up, immigration computed as a residual)
- The [UN World Population Prospects data](#) provide all of these for most countries from present with forecasts through 2100!

Population Dynamics

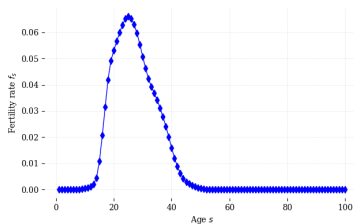
The population evolves according to the following laws of motion, where $\omega_{s,t}$ are the number of age s households at time t :

$$\omega_{1,t+1} = (1 - \rho_0) \sum_{s=1}^{E+S} f_s \omega_{s,t} + i_1 \omega_{1,t} \quad \forall t$$

$$\omega_{s+1,t+1} = (1 - \rho_s) \omega_{s,t} + i_{s+1} \omega_{s+1,t} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E + S - 1$$

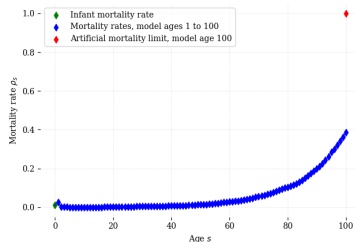
- The age-specific fertility rates, f_s , mortality rates, ρ_s , and immigration rates, i_s affect the distribution each period
- Given that these rates are constant, the population distribution will converge to a steady-state distribution

South Africa: Fertility and Mortality



Source: UN Population Data

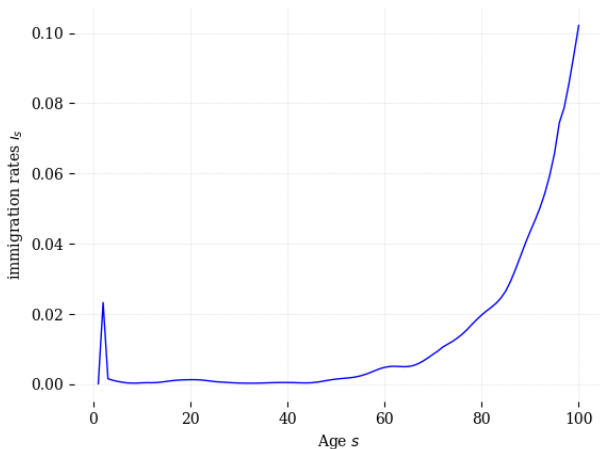
Fertility Rates



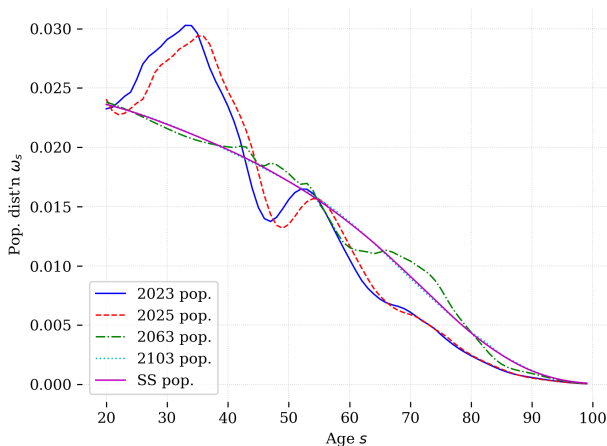
Source: UN Population Data

Mortality Rates

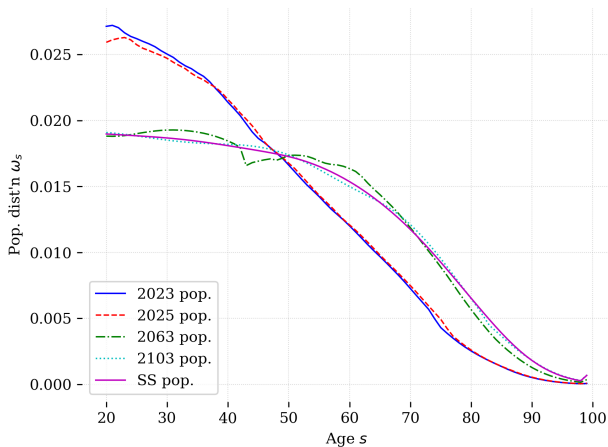
South Africa: Demographics, immigration rates



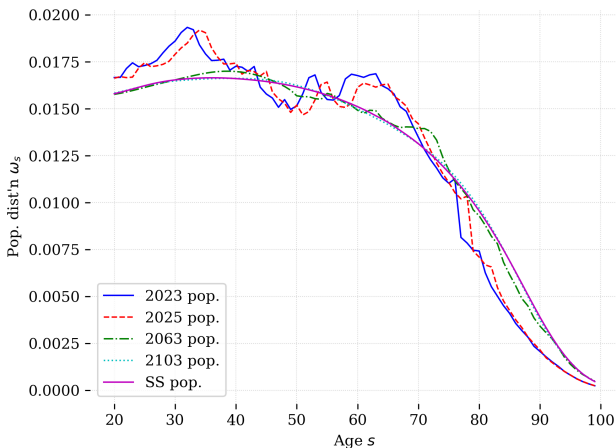
South Africa: Demographics, pop. distribution



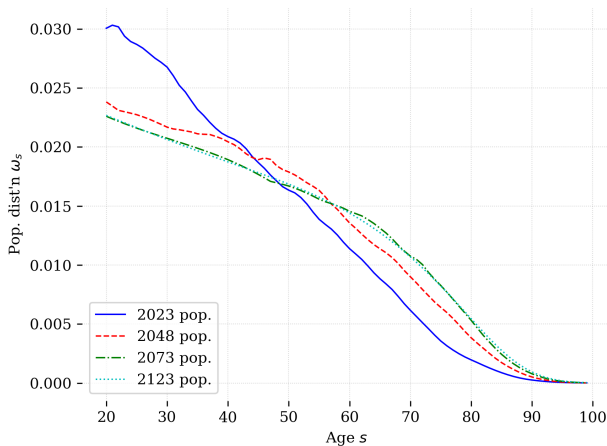
Compare to India: Demographics, pop. distribution



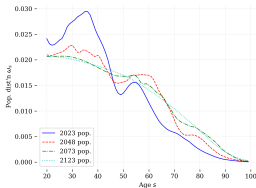
Compare to USA: Demographics, pop. distribution



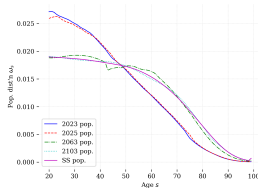
Compare to Philippines: Demographics, pop. distribution



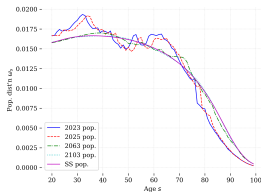
Population Distribution Comparison



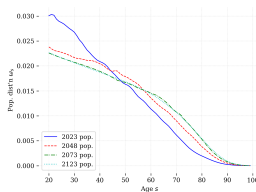
South Africa



India

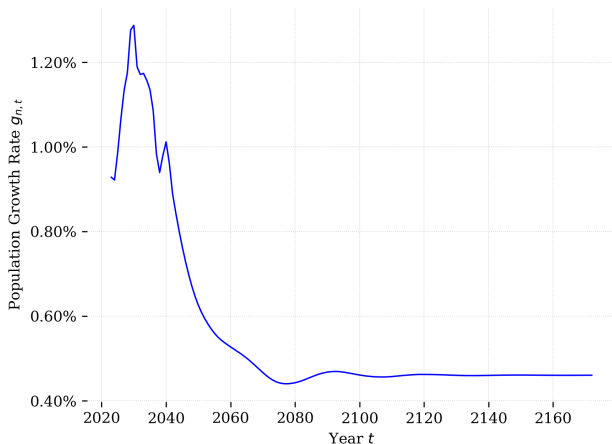


United States



Philippines

Demographics, pop. growth



Earnings Ability

- Households are heterogeneous earnings ability/effective labor units
 - There is a (endogenously determined) common wage rate per effective unit of labor, but households vary in the effective units of labor per unit of labor supply, giving rise to differences in hourly earnings

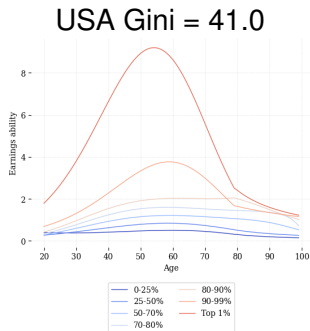
$$c_{j,s,t} + b_{j,s+1,t+1} = (1 + r_t)b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \dots$$

- Earnings ability varies across households and over the lifecycle within a household
- There is no earnings risk: while earnings vary over the lifecycle, this process is completely deterministic
- Effective labor units are homogeneous from the point of view of firms
- No human capital accumulation decisions: earnings ability profiles are exogenous

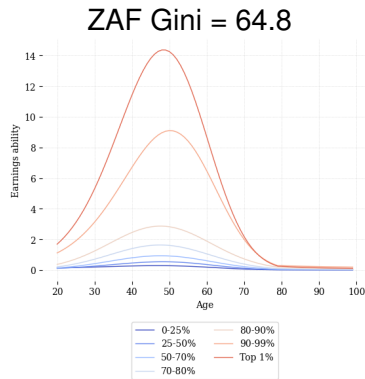
Calibrating Lifetime Earnings Profiles

- Ideally, one can estimate the lifetime earnings profiles from microdata that represents a long panel of households (see e.g., Fullerton and Rogers (1993) or DeBacker et al. (2016))
- However, these data are often hard to come by
- We've therefore devised a reasonable approximation that requires only minimal data:
 - Begin with the earnings profiles for U.S. household estimated by DeBacker et al. (2016)
 - Using single parameter function, adjust the shape of the profiles for each earnings group to match the gini coefficient in South Africa

OG-ZAF Lifetime Earnings Profiles



Earnings Profiles, USA



Earnings Profiles, ZAF

Calibrating Firm Parameters

- Marcelo was able to use the ZAF national accounts data + industry-level data from the South African Reserve Bank to find:
 - Output, labor, capital by industry \implies industry TFP
 - Labor's share of output by industry

Production Sector	TFP ($GVA/F(K, L)$)	TFP (normalized)	Capital Share Income
Primary	396.80	0.11	0.67
Secondary (ex energy)	3474.10	1.00	0.50
Tertiary	6604.17	1.90	0.45
Energy	551.63	0.16	0.53

Firm Production Technology

- Use national accounts data to calibrate the firm production function:

$$\begin{aligned} Y_{m,t} &= F(K_{m,t}, K_{g,m,t}, L_{m,t}) \\ &\equiv Z_{m,t} \left[(\gamma_m)^{\frac{1}{\varepsilon_m}} (K_{m,t})^{\frac{\varepsilon_m-1}{\varepsilon_m}} + (\gamma_{g,m})^{\frac{1}{\varepsilon_m}} (K_{g,m,t})^{\frac{\varepsilon_m-1}{\varepsilon_m}} + \right. \\ &\quad \left. (1 - \gamma_m - \gamma_{g,m})^{\frac{1}{\varepsilon_m}} (e^{g_y t} L_{m,t})^{\frac{\varepsilon_m-1}{\varepsilon_m}} \right]^{\frac{\varepsilon_m}{\varepsilon_m-1}} \quad \forall m, t \end{aligned}$$

- Capital share of output: $\gamma_m = 0.401$
- Set $\varepsilon_m = 1.0$ for all sectors (Cobb-Douglas production)

Mapping Production Goods to Consumption Goods

- Model households consume differentiated goods:

$$c_{j,s,t} \equiv \prod_{i=1}^I (c_{i,j,s,t} - c_{min,i})^{\alpha_i} \quad \forall j, s, t \quad \text{with} \quad \sum_{i=1}^I \alpha_i = 1$$

- Consumption goods are composites of production goods, determined via fixed proportions:

$$\tilde{p}_{i,t} = \sum_{m=1}^M \pi_{i,m} \tilde{p}_{m,t} \quad \forall i, t$$

Mapping Production Goods to Consumption Goods

- This leaves two parameter objects to calibrate:
 - 1 Household preferences (expenditure shares) over the differentiated production goods, α_j
 - 2 A matrix representing the shares of each output good in the composition of each consumption good, Π

Data Mapping Production Goods to Consumption Goods

- Data for these are contained in standard “social accounting matrices” used in CGE modeling
- These are readily available for most countries from [GTAP](#) or other sources
- For the ZAF calibration, we use [data from the UN University-Wider](#)

Calibration of IO Matrix and Consumption Shares

Consumption-Production Bridge Matrix

Cons/Prod	Primary	Energy	Tertiary	Secondary (ex energy)
Food	0.060	0.001	0.247	0.692
Energy	0.092	0.089	0.044	0.776
Non-durables	0.105	0.007	0.370	0.518
Durables	0.095	0.018	0.327	0.560
Services	0.084	0.011	0.729	0.176

Consumption Expenditure Shares

Food	Energy	Non-durables	Durables	Services
0.10	0.04	0.16	0.15	0.55

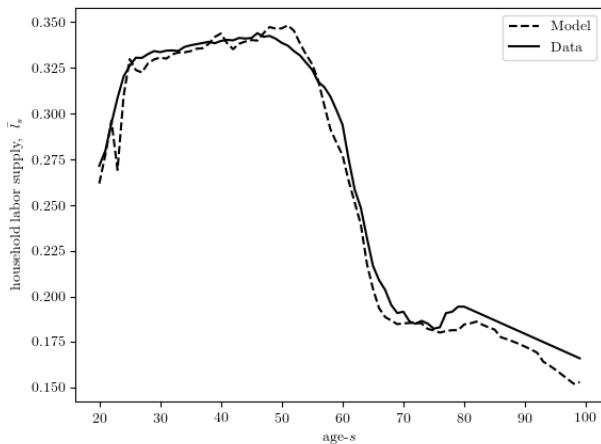
Macroeconomic parameters:

- Long run growth rate, $g_y = 0.0\%$ (no growth in GDP per capita since 2007)
- Initial period debt-to-GDP ratio, 73.4%
- Open economy parameters:
 - Initial percentage of debt held by foreigners, 26%
 - Percentage of newly issued debt purchased by foreigners, 23%
 - Openness of capital flow (0 to 1 scale), 0.9
- Government spending:
 - Non-pension transfers to GDP, 4%
 - Government consumption expenditures to GDP, 26.7%

To come:

- More detail with tax and benefit system
- Match distribution of wealth
- Calibrate labor supply to match rates in South Africa
 - Note: model is of cohorts of agents, so unemployment not directly modeled, but we can get at it through, e.g., low labor supply of the young

Matching labor supply, US Example



Labor Supply, ZAF (QLFS 2023)

