Project 0: Inaugural Project

Vision: The inaugural project teaches you to solve a simple economic model and present the results.

- **Objectives:** In your inaugural project, you should show that you can:
  1. Apply simple numerical solution and simulation methods
  2. Structure a code project
  3. Document code
  4. Present results in text form and in figures
- **Content:** In your inaugural project, you should:
  1. Solve and simulate a pre-specified economic model (see next page)
  2. Visualize results

  Example of structure: See this repository.
- **Structure:** Your inaugural project should consist of:
  1. A README.md with a short introduction to your project
  2. A single self-contained notebook (.ipynb) presenting the analysis
  3. Fully documented Python files (.py)
- **Hand-in:** On GitHub by uploading it to the subfolder *inaugralproject*, which is located in:

  github.com/NumEconCopenhagen/projects-YEAR-YOURGROUPNAME
- **Deadline:** See Calendar.
- **Peer feedback:** After handing in, you will be asked to give peer feedback on the projects of two other groups.
- **Exam:** Your inaugural project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Time Use of Couples

We consider a household with a *male* and a *female* member. They jointly maximize utility by choosing their time use:

1. Hours working *in the market*, L_M and L_F, at wages w_M and w_F.
2. Hours working *at home*, H_M and H_F.

Their choice of time use implies:

1. Consumption of market goods, $C = w_M L_M + w_F L_F$
2. Consumption of home production,

$$H = \begin{cases} \min\{H_M, H_F\} & \text{if } \sigma = 0 \\ H_M^{1-\alpha} H_F^{\alpha} & \text{if } \sigma = 1 \\ \left((1-\alpha) H_M^{\frac{\sigma-1}{\sigma}} + \alpha H_F^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} & \text{else} \end{cases}$$

   where σ is the elasticity of substitution, and $\frac{\alpha}{1-\alpha}$ is the productivity in home production for females relative to males.

3. Total consumption, $Q = C^{\omega} H^{1-\omega}$, where ω is the weight on market goods.

The household gets disutility from time spend working. The full maximization problem of the household is

$$\max_{L_M, H_M, L_F, H_F} \frac{Q^{1-\rho}}{1-\rho} - \nu \left(\frac{T_M^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} + \frac{T_F^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \right), \ \rho > 1, \nu > 0, \epsilon > 0$$

s.t.

$$\begin{aligned} C &= w_M L_M + w_F L_F \\ H &= \begin{cases} \min\{H_M, H_F\} & \text{if } \sigma = 0 \\ H_M^{1-\alpha} H_F^{\alpha} & \text{if } \sigma = 1 \\ \left((1-\alpha) H_M^{\frac{\sigma-1}{\sigma}} + \alpha H_F^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} & \text{else} \end{cases} \\ Q &= C^{\omega} H^{1-\omega} \\ T_M &= L_M + H_M \\ T_F &= L_F + H_F \\ L_M, H_M, L_F, H_F &\geq 0 \\ T_M, T_F &\leq 24 \end{aligned}$$

The baseline parameters are:

1. **Preferences:** $\rho = 2$, $\nu = 0.001$, $\epsilon = 1$, $\omega = 0.5$
2. **Household production:** $\alpha = 0.5$, $\sigma = 1$
3. **Wages:** $w_M = w_F = 1$

Questions

We first assume that the choice set is *discrete* in half hours, specifically

$$L_M, L_F, H_M, H_F \in \left[\frac{0 \cdot 24}{48}, \frac{1 \cdot 24}{48}, \frac{2 \cdot 24}{48}, \ldots, \frac{48 \cdot 24}{48}\right]$$

Code is provided in *IntroProg-lectures/projects/HouseholdSpecializationModel.py* for solving the model in this case when $\sigma = 1$.

1. Illustrate how $\frac{H_F}{H_M}$ changes when varying $\alpha \in \{0.25, 0.50, 0.75\}$ and $\sigma = \{0.5, 1.0, 1.5\}$.
2. Plot $\log \frac{H_F}{H_M}$ against $\log \frac{w_F}{w_M}$ for $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$.

We now assume that the choice set is *continuous*, i.e.

$$L_M, L_F, H_M, H_F \in [0, 24]$$

3. Plot $\log \frac{H_F}{H_M}$ against $\log \frac{w_F}{w_M}$ for $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$

In Siminski and Yetsenga (2022), *Specialization, Comparative Advantage, and the Sexual Division of Labor*, the following regression is run on time use data of couples,

$$\log \frac{H_F}{H_M} = \beta_0 + \beta_1 \log \frac{w_F}{w_M}$$

They find that $\beta_0 \approx 0.4$ and $\beta_1 \approx -0.1$. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ denote the same coefficients when estimated on data from the model for fixed w_M and $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$.

4. Choose α and σ such it minimizes $\left(\beta_0 - \hat{\beta}_0\right)^2 + \left(\beta_1 - \hat{\beta}_1\right)^2$. Illustrate how the model fit the data and discuss the economic interpretation of the results.
   Note: This might take be time consuming. Try to use a narrow interval for α and σ.

Assume that additional empirical evidence shows that men and women are equally productive in home production, i.e. $\alpha = 0.5$.

5. Suggest and implement an extension of the model, and analyze whether or not it can help match the data when $\alpha = 0.5$.