Reduce Exercise (25% of total marks)

Notes by James H. Davenport 4 September 2018

1 Aim

The aim of this exercise is to implement various g.c.d. algorithms in Reduce. You are writing in ordinary Reduce (not the "symbolic mode described in Chapter 17 of [HS18]. You should implement functions for g.c.d. of polynomials in several variables, using, recursively, these algorithms as defined in Lecture 2. Note that these algorithms (slide 11 etc.) fulfil the role of "Some PseudoEuclid" in the Gauss algorithm on slide 8. To do g.c.d. of polynomials in several variables (say x, y, z) you let them be in $\mathbf{Z}[z][y][x]$ and use the Gauss algorithm in R[x], where $R = \mathbf{Z}[z][y]$ etc. A lot of this infrastructure will be common to all implementations, and I expect you to cut and paste it from one file to the next.

2 Research Questions

Reduce has two g.c.d. algorithms already programmed:

Native This is almost the algorithm described as "Full" in [Hea79];

EZGCD This is the [MN81] version of the [MY73] algorithm, obtained after you say on ezgcd. See [HS18, p. 131].

So comparing your implementations with the built-in ones, using the sort of polynomials described in the appendix of [Hea79] gives these questions.

RQ1 How much faster is Reduce's built-in implementation than your HearnFull?

RQ2 How do your five implementations compare with each other?

3 Submission

You must submit, by e-mail to J.H.Davenport@bath.ac.uk by 18:00 on Saturday 8 September, a single zip file, called 3160567890.zip (if your student ID is 3160567890 - use YOUR OWN student number) containing these files.

- **Primgcd.red** Implements a function Primgcd to take the g.c.d. of two polynomials using the algorithm on slide 11. Marks: 10%.
- **HearnBasic.red** Implements a function HearnBasicgcd to take the g.c.d. of two polynomials using the algorithm on slide 15. Marks: 10%.
- HearnFull.red Implements a function HearnFullgcd to take the g.c.d. of two polynomials using the algorithm called "Hearn Primitive" on slide 16. Marks: 10%.
- **Subresgcd.red** Implements a function **Subresgcd** to take the g.c.d. of two polynomials using the algorithm on slide 13. **Marks:** 10%.
- JHD.red Implements a function JHDgcd to take the g.c.d. of two polynomials using the ideas on slide 16. I haven't given you an algorithm you have to work out what you think "better management of l" could mean. Marks: 20%.
- **Report.pdf** A report with your investgation of the research questions stated above. Marks: 40%.

4 Questions and Answers

There will undoubtedly be questions, and if they are general ones, I will put the questions and answers here. If it seems important, I will tell QQ that I have updated this file.

 $\mathbf{Q}\mathbf{1}$

 $\mathbf{A1}$

References

- [Hea79] A.C. Hearn. Non-Modular Computation of Polynomial Gcd Using Trial Division. In *Proceedings EUROSAM 79*, pages 227–239, 1979.
- [HS18] A.C. Hearn and R. Schöpf. REDUCE User's Manual (Free Version; June 8, 2018). http://reduce-algebra.sourceforge.net/, 2018.
- [MN81] P.M.A. Moore and A.C. Norman. Implementing a Polynomial Factorization and GCD Package. In *Proceedings SYMSAC 81*, pages 109–116, 1981.
- [MY73] J. Moses and D.Y.Y. Yun. The EZ GCD Algorithm. In *Proceedings* ACM 73, pages 159–166, 1973.