



given $\theta_1 = 30^\circ$ $\theta_2 = 45^\circ$ $\theta_3 = 60^\circ$ $L_1 = 2$ $L_2 = 2$ $L_3 = 1$

Forward

$$x_3 = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$x_3 = 2 \cos(30) + 2 \cos(30 + 45) + 1 \cos(30 + 45 + 60) = 1.54$$

$$y_3 = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$y_3 = 2 \sin(30) + 2 \sin(30 + 45) + 1 \sin(30 + 45 + 60) = 3.63$$

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$\theta = 135^\circ$$

Given $\theta = 135^\circ$ $x_3 = 1.54$ $y_3 = 3.63$ $L_1 = 2$ $L_2 = 2$ $L_3 = 1$

Inverse

$$x_2 = x_3 - L_3 \cos(\theta)$$

$$x_2 = 1.54 - 1 \cos(135) = 2.24$$

$$y_2 = y_3 - L_3 \sin(\theta)$$

$$y_2 = 3.63 - 1 \sin(135) = 2.92$$

$$\cos(\theta_2) = \frac{x_2^2 + y_2^2 - L_1^2 - L_2^2}{2 L_1 L_2}$$

$$\cos(\theta_2) = \frac{2.24^2 + 2.92^2 - 2^2 - 2^2}{2 \times 2 \times 2} = 0.693 \rightarrow \theta_2 = \cos^{-1}(0.70)$$

$$\theta_2 = 45^\circ$$

$$\cos(\theta_1) = \frac{(L_1 + L_2 \cos(\theta_2)) x_2 + (L_2 \sin(\theta_2)) y_2}{x_2^2 + y_2^2}$$

$$\cos(\theta_1) = \frac{(2 + 2 \cos(45)) 2.24 + (2 \sin(45)) 2.92}{2.24^2 + 2.92^2} = 0.86$$

$$\sin(\theta_1) = \frac{(L_1 + L_2 \cos(\theta_2)) y_2 - (L_2 \sin(\theta_2)) x_2}{x_2^2 + y_2^2}$$

$$\sin(\theta_1) = \frac{(2 + 2 \cos(45)) 2.92 - (2 \sin(45)) 2.24}{2.24^2 + 2.92^2} = 0.50$$

$$\theta_1 = \tan^{-1} \left(\frac{\sin(\theta_1)}{\cos(\theta_1)} \right)$$

$$\theta_1 = \tan^{-1} \left(\frac{0.50}{0.86} \right) = 30^\circ$$

$$\theta_3 = \theta - (\theta_1 + \theta_2)$$

$$\theta_3 = 135 - (30 + 45) = 60^\circ$$