$$AX = \lambda X$$

$\lambda \rightarrow$ eigen value

$X \rightarrow$ eigen vector

$$A = \begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix}$$

$$AX - \lambda X = 0$$

$$(A - \lambda I)X = 0$$

Let $(A - \lambda I) = 0$

$$\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Let $\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix} = 0$

$$4-\lambda \begin{vmatrix} -9-\lambda & -2 \\ 10 & 5-\lambda \end{vmatrix} - 8 \begin{vmatrix} -2 & -2 \\ 0 & 5-\lambda \end{vmatrix} + -1 \begin{vmatrix} -2 & -9-\lambda \\ 0 & 10 \end{vmatrix} = 0$$

$$4-\lambda\left[(-2-\lambda)(5-\lambda)+20\right] - 8(-10+2\lambda) + 20 = 0$$

$$4-\lambda\left[-45+9\lambda-5\lambda+\lambda^2+20\right] + 80 - 16\lambda + 20 = 0$$

$$(4-\lambda)(\lambda^2+4\lambda-25) - 16\lambda + 100 = 0$$

$$(4-\lambda)(\lambda^2+4\lambda-25)-16\lambda+100=0$$

$$4(\lambda^2+4\lambda-25)-\lambda(\lambda^2+4\lambda-25)-16\lambda+100=0$$

$$4\lambda^2+16\lambda-100-\lambda^3-4\lambda^2+25\lambda-16\lambda+100=0$$

$$4\lambda^2-4\lambda^2=0$$

$$16\lambda+25\lambda-16\lambda=25\lambda$$

$$-100+100=0$$

$$-\lambda^3+25\lambda=0$$

$$-\lambda(\lambda^2-25)=0$$

$$-\lambda(\lambda-5)(\lambda+5)=0$$

$$\lambda_1=-5$$

$$\lambda_2=5$$

$$\lambda_3=0$$

$$\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

If $\lambda=0$
$$\begin{pmatrix} 4 & 8 & -1 \\ -2 & -9 & -2 \\ 0 & 10 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Echelon form

Augmented matrix
$$\begin{pmatrix} 4 & 8 & -1 & 10 \\ -2 & -9 & -2 & 10 \\ 0 & 10 & 5 & 10 \end{pmatrix} \quad r_2 \leftrightarrow r_3$$

$$\begin{pmatrix} 4 & 8 & -1 & 10 \\ 0 & 10 & 5 & 10 \\ -2 & -9 & -2 & 10 \end{pmatrix} \; 2r_3 + r_1 \begin{pmatrix} 4 & 8 & -1 & 10 \\ 0 & 10 & 5 & 10 \\ 0 & -10 & -5 & 10 \end{pmatrix} \; r_3 + r_2$$

$$\begin{pmatrix} 4 & 8 & -1 \\ 0 & 10 & 5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$4x + 8y - z = 0$$
$$10y + 5z = 0$$
$$0z = 0$$

for $z = 1$

$$10y + ((1)5) = 0$$
$$10y + 5 = 0$$
$$\frac{10y}{10} = \frac{-5}{10}$$

$$y = -1/2$$

$$4x + 8y - z = 0$$
$$4x - \left(\frac{8}{2}\right) - 1 = 0$$
$$4x - 4 - 1 = 0$$
$$4x = 5$$
$$x = 5/4$$

Eigen Vector $= (5/4, -1/2, 1)$

$$\therefore \; x_1 = \begin{pmatrix} 5/4 \\ -1/2 \\ 1 \end{pmatrix}$$

for $\lambda_2 = 5$ $\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -1 & 8 & -1 \\ -2 & -14 & -2 \\ 0 & 10 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Gaussian elimination
Augmented matrix $\begin{pmatrix} -1 & 8 & -1 & 10 \\ -2 & -14 & -2 & 10 \\ 0 & 10 & 0 & 10 \end{pmatrix}$ $r_2 \leftrightarrow r_3$

$$\begin{pmatrix} -1 & 8 & -1 & 10 \\ 0 & 10 & 0 & 10 \\ -2 & -14 & -2 & 10 \end{pmatrix} 2r_1 - r_3 \begin{pmatrix} -1 & 8 & -1 & 10 \\ 0 & 10 & 0 & 10 \\ 0 & 30 & 0 & 10 \end{pmatrix} 3r_2 - r_3$$

$$\begin{pmatrix} -1 & 8 & -1 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x + 8y - z = 0$$
$$10y = 0$$
$$0z = 0$$

when $z = 1$, $y = 0$

$$-x + 8(0) - 1 = 0$$
$$x = -1$$

Eigen Vector $= X_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

Eigen Value $\lambda_3 = -5$, $(A - \lambda I)(x) = 0$

$$\begin{pmatrix} 4-\lambda & 8 & -1 \\ -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 8 & -1 \\ -2 & -4 & -2 \\ 0 & 10 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Gaussian elimination
Augmented matrix $\begin{pmatrix} 9 & 8 & -1 & 10 \\ -2 & -4 & -2 & 10 \\ 0 & 10 & 10 & 10 \end{pmatrix}$ $r_2 \leftrightarrow r_3$

$$\begin{pmatrix} 9 & 8 & -1 & 10 \\ 0 & 10 & 10 & 10 \\ -2 & -4 & -2 & 10 \end{pmatrix} \; \tfrac{9}{2} r_3 + r_1 \; \begin{pmatrix} 9 & 8 & -1 & 10 \\ 0 & 10 & 10 & 10 \\ 0 & -10 & -10 & 10 \end{pmatrix} \; r_2 + r_3$$

$$\begin{pmatrix} 9 & 8 & -1 \\ 0 & 10 & 10 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$9x + 8y - z = 0$$
$$10y + 10z = 0$$
$$0z = 0$$

For $z = 1$,
$$10y + 10 = 0$$
$$y = -1$$

$$9x - 8 - 1 = 0$$
$$9x - 9 = 0$$
$$x = 1$$

Eigen Vector

$$\gamma_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Importance of Eigen Values

$$\frac{(\lambda_i)}{\text{Sum } \lambda} \times 100$$

$$\text{Sum} = |0| + |-5| + |5| = 10$$

If $\lambda_1 = 0$
$$\frac{|0|}{10} \times 100 = 0\%$$

If $\lambda_2 = 5$ $\quad \frac{|5|}{10} \times 100 = 50\%$ $\quad$ if $\lambda_3 = 5$ $\quad \frac{|-5|}{10} \times 100 = 50\%$