Numerical Methods in Physics
Numerical Solution of Poisson's Equation using Numerov's Method

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March 27, 2015

Numerical Methods in Physics
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1 Exercise 1

By substituting Φ with φ/r in equation 15 from the lab instruction, substituting in the expression for $\rho(r)$ and simplifying we get an expression

$$\frac{\partial^2\varphi(r)}{\partial r^2} = -\frac{1}{2}e^{-r}r \tag{1}$$

Putting this into the Numerov scheme we get

$$\varphi_{j+1} = 2\varphi_j - \varphi_{j-1} + \frac{h^2}{12}(S_{j+1} + 10S_j + S_{j-1}) \tag{2}$$

where S_j is

$$S_j = \frac{1}{2}e^{-r}r \tag{3}$$

Which is slightly confusing since S from the Numerov's method should be $-\frac{1}{2}e^{-r}r$. But due to it being simpler to write we defined S in this slightly obfuscated way.

2 Exercise 2

The following three tables shows the values requested in the lab instructions.

Table 1 – Table with values using $\varphi(1)$ as the analytical value

r (cm)	$\varphi(r)$ (statvolt/cm)	error
2	0.72933	-7.529e-07
4	0.945055	-1.91455e-06
6	0.990088	-3.08937e-06
8	0.998327	-4.26028e-06
10	0.999733	-5.42988e-06
12	0.999964	-6.5992e-06
14	1	-7.76847e-06
16	1.00001	-8.93773e-06
18	1.00001	-1.0107e-05
20	1.00001	-1.12762e-05

Table 2 – Table with values using $\varphi(1)$ as the numerical value of $\Phi(0)$ multiplied with the step length

r (cm)	$\varphi(r)$ (statvolt/cm)	error
2	0.730916	-0.00158653
4	0.948227	-0.00317347
6	0.994845	-0.00476043
8	1.00467	-0.00634738
10	1.00766	-0.00793432
12	1.00948	-0.00952127
14	1.0111	-0.0111082
16	1.01269	-0.0126952
18	1.01428	-0.0142821
20	1.01587	-0.0158691

Table 3 – Table with values using $\varphi(1)$ as 95% of the analytical value

r (cm)	$\varphi(r)$ (statvolt/cm)	error
2	0.679409	0.04992
4	0.845214	0.0998395
6	0.840326	0.149759
8	0.798644	0.199679
10	0.750129	0.249598
12	0.700439	0.299518
14	0.650556	0.349437
16	0.600642	0.399357
18	0.550724	0.449276
20	0.500804	0.499196

3 Exercise 3

In figure 1 and 2 we can see the difference between only using the $\varphi(h)$ value that is off by 5% and using the value compensated according how the lab instruction specified. The error has also been plotted for illustration. Lastly we have table with values for the compensated plots as done in the previous exercise.

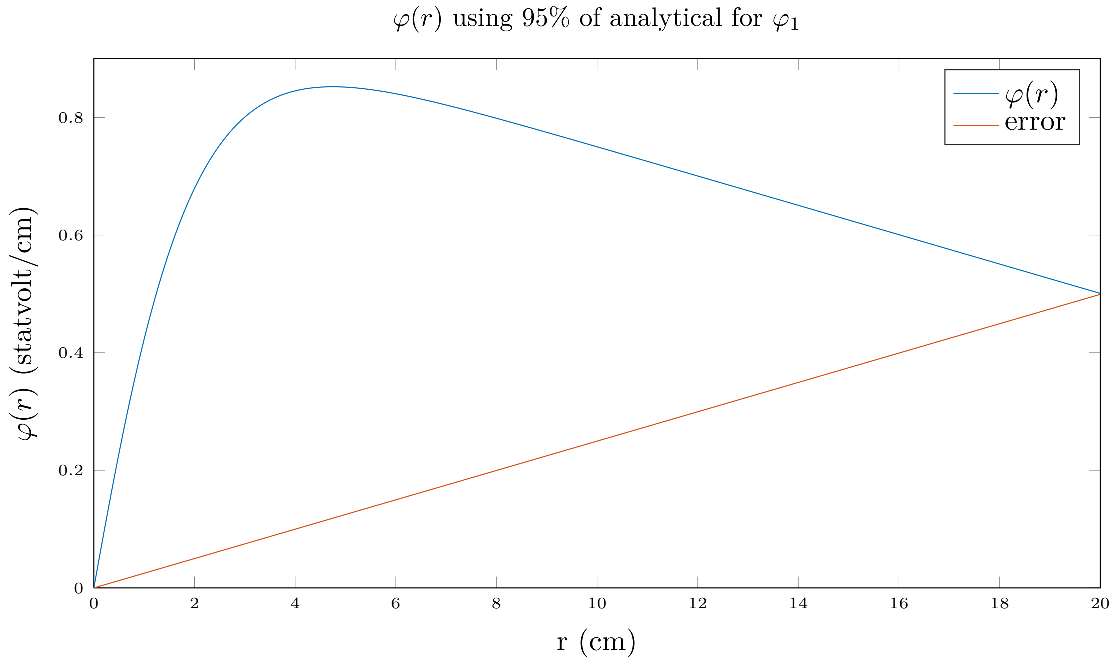

Figure 1 – Showing the value for φ when $\varphi(h)$ was off by 5%.

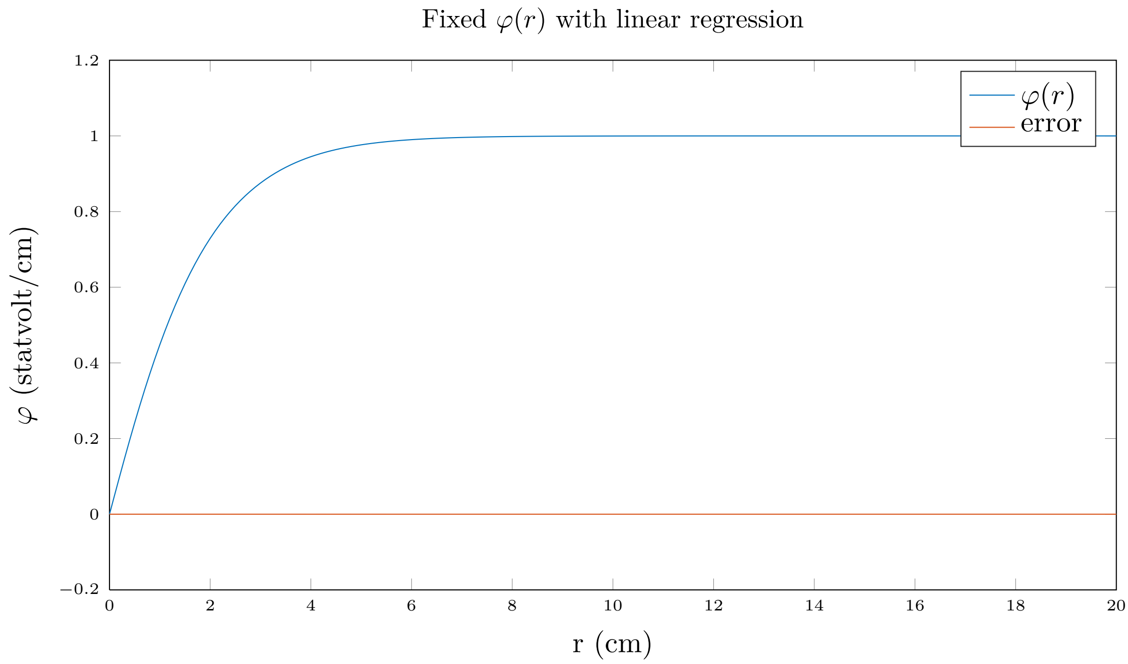

Figure 2 – Showing the compensated φ.

Table 4 – Table with values using $\varphi(1)$ as 95% of the analytical value, but compensated for the error.

r (cm)	$\varphi(r)$ (statvolt/cm)	error
2	0.729329	4.33527e-07
4	0.945054	-9.16666e-07
6	0.990086	-1.00871e-06
8	0.998324	-1.31314e-06
10	0.999729	-1.39958e-06
12	0.999959	-2.00949e-06
14	0.999996	-2.65223e-06
16	1	-3.01282e-06
18	1	-4.1523e-06
20	1	-4.02267e-06

4 How to Run and Find Code

The `c` code developed for this assignment can be found in the directory

`~jeve0010/Documents/jesper/fnm/lab4/src`

To compile the code simply use the `make` command. To run the `c` code execute the command `./num`. This program generate the plot and table values in the subdirectory `plots/data`. In order to plot this data and generate the table for the compensated φ values when using a bad start value simple execute the matlab script `phiPlot.m` that resides in the subdirectory `plots/code`.