# Laboration 4: The Poisson equation and the time-independent Shrodinger equation in 2D

Jesper Vesterberg (jumziey@gmail.com)

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Supervisor: Juan Carlos Araujo-Cabarcas

# 1 Problem 1

## 1.1 Poisson

For the poisson equation problem we have

$$-\nabla(a(x,y)\nabla u) = f, \quad x \in \Omega \tag{1}$$
$$u = 0, \quad x \in \delta\Omega. \tag{2}$$

In order to rewrite this equation on weak, or variational, form we start with multiplying the integral of $f$ with a test function $v$ where $v \in V_0$ where $V_0 = \{||v||_{L^2}^2 + ||\nabla v||^2 < \infty, v|_{\nabla\Omega} = 0\}$. This results in

$$\int_\Omega fvds = -\int_\Omega \nabla(a\nabla u)vds \tag{3}$$
$$= \int_\Omega a\nabla u\nabla vds - \int_{\delta\Omega} v\nabla ud\bar{l} \tag{4}$$
$$= \int_\Omega a\nabla u\nabla vds \tag{5}$$

where we used greens identity in the first step and the fact that the test function is zero at the boundary in the second step.

## 1.2 Shroedinger

When we look at the time dependent solution we have a significantly different problem.

$$\nabla^2 u + a(x,y)u = f, \quad x \in \Omega \tag{6}$$
$$u = 0, \quad x \in \delta\Omega \tag{7}$$

We continue and do the same thing as we did for the poisson problem. Starting with multiplying the integral of f with a test function $v \in V_0$.

$$\int_\Omega fvds = -\int_\Omega \nabla^2 uvds + \int_\Omega auvds \tag{8}$$
$$= \int_\Omega \nabla u\nabla vds - \int_{\delta\Omega} v\nabla ud\bar{l} + \int_\Omega auvds \tag{9}$$
$$= \int_\Omega \nabla u\nabla vds + \int_\Omega auvds \tag{10}$$

here the exact same steps as for the poisson derivation was made. The difference of course is the extra term $\int_\Omega auvds$ which is not within the derivate sign in the initial problem setup.

# 2 Problem 2