Hw8

Wednesday, April 1, 2020 11:14 PM

$$Lu = \begin{cases} -u'' + \frac{1}{\varepsilon} u & , 0 < x < 1 \\ u & x = 0, 1 \end{cases}$$

① Consistency. N.T.S. $\| L^h R^h v - R^h L v \|_\infty \le K \cdot h^\alpha$, where $v \in C^2[0,1]$

Here, we have

$$L^h = \begin{pmatrix} 1 & & & \\ -N^2 & 2N^2 + \frac{1}{\varepsilon} & -N^2 & \cdots \\ & \cdots & \cdots & \\ & \cdots & \cdots & 1 \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_N \end{pmatrix}$$

1° $(L^h R^h v)_0 = v_0$

$(R^h L v)_0 = Lv(x_0) = Lv(0) = v(0)$

thus $|(L^h R^h v)_0 - (R^h L v)_0| = 0$

2° Similarly $|(L^h R^h v)_N - (R^h L v)_N| = 0$

3° for $1 \le i \le N-1$

$(L^h R^h v)_i = -\delta_h \delta_{-h} v_i + \frac{1}{\varepsilon} v_i$

$(R^h L v)_i = Lv(x_i) = -v''(x_i) + \frac{1}{\varepsilon} v(x_i)$

$|(L^h R^h v)_i - (R^h L v)_i| = O(h^2)$

Thus L^h is consitent with order $\alpha = 2$.

② Stability. NTS: $\|v\|_\infty \le K \|L^h v\|_\infty \quad \forall v \in \mathbb{R}^{N+1}$

1° If $|v_0| = \|v\|_\infty$, then

$\|L^h v\|_\infty \ge |(L^h v)_0| = |v_0| = \|v\|_\infty$

2° If $|v_N| = \|v\|_\infty$, then

$\|L^h v\|_\infty \ge |(L^h v)_N| = |v_N| = \|v\|_\infty$

3° If $v_i = \|v\|_\infty$, for some $1 \le i \le N-1$, then

$(L^h v)_i = -r v_{i-1} + s v_i - t v_{i+1}$

$= r(v_i - v_{i-1}) + t(v_i - v_{i+1}) + (s - r - t) v_i$

$= r(v_i - v_{i-1}) + t(v_i - v_{i+1}) + 2 v_i \ge 2 v_i$

$\|L^h v\|_\infty \ge |(L^h v)_i| \ge |2 v_i| = 2\|v\|_\infty$

4° If $v_i = -\|v\|_\infty$ for some $1 \le i \le N-1$, then

$$\|L^h v\|_\infty \geq |(L^h v)_i| \geq |2v_i| = 2\|v\|_\infty$$

4° If $-v_i = \|v\|_\infty$ for some $1 \leq i \leq N-1$, then

$$(L^h v)_i = -r v_{i-1} + s v_i - t v_{i+1}$$

$$= -r(v_{i-1} - v_i) - t(v_{i+1} - v_i) + 2v_i \leq 2v_i \leq 0$$

$$\|L^h v\|_\infty \geq |(L^h v)_i| \geq |2v_i| = 2\|v\|_\infty$$

To sum up $\|v\|_\infty \leq \frac{1}{2}\|L^h v\|_\infty$ #