

known: $a, x_{\text{tot}}, t_{\text{tot}}$

$$x_{\text{tot}} = x_1 + x_2 + x_3$$

initial acceleration:

$$V_1(t) = at \Rightarrow x_1 = \int_0^{t_1} at dt = \frac{1}{2} at_1^2$$

constant velocity:

$$V_2 = V_2 \Rightarrow x_2 = \int_0^{t_2} V_2 dt = V_2 t_2$$

final deceleration:

$$V_3(t) = V_2 - at \Rightarrow x_3 = \int_0^{t_3} (V_2 - at) dt = V_2 t - \frac{1}{2} at^2 \Big|_0^{t_3} = V_2 t_3 - \frac{1}{2} at_3^2$$

assume $t_1 = t_3$:

$$x_{\text{tot}} = \frac{1}{2} at_1^2 + V_2 t_2 + V_2 t_1 - \frac{1}{2} at_1^2 \\ = V_2 (t_1 + t_2)$$

since $t_{\text{tot}} = 2t_1 + t_2$:

$$x_{\text{tot}} = V_2 (t_1 + t_{\text{tot}} - 2t_1) = V_2 (t_{\text{tot}} - t_1)$$

and $V_2 = at_1$:

$$x_{\text{tot}} = V_2 t_{\text{tot}} - \frac{V_2^2}{a} \Rightarrow \underline{V_2^2 - at_{\text{tot}} V_2 + ax_{\text{tot}} = 0}$$

Solve for const. velocity term:

$$V_2 = \frac{at \pm \sqrt{a^2 t^2 - 4ax}}{2}$$

only real when determinant > 0 so:

$$a^2 t_{\text{min}}^2 - 4ax = 0$$

$$\Rightarrow t_{\text{min}} = \sqrt{\frac{4x}{a}}$$

$$V = \frac{at - \sqrt{a^2 t^2 - 4ax}}{2}$$

use these two

