Exercise 1

Given Conditions

- W = 12 (total weeks)
- r = 3 (points earned per week - 1 theory + 2 programs)
- $p \in \{0, 0.25, 0.5, 0.7\}$
- n = 5 (samples each week)
- k = number of times you present
- T_j : temporary points before j^{th} presentation
- P = total permanent points after week W
- P_final = min (P,30) ... because cap is 30

Expected Presentations

CASE A : 60 Participants

Probability you are in the sample of 5 in a given week:

$$\frac{5}{60} = \frac{1}{12}$$

Thus, over 12 weeks :

$$E[k] = 12 \cdot \frac{1}{12} = 1$$

CASE B : 30 Participants with acceptable submissions

Now the 5 are chosen from only 30 people. Probability you are selected in a week :

$$\frac{5}{30} = \frac{1}{6}$$

Expected Presentations:

$$E[k] = 12 \cdot \frac{1}{6} = 2$$

Point Systems

Each wek : earn 3 tentative points. If you present, your current tentative points are multiplied by p and rounded up and become permanent.

Then tentative points reset to 0. At week 12, all leftover tentative points become permanent.

General Expected Value Calculation

- k : number of presents
- weeks t_1, t_2, ...

$$P = \Sigma_{j=1}^{k}[p \cdot 3(t_j - t_{j-1})] + 3(12 - t_k)$$

- t_j : current presentation week
- t_{j-1} : last presentation week
- $t_j - t_{j-1}$: how long has it passed after last presentation
- $3 \cdot (t_j - t_{j-1})$: 3 pts x temporary accumulated scores
- $p \cdot 3(t_j - t_{j-1})$: permanent score
- $+3 \cdot (12 - t_k)$: scores after the last presentations (because, these scores will be permanent without further presentation)

where $t_0 = 0$

Example

Let's say there will be 2 presentations ($t_1 = 4, t_2 = 8$)

1. j = 1 p $t_1 - t_0$ = 4 weeks
   - temporary score = 3 x 4 = 12
   - permanent score = p x 12
2. j =2
   - $t_2 - t_1$ = 8 - 4= 4 weeks
   - temporary score = 3 x 4 = 12
   - permanent score = p x 12
3. Last
   - $12 - t_2$ = 12 - 8 = 4 weeks
   - temporary score = 3 x 4 = 12 -> permanent

Total = 12 + 12 + 12 = 36 (but the cap is 30)

Computing E[P]

We find the expected value by considering all possible values of k :

$$E[P] = \Sigma_{k=0}^{1} 2P(kpresentations) \cdot E[P|kpresentations]$$

- if k = 0 : You never present. So all tentative points become permanent at the end

$$E[P|k=0] = 12 \cdot 3 = 36$$

- if k>=1 : k presentation times are a random uniform subset of {1,2,3,..,12}. By symmetry, the expected length of each interval between presentation is :

$$E[intervallength] = \frac{12}{m+1} weeks$$

Therefore :

- for k presentation intervals : $[p \cdot 3, \frac{12}{m+1}]$
- for final interval : $3 \cdot \frac{12}{m+1}$

Thus, expected total points(P) from k presentations :

$$E[P|k] = k \cdot [\frac{36p}{k+1}] + \frac{36}{k+1}$$

- each week gives 3 temp points => 12 weeks = 36
- multiply accumulated points by p
- Last interval becomes permanent without multiplying p

$$E[P|k] = [\frac{36pk}{k+1}] + \frac{36}{k+1}$$

$$E[P|k] = 36 \cdot \frac{pk+1}{k+1}$$

Apply points cap

Max points = 30

$$E[P_{final}|k] = min(E[P|k], 30)$$

Applying to Both Cases

Recap,

$$E[P|k] = 36 \cdot \frac{pk+1}{k+1}$$

CASE 60 Participants

`k=1` from Expected Presentation

$$E[P|1] = 36 \cdot \frac{p+1}{1+1}$$

Thus,

$$E[P|k=1] = 18(p+1)$$

- p = 0 -> 18
- p = 0.25 -> 22.5
- p = 0.5 -> 27
- p = 0.75 -> 31.5 -> 30

CASE 30 Participants

`k=2` from Expected Presentation

$$E[P|2] = 36 \cdot \frac{p+1}{1+2}$$

Thus,

$$E[P|k=2] = 12(p+1)$$

- p = 0 -> 12
- p = 0.25 -> 18
- p = 0.5 -> 24
- p = 0.75 -> 30

Conclusion

When only 30 people submit acceptable work, students present more frequently (E[k]=2), resulting in lower expected points for p<1 compared to the 60-participant case (E[k]=1), due to more frequent point resetting with p<1.