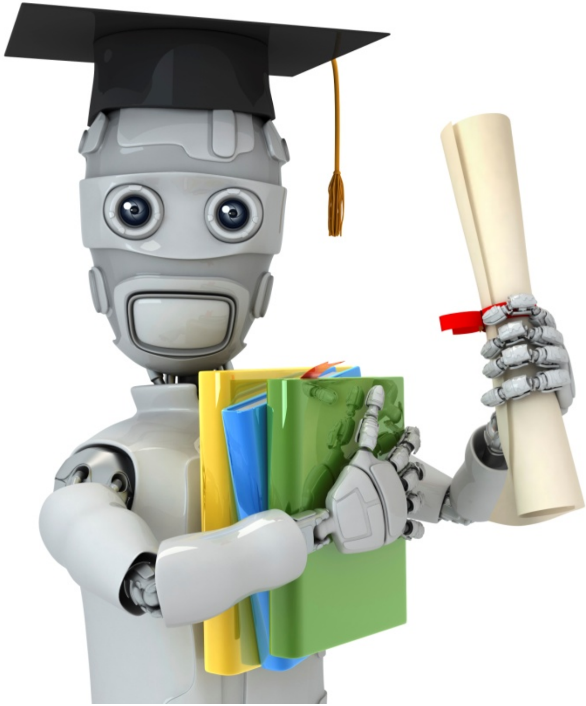


Machine Learning

# Linear Algebra review (optional)

# Matrices and vectors

**Matrix:** Rectangular array of numbers:

$$\begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

4 x2 matrix

$$\boxed{\mathbb{R}^{4\times 2}}$$

$$\begin{array}{cc} 2 & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\ & 3 \end{array}$$

2 x 3 matrix

$$\boxed{\mathbb{R}^{2\times 3}}$$

Dimension of matrix: number of rows x number of columns

# **Matrix Elements** (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$A_{ij} =$ "$i, j$ entry" in the $i^{th}$ row, $j^{th}$ column.

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

$\cancel{A_{43}}$ = undefined (error)

**Vector:** An n x 1 matrix.

$\mathbb{R}^{3\times2}$

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix} \leftarrow \text{4-dimensional vector.} \quad \mathbb{R}^4$$

$n = 4$

$y_i = i^{th}$ element

$y_1 = 460$

$y_2 = 232$

$y_3 = 315$

$\boxed{A, B, C, X}$ $\boxed{a, b, x, y}$

1-indexed vs 0-indexed:

y[1]

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

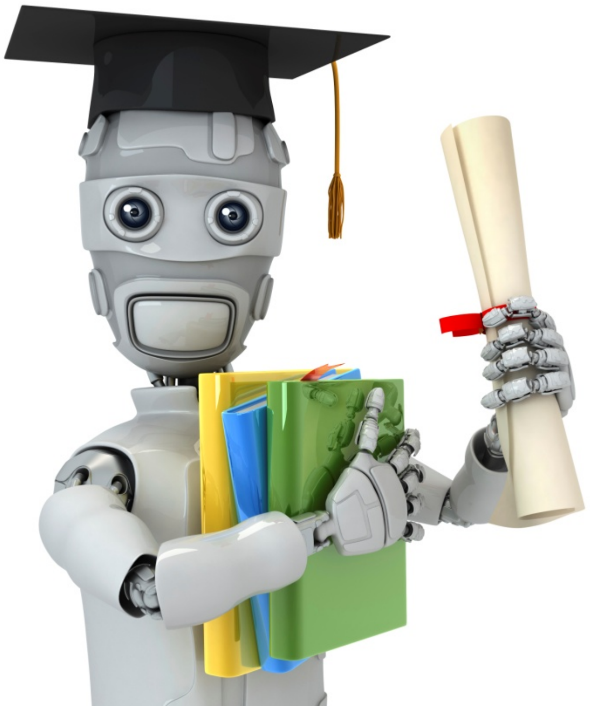
y[0]

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

0-indexed

we usually use this 1-indexed

Andrew Ng



Machine Learning

# Linear Algebra review (optional)

## Addition and scalar multiplication

# Matrix Addition

we can only add 2 matrix with same dimension

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{3 \times 2 \text{ matrix}} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix}}_{3 \times 2} = \underbrace{\begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}}_{3 \times 2}$$

$$\cancel{\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{3 \times 2} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix}}_{2 \times 2}} = \text{error}$$

# Scalar Multiplication

real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

$3 \times 2$ $\qquad$ $3 \times 2$

$$\boxed{\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4} = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

Andrew Ng

# Combination of Operands

$$\boxed{3 \times \begin{bmatrix}1\\4\\2\end{bmatrix}} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \boxed{\begin{bmatrix}3\\0\\2\end{bmatrix} /3}$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix}3\\12\\6\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}1\\0\\\frac{2}{3}\end{bmatrix}$$

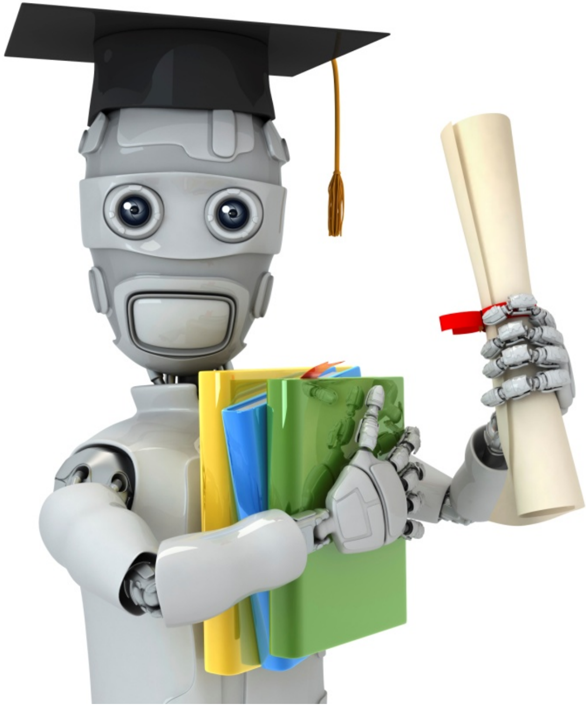
matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix}2\\12\\10\frac{1}{3}\end{bmatrix}$$

3x1 matrix

3-dimensional vector



Machine Learning

# Linear Algebra review (optional)

## Matrix-vector multiplication

# Example

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}}_{3\times 1 \text{ matrix}}$$

$1\times 1 + 3\times 5 = 16$

$4\times 1 + 0\times 5 = 4$

$2\times 1 + 1\times 5 = 7$

# Details:

$$A \times x = y$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \times \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

m x n matrix (m rows, n columns) | n x 1 matrix (n-dimensional vector) | m-dimensional vector

To get $y_i$, multiply $A$'s $i^{th}$ row with elements of vector $x$, and add them up.

# Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{14} \\ \boxed{13} \\ \boxed{-7} \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$\boxed{3} \times 4$ $\qquad 4 \times 1 \qquad 3 \times 1$

$$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$$

$$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$$

$$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$$

House sizes:

- → 2104
- → 1416
- → 1534
- → 852

$$h_\theta(x) = -40 + 0.25x$$

$h_\theta(x)$

Matrix 4x2

2x1 Vector

4x1 matrix

$h_\theta(2104)$

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -40\times 1 + 0.25\times 2104 \\ -40\times 1 + 0.25\times 1416 \\ \\ \\ \end{bmatrix}$$

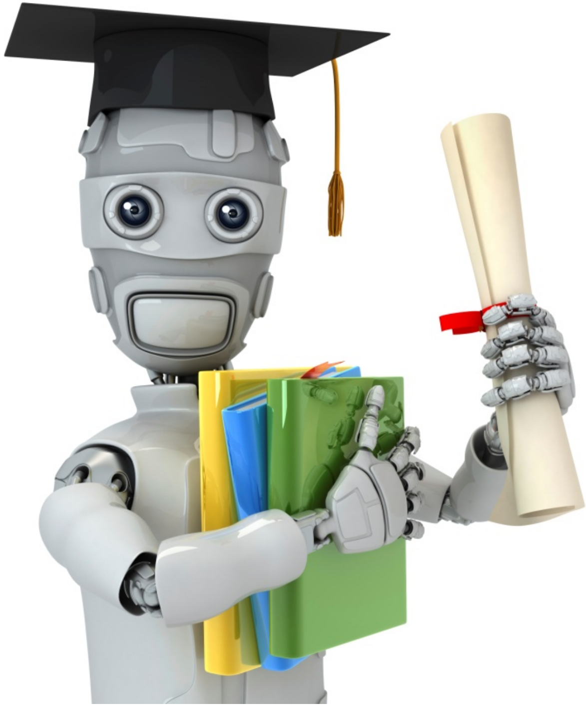
$h_\theta(1416)$

we can implement this one line of code

prediction = DataMatrix ⊗ parameters.

4x1

for i = 1:1000,
prediction (i) = ....



Machine Learning

# Linear Algebra review (optional)

## Matrix-matrix multiplication

# Example

$$\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}}_{2 \times 3} \underbrace{\begin{bmatrix} \boxed{\begin{matrix}1\\0\\5\end{matrix}} & \boxed{\begin{matrix}3\\1\\2\end{matrix}} \end{bmatrix}}_{3 \times 2} = \underbrace{\begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}}_{2 \times 2}$$

$$\underline{\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}} \times \underline{\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\underline{\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}} \times \underline{\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

# Details:

$$A \times B = C$$

$$\begin{bmatrix} \; \end{bmatrix} \times \begin{bmatrix} \; \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

| $A$ | $B$ | $C$ |
| --- | --- | --- |
| m x n matrix (m rows, n columns) | n×1<br>n x o matrix (n rows, o columns) | m x o matrix |

The $i^{th}$ column of the matrix $C$ is obtained by multiplying $A$ with the $i^{th}$ column of $B$. (for $i$ = 1,2,…,o)

# Example

$$\underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 3\times 3 \\ 2\times 0 + 5\times 3 \end{bmatrix} = \cancel{\begin{bmatrix} 9 \\ 15 \end{bmatrix}} \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 2 \\ 2\times 1 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

Matrix

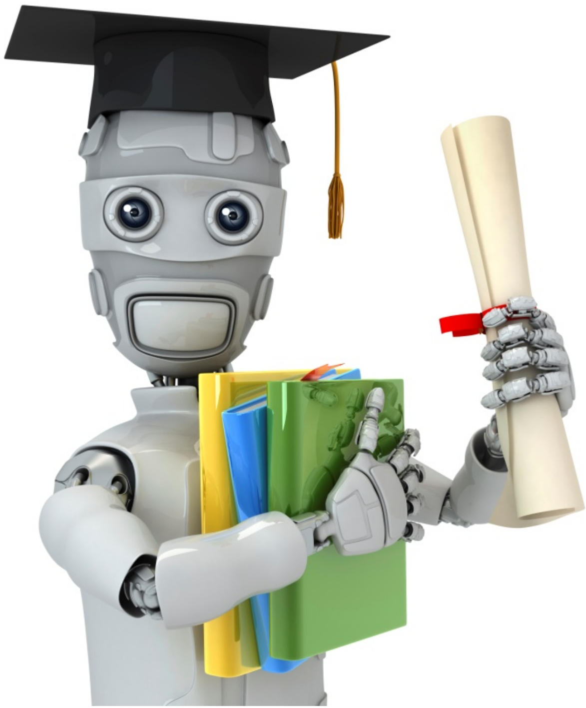
Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Prediction of first $h_\theta$

Predictions of $2^{nd}$ $h_\theta$

Andrew Ng



Machine Learning

# Linear Algebra review (optional)

## Matrix multiplication properties

$3 \times 5 = 5 \times 3$ "Commutative"

Let $A$ and $B$ be matrices. Then in general,

$$A \times B \neq B \times A. \text{ (not commutative.)}$$

E.g.
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\neq$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

$3 \times 5 \times 2$

$3 \times 10 = 30 = 15 \times 2$

$3 \times (5 \times 2) = (3 \times 5) \times 2$

"Associative"

$A \times (B \times C)$

$(A \times B) \times C$

$$A \times B \times C.$$

Let $D = B \times C$. Compute $A \times D$.

Let $E = A \times B$. Compute $E \times C$.

$A \times (B \times C)$

$(A \times B) \times C$

Same answer.

matrix computation is also associative !!

# Identity Matrix

1 is identity

$\boxed{1 \times z = z \times 1 = z}$ for any $z$

Denoted $I$ (or $\boxed{I_{n \times n}}$).

Examples of identity matrices:

$[1]$ $1 \times 1$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
2 x 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
3 x 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
4 x 4

Informally:

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots \\ 0 & & 1 \end{bmatrix}$$

For any matrix $A$,

$$\underset{m \times n}{A} \cdot \underset{n \times n}{\boxed{I}} = \underset{m \times m}{\boxed{I}} \cdot \underset{m \times n}{A} = \underset{m \times n}{A}$$

different dimension !!

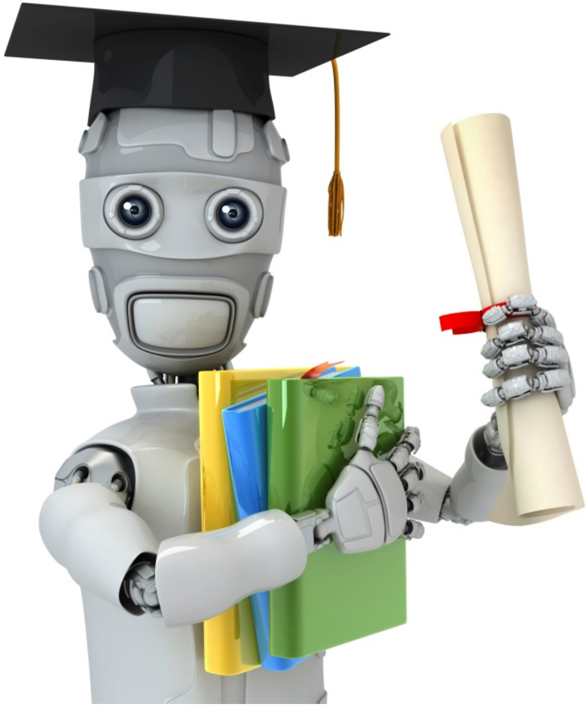
$I_{n \times n}$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓

different dimension !!



Machine Learning

# Linear Algebra review (optional)

## Inverse and transpose

$1 =$ "identity"

$3 \boxed{(3^{-1})} = 1$

$\frac{1}{3}$

$12 \times (12^{-1}) = 1$

$\frac{1}{12}$

$0 \; (0^{-1})$ undefined

zero has no inverse
square matrix of all zero elements has no inverse

Not all numbers have an inverse.

---

**Matrix inverse:**

square matrix
(#rows = #columns)

$A^{-1}$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If A is an m x m matrix, and if it has an inverse,

$$\rightarrow \quad A(A^{-1}) = A^{-1}A = I.$$

e.g.

$$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times 2 \\ A}} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

$A^{-1}A$

only square matrix
has inverse

square

Matrices that don't have an inverse are "singular" or "degenerate"

# Matrix Transpose

Example:

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix} \qquad B = \underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$

2 x 3 3 x 2

Let $A$ be an m x n matrix, and let $B = A^T$.
Then $B$ is an n x m matrix, and

$$B_{\underline{ij}} = A_{\underline{ji}}.$$

$B_{12} = A_{21} = 2$

$B_{32} = 9$ $A_{23} = 9$.