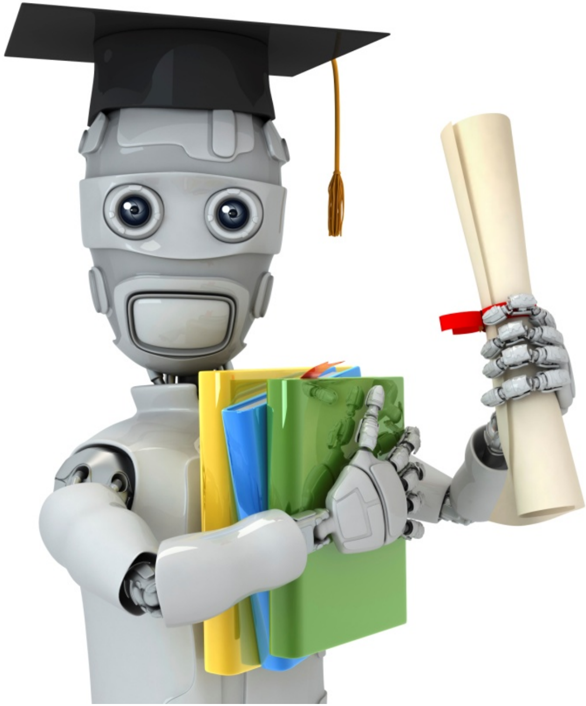


Machine Learning

Logistic Regression

Classification

Classification

- Email: Spam / Not Spam?
- Online Transactions: Fraudulent (Yes / No)?
- Tumor: Malignant / Benign ?

variable we try to predict is 0,1,...

$y \in \{0, 1\}$

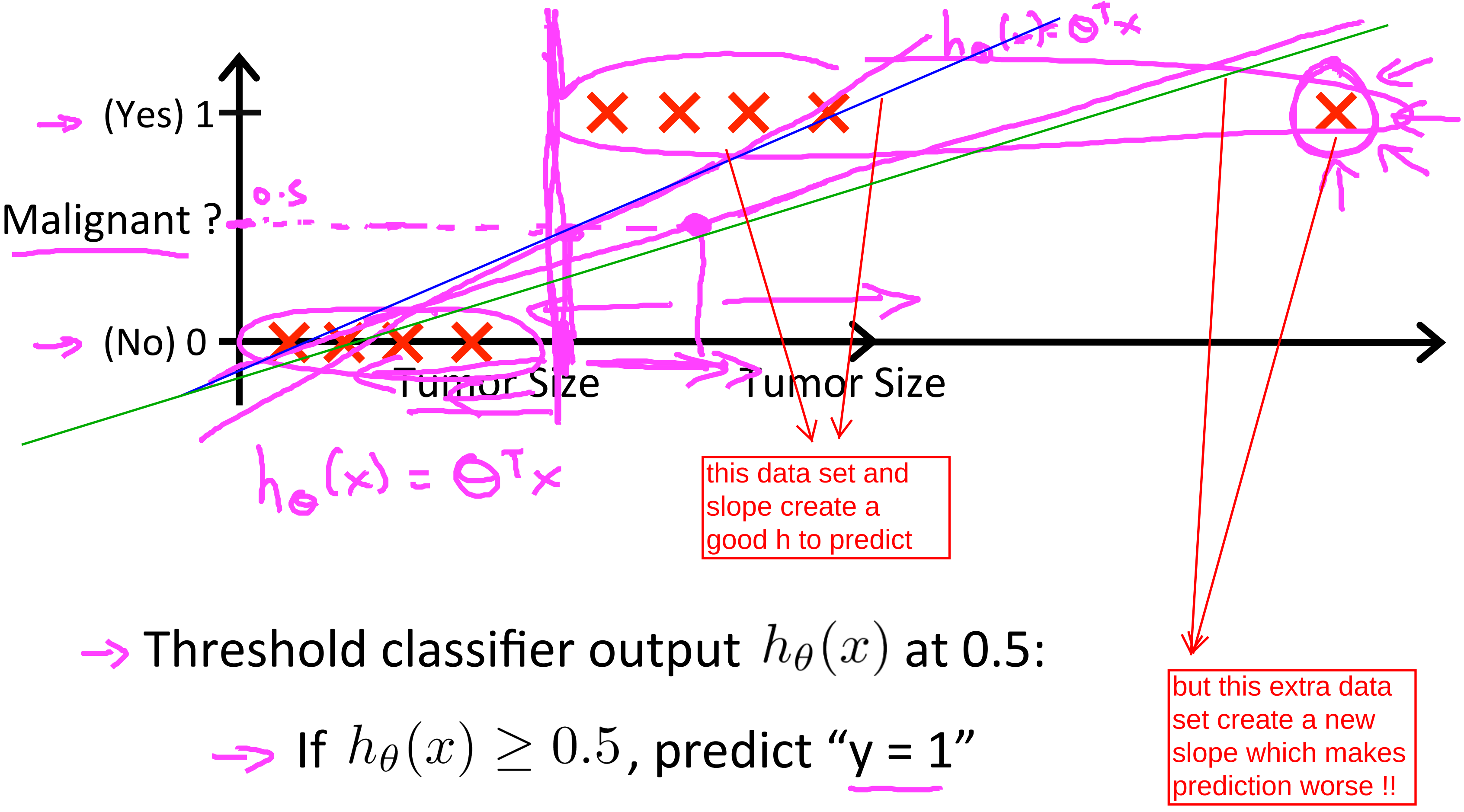
0: "Negative Class" (e.g., benign tumor)

sometimes = absence of xxx

1: "Positive Class" (e.g., malignant tumor)

presence of xxx

$y \in \{0, 1, 2, 3\}$



Threshold classifier output $h_\theta(x)$ at 0.5:

If $h_\theta(x) \geq 0.5$, predict "y = 1"

If $h_\theta(x) < 0.5$, predict "y = 0"

apply linear regression to classification problem isn't a good idea

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Classification: $\boxed{y = 0 \text{ or } 1}$

$h_\theta(x)$ can be > 1 or < 0

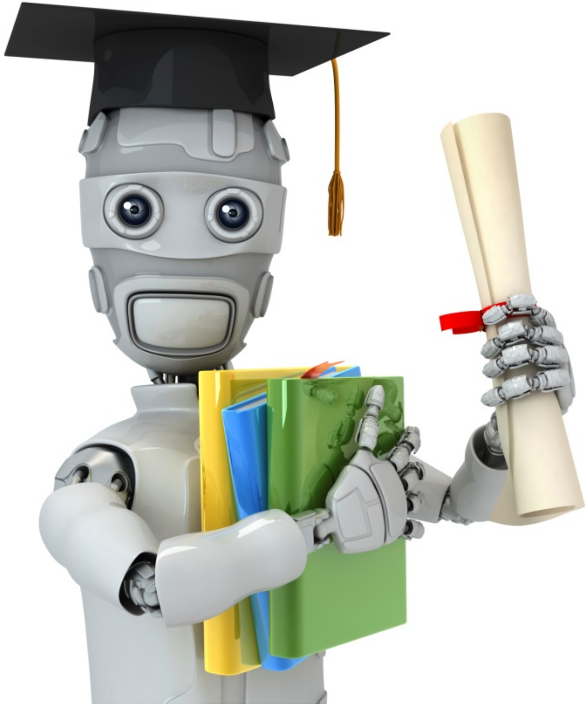
if all examples are 0 or 1, linear regression (h) can still give results >> 1 or << 0

Logistic Regression: $0 \leq h_\theta(x) \leq 1$

the prediction will always >= 0 and <= 1

Classification

Logistic regression is actually a classification algorithm that we apply to setting that label y is discrete value !!!!



Logistic Regression

Hypothesis Representation

Machine Learning

Logistic Regression Model

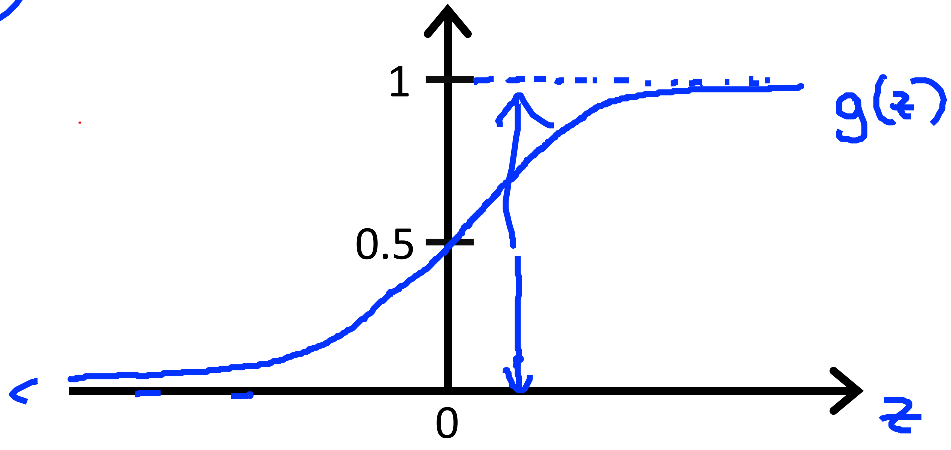
Want $\boxed{0 \leq h_\theta(x) \leq 1}$

$$h_\theta(x) = g(\theta^T x)$$

$$\boxed{h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}}$$

$$g(z) = \frac{1}{1+e^{-z}}$$

$\theta^T x$



Sigmoid function

Logistic function

$= g$

Parameters θ.

$h_\theta(x)$

Interpretation of Hypothesis Output

$h_\theta(x)$ = estimated probability that $\boxed{y = 1}$ on input x

Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

for theta_0

$h_\theta(x) = 0.7$ $\quad$ $y=1$

Tell patient that 70% chance of tumor being malignant

$h_\theta(x) = P(y=1|x;\theta)$

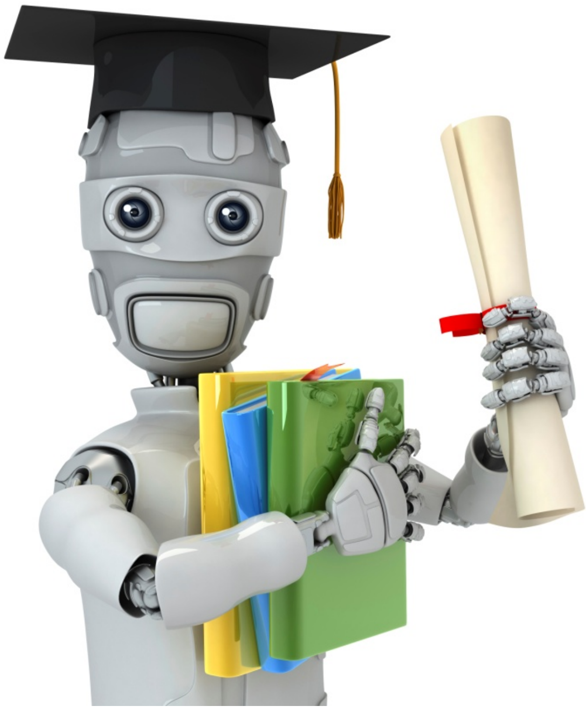
$y = 0$ or 1

since its a classification problem, y needs to be 0 or 1

"probability that y = 1, given x, parameterized by θ"

$$P(y=0|\cancel{x;}\theta) + \boxed{P(y=1|\cancel{x;\theta})} = 1$$

$$P(y=0|x;\theta) = 1 - \boxed{P(y=1|x;\theta)}$$



Logistic Regression

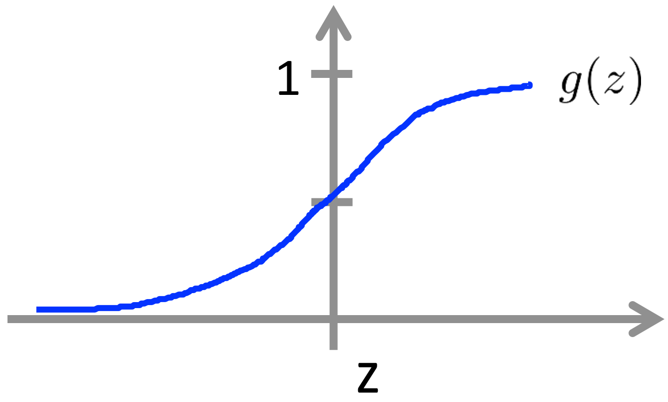
Decision boundary

Machine Learning

Logistic regression

$$h_\theta(x) = g(\theta^T x)$$ $\boxed{\text{= P (y=1 | x;theta)}}$

$$g(z) = \frac{1}{1+e^{-z}}$$



Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

$\theta^T x \geq 0$

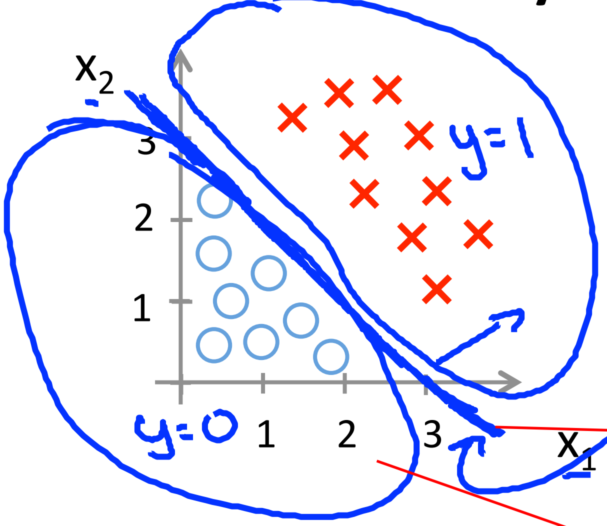
$g(z) \geq 0.5$
when $z \geq 0$
$h_\theta(x) = g(\theta^T x$

predict "$y = 0$" if $h_\theta(x) < 0.5$

$\theta^T x < 0$

$g(z) < 0.5$
when $z < 0$

Decision Boundary



$$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

$\theta_0 = -3$, $\theta_1 = 1$, $\theta_2 = 1$

Decision boundary

Predict "$y = 1$" if $\underbrace{-3 + x_1 + x_2}_{\theta^T x} \geq 0$

$\rightarrow x_1 + x_2 \geq 3$

x_1, x_2

$\rightarrow h_\theta(x) = 0.5$

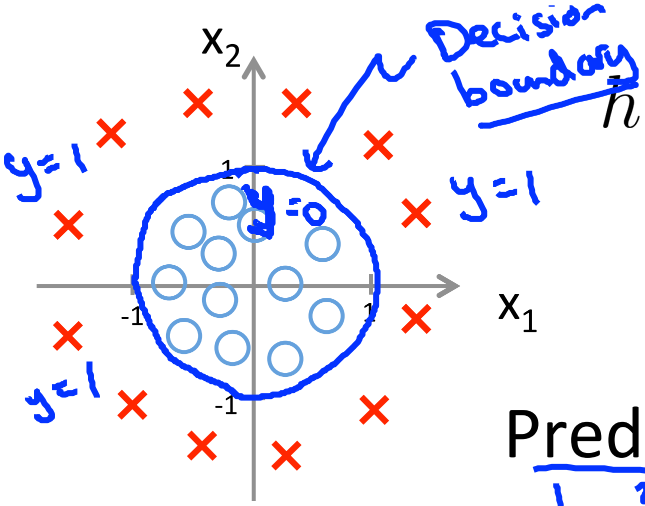
$\boxed{x_1 + x_2 = 3}$

$x_1 + x_2 < 3$

$\rightarrow y = 0$

Non-linear decision boundaries

the theta define the decision boundary, the training set is not what we use to define the decision boundary, it is used to fit the theata

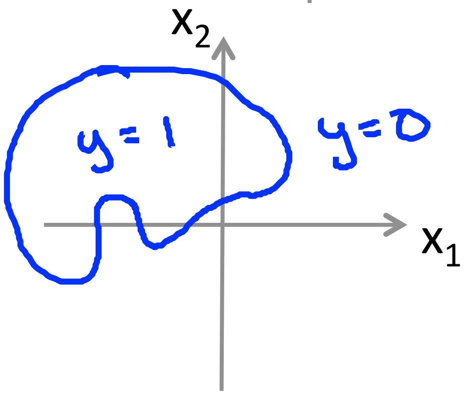


$$h_\theta(x) = g(\overset{=-1}{\theta_0} + \overset{=0}{\theta_1} x_1 + \overset{=0}{\theta_2} x_2 + \underset{=1}{\theta_3} x_1^2 + \underset{=1}{\theta_4} x_2^2)$$

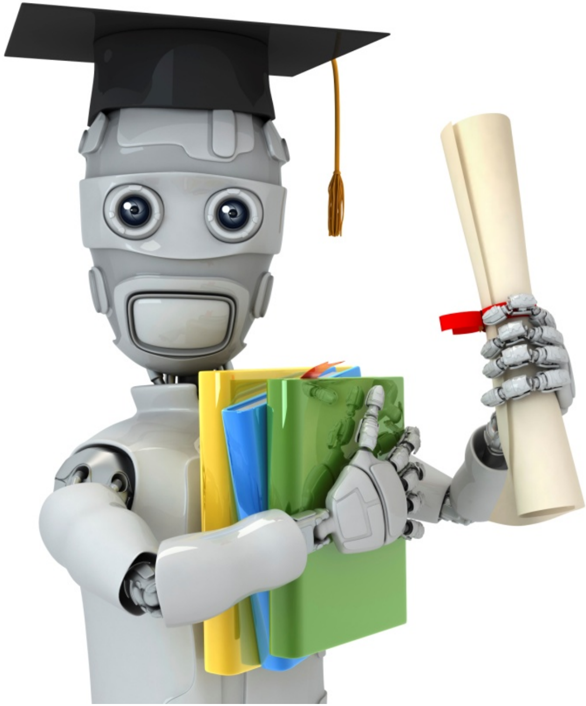
$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Predict "$y = 1$" if $\underbrace{-1 + x_1^2 + x_2^2 \geq 0}_{x_1^2 + x_2^2 \geq 1}$

$\boxed{x_1^2 + x_2^2 = 1}$



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 \underline{x_1^2} + \theta_4 \underline{x_1^2 x_2} + \theta_5 \underline{x_1^2 x_2^2} + \theta_6 \underline{x_1^3 x_2} + \dots)$$



Machine Learning

Logistic Regression

Cost function

Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \cdots, (x^{(m)}, y^{(m)})\}$

m examples

$$x \in \underbrace{\begin{bmatrix} x_0 \\ x_1 \\ \cdots \\ x_n \end{bmatrix}}_{\mathbb{R}^{n+1}} \qquad x_0 = 1, y \in \{0, 1\}$$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

give this train set, how do we choose or how do we fit the parameter theta

How to choose parameters θ ?

it is the cost of my learning algorithm to have to pay if it outputs that value, if its prediction is h of x, and the actual label was y,

Cost function

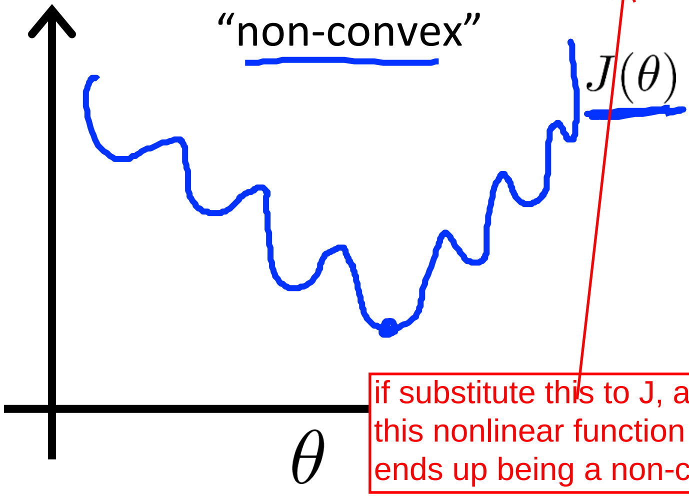
→ ~~Linear~~ regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \cancel{\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}$

Logistic

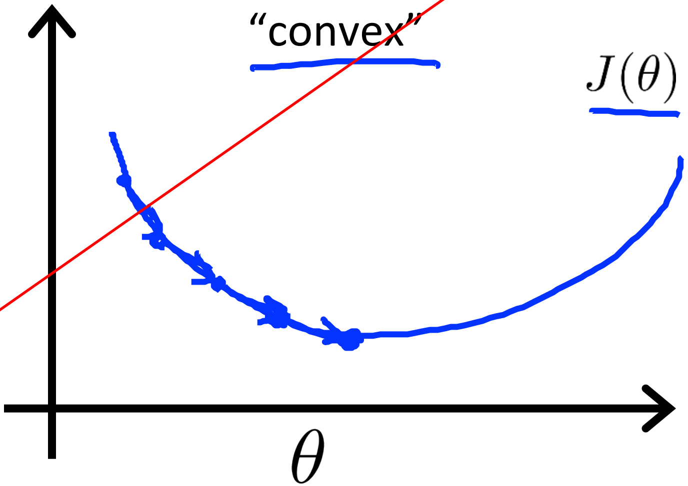
→ $\mathrm{cost}(h_\theta(x^{(i)}), y)$

$$\mathrm{Cost}(h_\theta(x), y) = \boxed{\frac{1}{2}\left(h_\theta(x) - y\right)^2} \leftarrow \boxed{\frac{1}{1+e^{-\theta^T x}}}$$

"non-convex"



"convex"



if substitute this to J, and because of this nonlinear function of h, J(theta) ends up being a non-convex function

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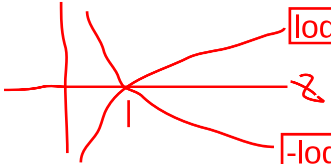
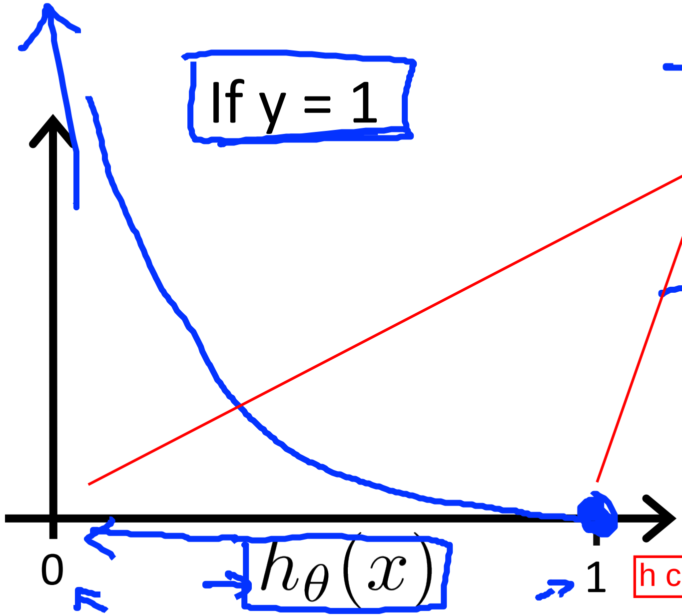
Logistic regression cost function



$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

log is nature log !!!!!!



h can be only between o and 1

- Cost $= 0$ if $y = 1, h_\theta(x) = 1$
  But as $h_\theta(x) \to 0$
  $Cost \to \infty$
- Captures intuition that if $h_\theta(x) = 0$, (predict $P(y = 1|x;\theta) = 0$), but $\boxed{y = 1}$, we'll penalize learning algorithm by a very large cost.

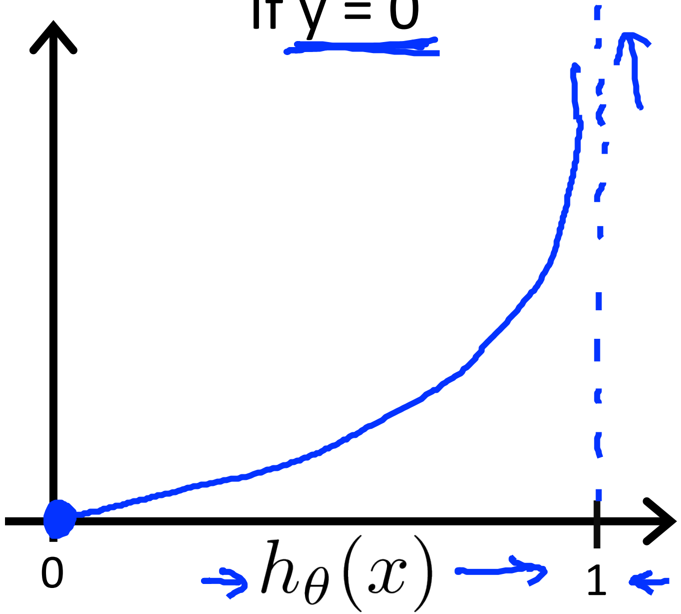
Logistic regression cost function

* if h=y, then cost = 0 for y=0 and y=1
* if y=0, then cost -> inf as h -> 1
* regardless of whether y=0 or y=1, if h=0.5, the cost > 0

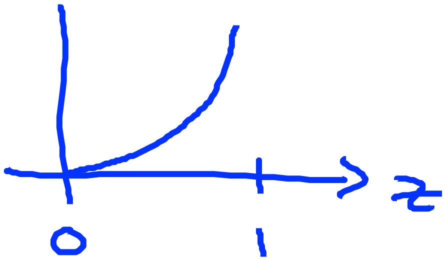
$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ \boxed{-\log(1 - h_\theta(x))} & \text{if } y = 0 \end{cases}$$

with the choice of cost function, this will give us a convex optimization problem,the overall cost function will be convex and local minimum free.

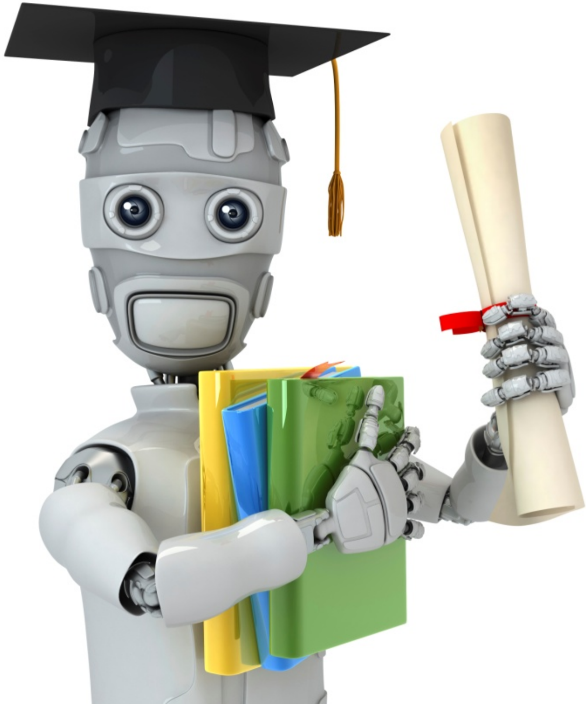
If y = 0



$-\log(1-z)$



if h=0,and y=0,then cost =0
if h=1,and y=0, then cost = inf



Machine Learning

Logistic Regression

Simplified cost function and gradient descent

Logistic regression cost function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Note: $y = \boxed{0}$ or $\boxed{1}$ always

$$\mathrm{Cost}(h_\theta(x), y) = -\underbrace{y}_{=0}\log(h_\theta(x)) - \underbrace{(1-y)}_{=0}\underbrace{\log(1-h_\theta(x))}_{=1}$$

If $y=1$: $\mathrm{Cost}(h_\theta(x), y) = -\log h_\theta(x)$

If $y=0$: $\mathrm{Cost}(h_\theta(x), y) = -\log(1-h_\theta(x))$

principle of maximumlikelihood estimation

Logistic regression cost function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log\left(1 - h_\theta(x^{(i)})\right)\right]$$

To fit parameters θ :

$$\boxed{\min_\theta J(\theta) \quad \text{Get } \theta}$$

To make a prediction given new x:

Output $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

$p(y=1 \mid x; \theta)$

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1-y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

$\}$ (simultaneously update all θ_j)

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

Gradient Descent

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))\right]$$

Want $\min_\theta J(\theta)$:

$$\Theta = \begin{bmatrix} \Theta_0 \\ \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_n \end{bmatrix}$$

for i = 0 to n

use a for loop or vectorize implementation

Repeat {

$$\boxed{\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (\boxed{h_\theta(x^{(i)})} - y^{(i)}) x_j^{(i)}}$$

(simultaneously update all θ_j)

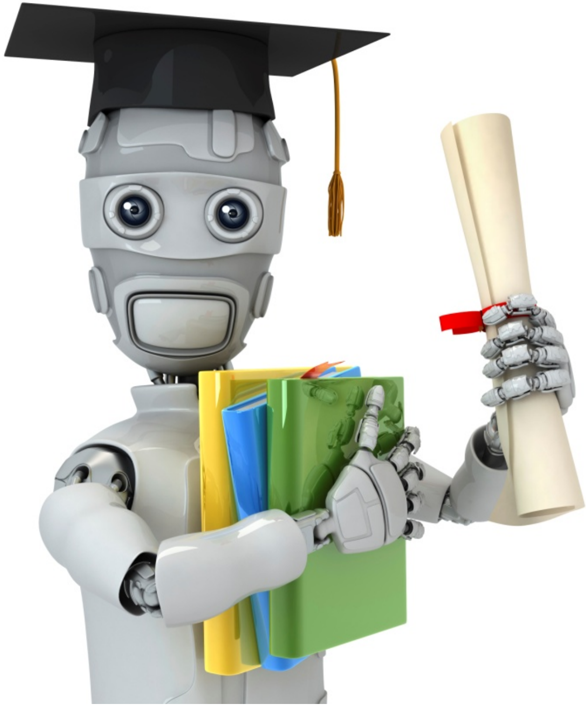
}

$h_\Theta(x) = \Theta^T x$

$h_\Theta(x) = \frac{1}{1+e^{-\Theta^T x}}$

Algorithm looks identical to linear regression!

compute and visualize the J to make sure that gradient decent converge !!
idea of feature scaling also applies to gradient decent for logistic regression which help it run faster



Machine Learning

Logistic Regression

Advanced optimization

Optimization algorithm

Cost function $J(\theta)$. Want $\min_\theta J(\theta)$.

Given θ, we have code that can compute

- $\boxed{J(\theta)}$
- $\boxed{\frac{\partial}{\partial\theta_j} J(\theta)}$ (for $j = 0, 1, \ldots, n$)

Gradient descent:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial\theta_j} J(\theta)}$$

$\}$

Optimization algorithm

Given θ, we have code that can compute

- $J(\theta)$
- $\frac{\partial}{\partial\theta_j} J(\theta)$ (for $j = 0, 1, \ldots, n$)

Optimization algorithms:

- Gradient descent
- Conjugate gradient
- BFGS
- L-BFGS

Advantages:

- No need to manually pick α
- Often faster than gradient descent.

Disadvantages:

- More complex

Example: $\min_\theta J(\theta)$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \qquad \theta_1 = 5, \theta_2 = 5.$$

$$\boxed{J(\theta) = (\theta_1 - 5)^2 + (\theta_2 - 5)^2}$$

$$\frac{\partial}{\partial \theta_1} J(\theta) = 2(\theta_1 - 5)$$

$$\frac{\partial}{\partial \theta_2} J(\theta) = 2(\theta_2 - 5)$$

```
function [jVal, gradient]
              = costFunction(theta)
  jVal = (theta(1)-5)^2 + ...
              (theta(2)-5)^2;
  gradient = zeros(2,1);
  gradient(1) = 2*(theta(1)-5);
  gradient(2) = 2*(theta(2)-5);
```

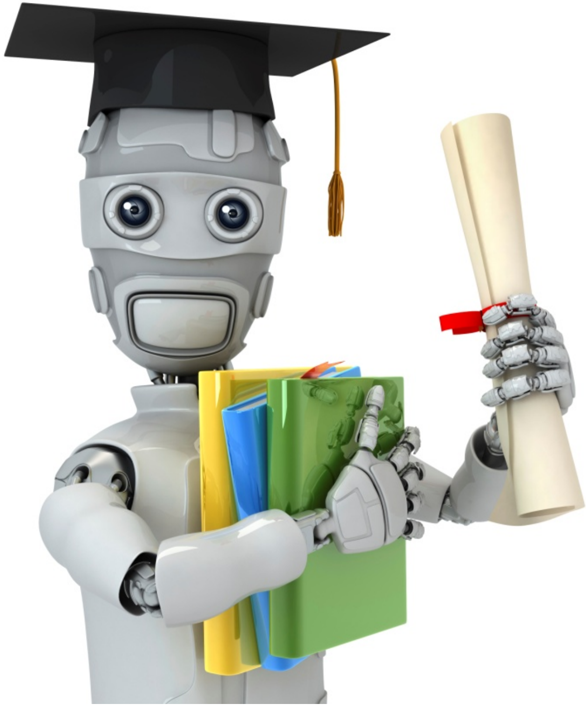
```
options = optimset('GradObj', 'on', 'MaxIter', '100');
initialTheta = zeros(2,1);
[optTheta, functionVal, exitFlag] ...
      = fminunc(@costFunction, initialTheta, options);
```

$\theta \in \mathbb{R}^d \quad d \geq 2.$

theta $= \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$ — theta(1), theta(2), theta(n+1)

```
function [jVal, gradient] = costFunction(theta)

    jVal = [code to compute J(θ)];

    gradient(1) = [code to compute ∂/∂θ_0 J(θ)];

    gradient(2) = [code to compute ∂/∂θ_1 J(θ)];

    ⋮

    gradient(n+1) = [code to compute ∂/∂θ_n J(θ)];
```

$J(\theta)$, $\frac{\partial}{\partial\theta_0}J(\theta)$, $\frac{\partial}{\partial\theta_1}J(\theta)$, $\frac{\partial}{\partial\theta_n}J(\theta)$



Machine Learning

Logistic Regression

Multi-class classification: One-vs-all

Multiclass classification

Email foldering/tagging: Work, Friends, Family, Hobby

$y=1$ $y=2$ $y=3$ $y=4$

Medical diagrams: Not ill, Cold, Flu

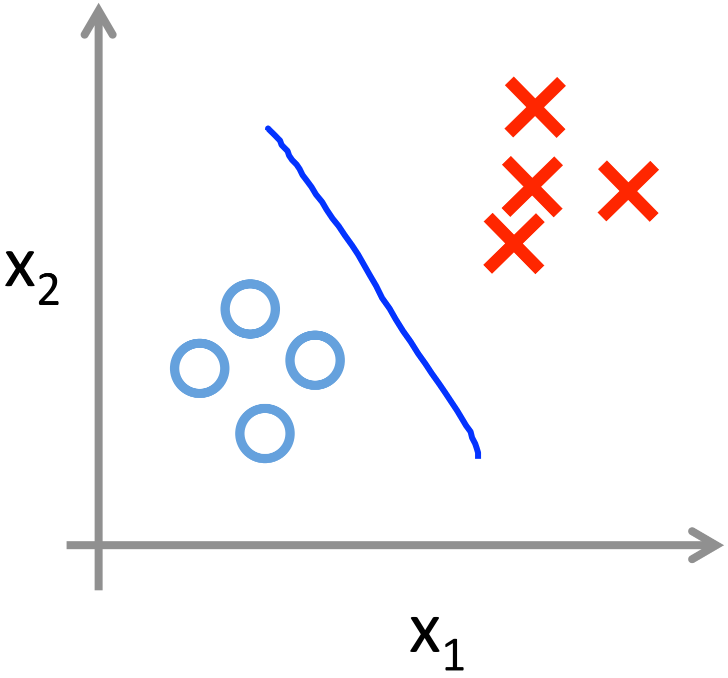
$y=1$ 2 3

Weather: Sunny, Cloudy, Rain, Snow

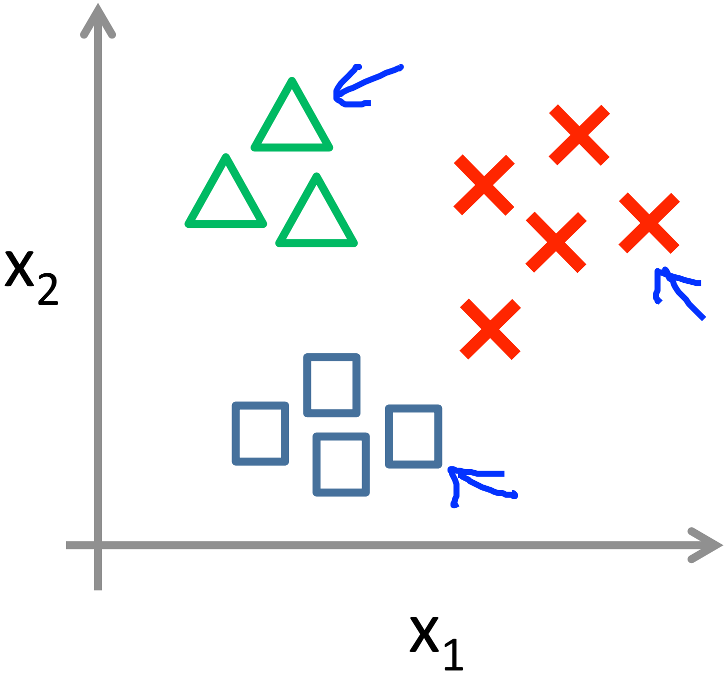
$y=1$ 2 3 4

0 1 2 3

Binary classification:



Multi-class classification:



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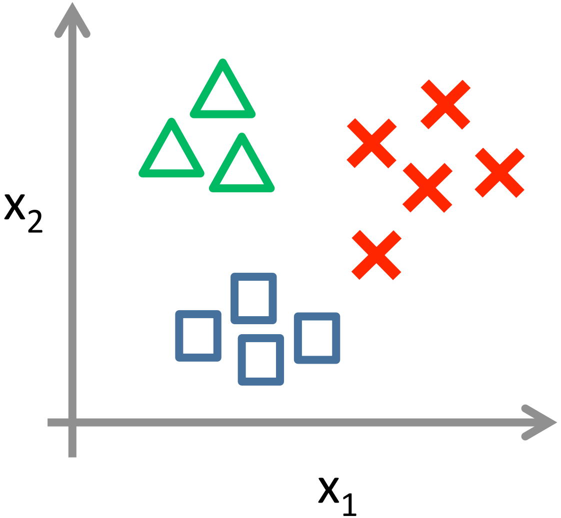
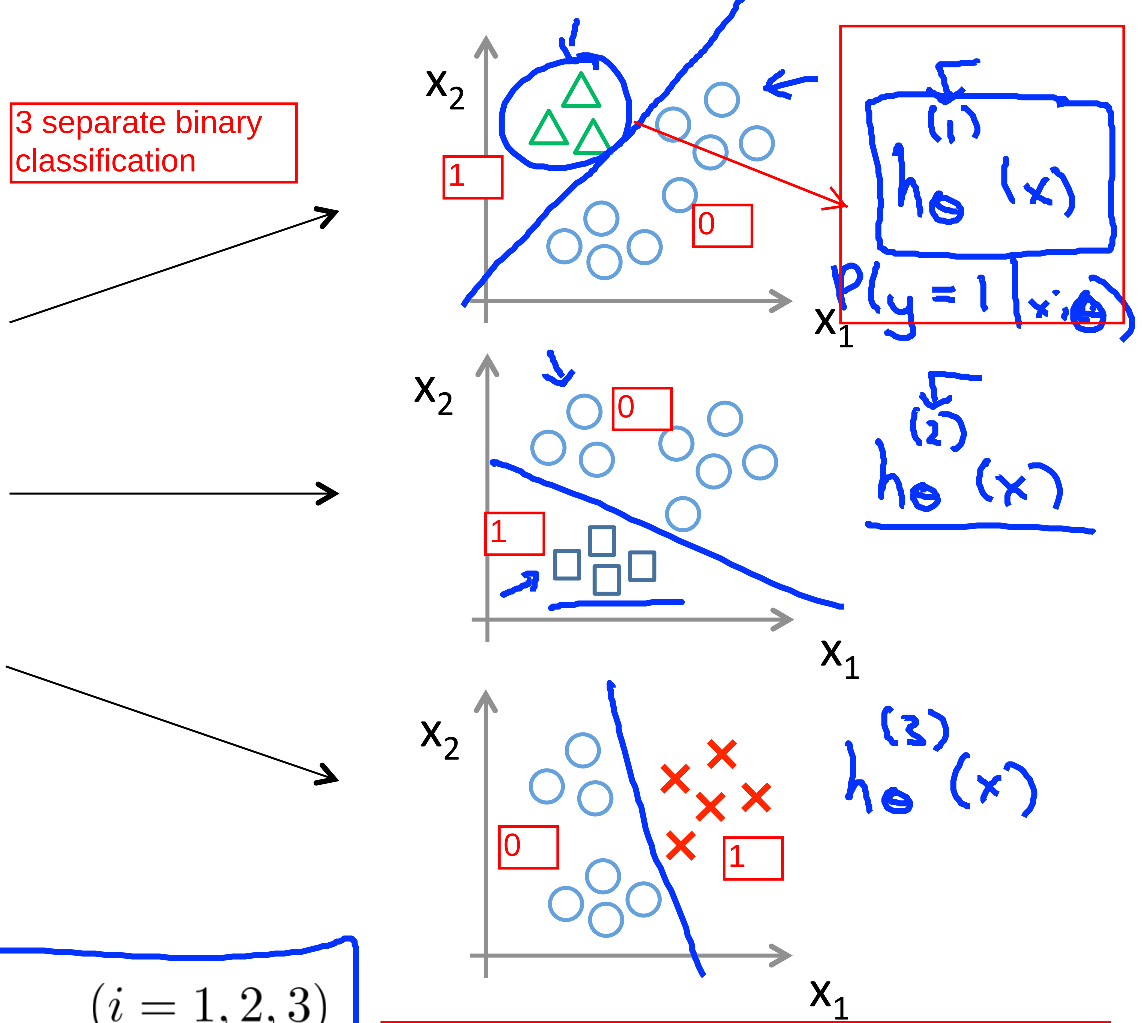
One-vs-all (one-vs-rest):

3 separate binary classification





Class 1: △

Class 2: □

Class 3: ✕

$$\boxed{h_\theta^{(i)}(x) = P(y=i|x;\theta) \qquad (i=1,2,3)}$$

fit 3 classifiers (i=1,2,3)

we train n classifiers when there are n classes

One-vs-all

Train a logistic regression classifier $h_\theta^{(i)}(x)$ for each class i to predict the probability that $y = i$.

On a new input x, to make a prediction, pick the class i that maximizes

$$\max_i h_\theta^{(i)}(x)$$