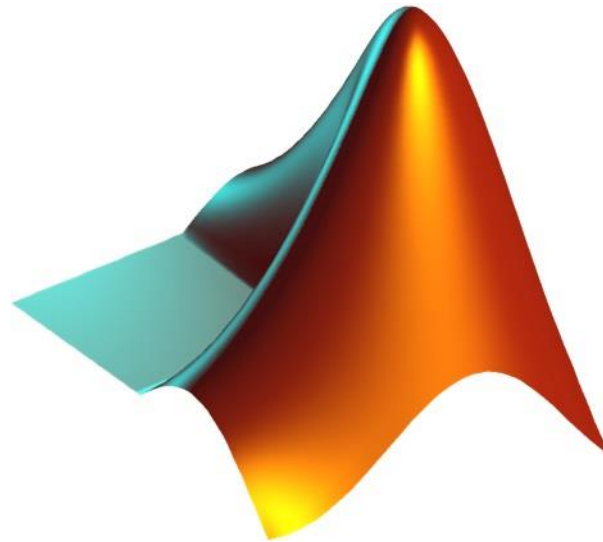


# MATLAB / Simulink Lab Course

## Control System Toolbox

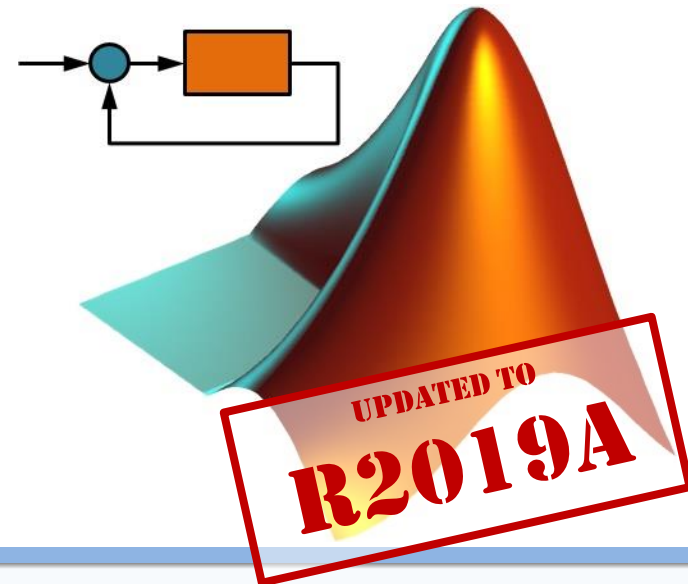


# Objectives & Preparation “Control System Toolbox”

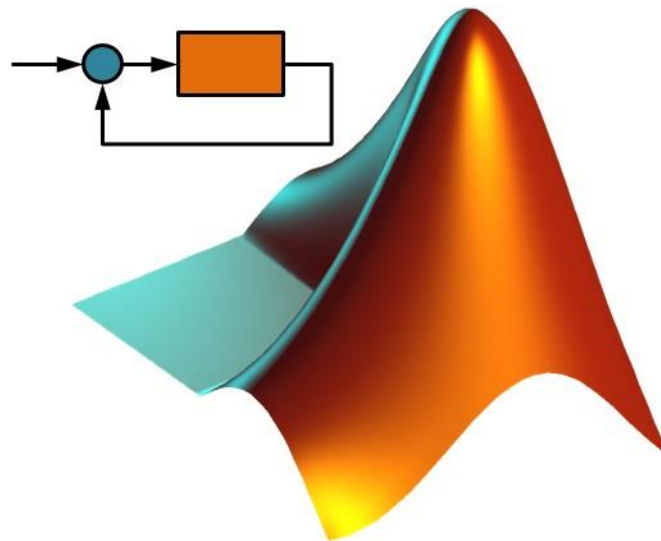
- Which MathWorks products are covered?
  - ⇒ Control System Toolbox
  
- What skills are learnt?
  - ⇒ Linear system representation
  - ⇒ Model interconnection and transformation
  - ⇒ Stability analysis, time domain analysis and frequency domain analysis
  - ⇒ Control design and controller tuning
  
- How to prepare for the session?
  - ⇒ MathWorks Tutorials:
    - [https://de.mathworks.com/help/control/getting-started-with-control-system-toolbox.html?s\\_tid=CRUX\\_lftnav](https://de.mathworks.com/help/control/getting-started-with-control-system-toolbox.html?s_tid=CRUX_lftnav)
    - <https://de.mathworks.com/help/control/getstart/linear-lti-models.html>
    - <https://de.mathworks.com/help/control/ug/control-system-modeling-with-model-objects.html>

# Outline

1. Introduction
2. Linear System Representation
  - Basic Models, Tunable Models and PID Models
  - Model Attributes
3. Model Interconnection
4. Model Transformation
  - Type and Continuous-Discrete Conversion
  - Simplification, State-Coordinate Transform and Modal Decomposition
5. Linear Analysis
  - Stability Analysis
  - Time Domain and Frequency Domain Analysis
6. Control Design
  - PID Controller Tuning
  - SISO Feedback Loops
  - Pole Placement, LQR, LQG and Kalman Estimator
7. Matrix Computations
8. List of Commands

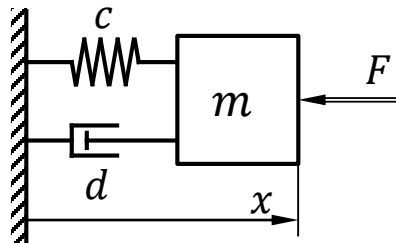


# 1. Introduction



# Introduction

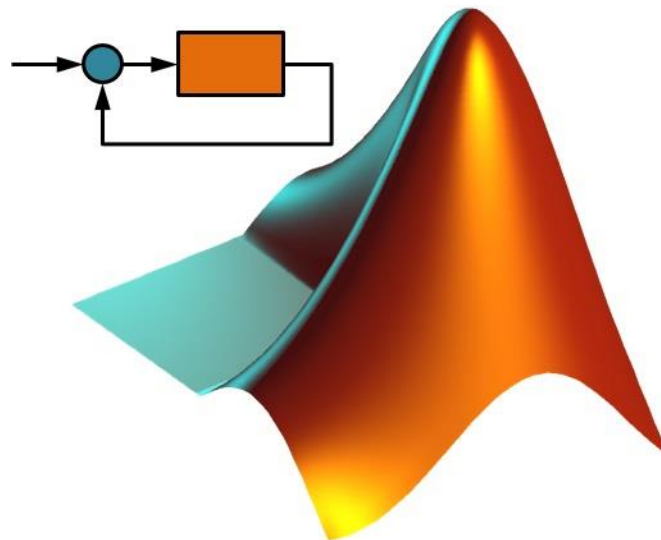
- MATLAB's **Control System Toolbox** is useful for
  - modeling,
  - analysis and
  - design of **linear** dynamic systems
- Dynamic systems change over time, like the mass-spring-damper system below.



- The toolbox provides key functionalities for the design of **control systems**. Control engineering aims at modifying dynamic behavior using feedbacks of system outputs.

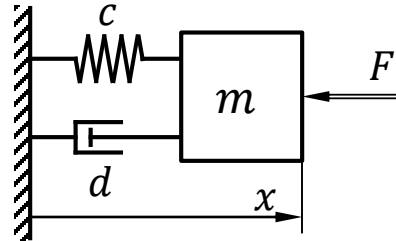


## 2. Linear System Representation



# Linear System Representation

- Consider the following linear spring-mass-damper system:



- Its motion can be described by the following equation:

$$m\ddot{x} + d\dot{x} + cx = F$$

- Using the Laplace transform, the transfer function from input  $F$  to output  $x$  can be easily obtained:

$$\frac{x}{F} = \frac{1}{m \cdot s^2 + d \cdot s + c}$$

- There are various other possible representations or **models** of this linear system. The following slide shows those models provided by the Control System Toolbox.

# Linear System Representation – Basic Models

Model	Example	Characterized by	MATLAB Function
Transfer Function	$\frac{n_2s^2 + n_1s + n_0}{d_3s^3 + d_2s^2 + d_1s + d_0}$	Numerator and denominator coefficients $n_i$ and $d_i$	tf
Zero-Pole-Gain Model	$\frac{k \cdot (s - z_1) \cdot (s - z_2)}{(s - p_{12})^2 \cdot (s - p_3)}$	Zeros $z_i$ , poles $p_i$ , gain $k$	zpk
State-Space Model (Explicit)	$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$	System matrices $A, B, C, D$	ss
Descriptor State-Space Model (Implicit)	$\begin{aligned}E\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$	System matrices $A, B, C, D, E$	dss
Discrete Transfer Function (Z-Transform)	$\frac{n_0 + n_1z^{-1}}{d_0 + d_1z^{-1} + d_2z^{-2}}$	Numerator and denominator coefficients $n_i$ and $d_i$	filt
Frequency Response	$\begin{aligned}f_1 &\rightarrow c_1 \\ f_2 &\rightarrow c_2 \\ f_3 &\rightarrow c_3\end{aligned}$	Gain and phase (complex number $c_i$ ) over frequency $f_i$	frd



## Linear System Representation – Basic Models

- Assuming the following numeric values for the spring-mass-damper system above

$$m = 50, \quad d = 35, \quad c = 10$$

the following models of the system can be defined in MATLAB:

```
%      tf(numerator_coefficients, denominator_coefficients)
sys_tf = tf(1, [50, 35, 10]);
```

$$= \frac{1}{50s^2 + 35s + 10}$$

```
%      zpk(zeros, poles, gain)
sys_zpk = zpk([], [0.35 + 0.278i, 0.35 - 0.278i], 0.1);
```

Note: this gain does not always equal the steady-state gain!

```
%      ss(A, B, C, D)
sys_ss = ss([0, 1; -0.2, -0.7], [0; 0.02], [1, 0], 0);
```

- Note that in this case only the transfer function can be defined straightforward, without further calculations. Should you need a different type of model, you can first define a transfer function and then transform it. (See chapter “Model Transformation”)

## Linear System Representation – Basic Models

- A Descriptor State-Space Model can be created much like a State-Space Model

```
sys_dss = ss(A, B, C, D, E);
```

- To create a digital filter  $H(z) = \frac{2+z^{-1}}{1+0.4z^{-1}+2z^{-2}}$  use the `filt` command

```
digital_filter = filt([2, 1], [1, 0.4, 2]);
```

- The `filt` command lets you specify a discrete-time transfer function in what MATLAB calls the DSP format. DSP stands for digital signal processing. In this case, it means that the transfer function is described by coefficients to  $z^{-n}$ , instead of  $z^n$ , with  $n \in \mathbb{N}_0$ .

Remark on Z-Transform and digital filtering:

$$\frac{y}{u} = H(z)$$

$$(1 + 0.4z^{-1} + 2z^{-2})y = (2 + z^{-1})u$$

$$y[t] + 0.4y[t - T_s] + 2y[t - 2T_s] = 2u[t] + u[t - T_s]$$

## Linear System Representation – Discrete-Time Systems

- All models presented above, with the exception of the digital filter, are time-continuous by default. To create discrete-time models, the sample time needs to be provided as an additional argument:

```
discrete_sys_tf = tf(num, den, Ts);
discrete_sys_zpk = zpk(zeros, poles, gain, Ts);
discrete_sys_ss = ss(A, B, C, D, Ts);
discrete_sys_dss = ss(A, B, C, D, E, Ts);
discrete_sys_frd = frd(response, frequency, Ts);
```

- To create a discrete-time model without specifying the sample time, set  $T_s = -1$
- The `filt` function always returns a discrete-time transfer function. The sample time argument is optional. If it is not provided, the sample time remains unspecified.

```
digital_filter = filt([2, 1], [1, 0.4, 2], 0.3);
```

Note:

$$\begin{aligned} \text{filt}([2, 1], [1, 0.4, 2], 0.3) &\leftrightarrow \frac{2+z^{-1}}{1+0.4z^{-1}+2z^{-2}} \\ \text{tf}([2, 1], [1, 0.4, 2], 0.3) &\leftrightarrow \frac{2z+1}{z^2+0.4z+2} \end{aligned}$$

## Linear System Representation – Systems with Time Delay

- Time delays can be added to all model types by specifying the properties `InputDelay` and/or `OutputDelay`:

```
sys_tf = tf(num, den, 'InputDelay', delay);  
sys_zpk = zpk(zeros, poles, gain, 'OutputDelay', delay);  
sys_frd = filt(num, den, Ts, 'InputDelay', delay);
```

Note: in SISO systems, input and output delays are usually equivalent.

- Input and output delays need to be distinguished in the case of State-Space Models, especially if the states have physical meaning!

```
sys_ss = ss(A, B, C, D, 'InputDelay', delay_in, 'OutputDelay', delay_out);
```

- Moreover, delays can be specified for each channel of a MIMO system separately.

```
sys_ss = ss(-2, 3, [1; -1], 0, 'InputDelay', 1.5, 'OutputDelay', [0.7; 0]);
```

- In discrete-time models, time delay is specified in numbers of sampling periods. In the following case, a 2.5 second time delay is introduced:

```
sys_discrete = tf(2, [1, -0.95], 0.1, 'InputDelay', 25);
```

## Linear System Representation – Time Delay Approximation

- In some cases, actual time delays may need to be approximated by linear functions. MATLAB can generate the common Padé approximation, here of second order for  $\tau = 0.3s$

```
[num,den] = pade(0.3, 2)
```

```
num =
```

```
    1.0000   -20.0000   133.3333
```

```
den =
```

```
    1.0000    20.0000   133.3333
```

Reminder – Padé approximation:

$$e^{-\tau s} \approx \frac{1 - k_1 s + k_2 s^2 + \dots \pm k_n s^n}{1 + k_1 s + k_2 s^2 + \dots + k_n s^n}$$

- To replace all time delays in an existing system `sys` by a 5<sup>th</sup> order Padé approximation:

```
sys_pade = pade(sys, 5)
```

- In the case of discrete-time models, use `absorbDelay` to replace a delay of `k` sampling periods by `k` poles at `z=0`:

```
sys_absorbed = absorbDelay(sys_discrete)
```

## Linear System Representation – Tunable Models

- **Tunable models** can be useful to represent systems that have both fixed and tunable (or parametric) coefficients. If, for instance, your control system comprises a tunable low-pass filter, you can use MATLAB tunable models to conduct parameter studies.
- To create a tunable low-pass filter, a tunable real parameter has to be created first.

```
a = realp('a',10); % 10 is the initial value  
F = tf(a,[1, a]);
```

- F will not be a TF object, but a GENSS object. To convert existing models to GENSS:

```
sys_gen = genss(sys);
```

- Such generalized models have various properties, one of which allows you to access all tunable parameters:

```
F.Blocks
```

```
F_blocks =  
    a: [1x1 realp]
```

## Linear System Representation – Tunable Models

- Similarly, a generalized FRD model can be created.

```
frd_gen = genfrd(sys, frequencies, frequency_units);
```

- All tunable parameters can be changed using the following function:

```
F_new = replaceBlock(F, 'a', 5);
```

- It is even possible to hand over multiple parameter values. The result is an array of state-space models:

```
F_new = replaceBlock(F, 'a', [2, 4, 7, 12]);
```

- To access the model where a=7

```
F_new(:, :, 1, 3)
```

or where a=12

```
F_new(:, :, 1, 4)
```

## Linear System Representation – PID Controllers

- PID controllers are standard in linear control. Therefore, the Control System Toolbox provides dedicated models.

```
C = pid(Kp);           % creates a proportional controller  
C = pid(Kp, Ki);       % creates a PI controller  
C = pid(Kp, Ki, Kd, Tf); % creates a PID controller
```

Reminder – PID controller:

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}$$

- To create a PID controller in standard form:

```
C = pidstd(Kp, Ti, Td, N);
```

PID controller in standard form:

$$C(s) = K_p \left( 1 + \frac{1}{T_i s} + \frac{T_d s^2}{\frac{T_d}{N} s + 1} \right)$$

- PID controllers can also be discrete-time:

```
C = pid(Kp, 0, 0, 0, Ts); % creates a discrete-time proportional controller  
C = pidstd(Kp, Ti, Td, N, Ts); % creates a discrete-time PID controller (standard form)
```



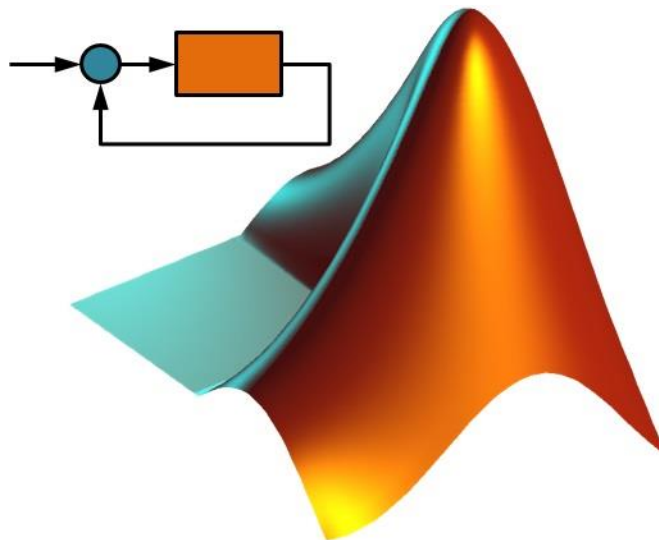
## Linear System Representation – Model Attributes

- The following functions determine various model attributes:

```
isct(sys)      % determine if the model is in continuous time
isdt(sys)      % determine if the model is in discrete time
isstable(sys)  % determine whether the model is stable
isproper(sys)  % determine whether the model is proper
issiso(sys)    % determine if the model is single-input/single-output (SISO)
hasdelay(sys)  % determine whether the model contains any delays
order(sys)     % query model order
```

- All these functions return Boolean values, with the exception of order.

# 3. Model Interconnection

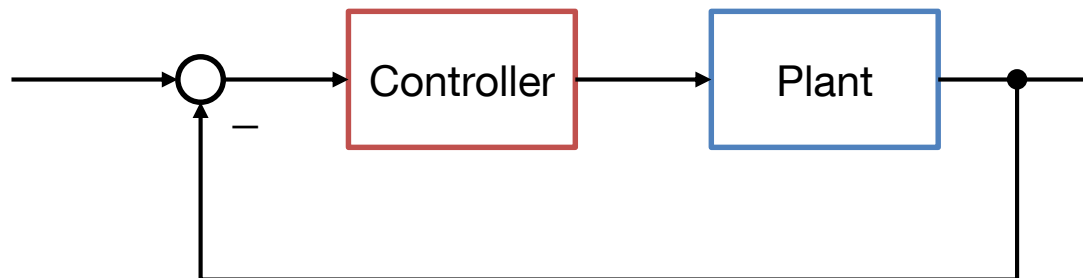


## Model Interconnection

- Suppose you have created the model of a plant and the model of a controller.



- Now you want to combine those two models to form a closed control loop.



- This is where you need the tools for model interconnection.

## Model Interconnection

- Series connection of two models:

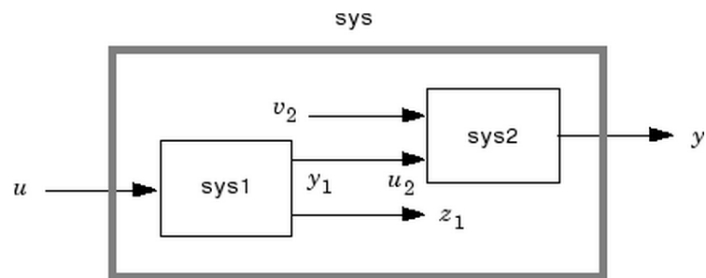
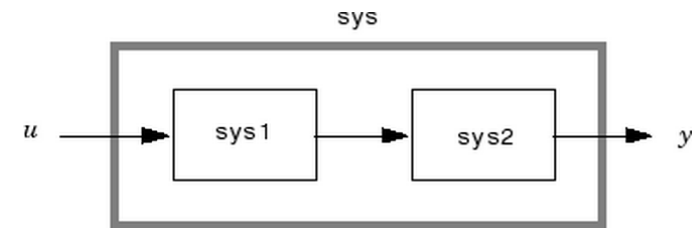
```
sys = series(sys1, sys2);
```

- Both systems must be either both continuous or both discrete with identical sample time.
- This command is equivalent to:

```
sys = sys2 * sys1;
```

- In the case of MIMO systems, you can specify which inputs and outputs to connect.

```
sys = series(sys1, sys2, outputs1, inputs2);
```



# Model Interconnection

- Parallel connection of two models:

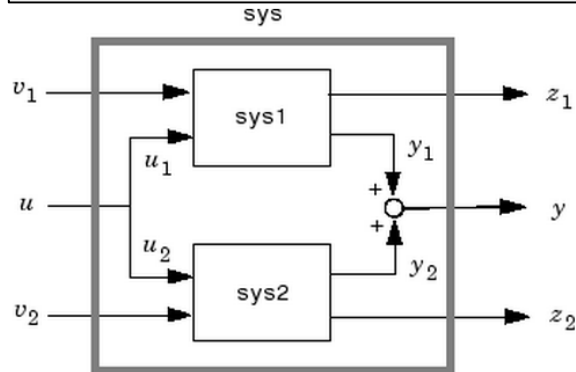
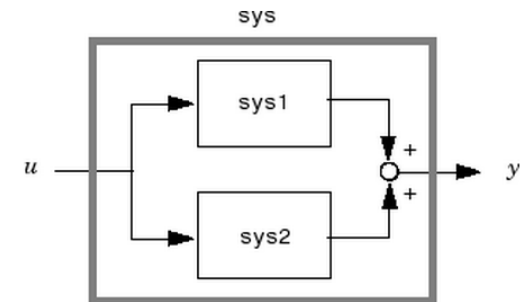
```
sys = parallel(sys1, sys2);
```

- Both systems must be either both continuous or both discrete with identical sample time.
- This command is equivalent to:

```
sys = sys1 + sys2;
```

- In the case of MIMO systems, you can specify which inputs and outputs to connect.

```
sys = parallel(sys1, sys2, input1, input2, output1, output2);
```



# Model Interconnection

- Feedback interconnection of two models:

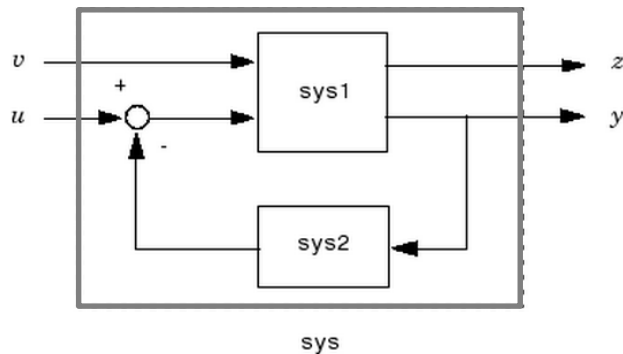
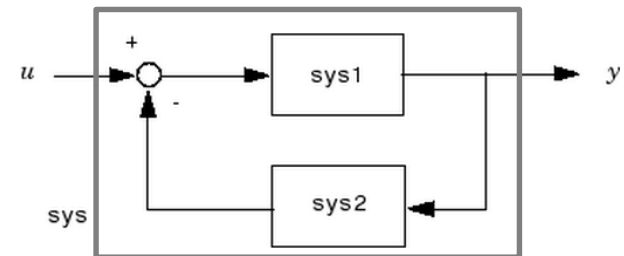
```
sys = feedback(sys1, sys2);
```

- Both systems must be either both continuous or both discrete with identical sample time.
- Negative feedback is default. To apply positive feedback:

```
sys = feedback(sys1, sys2, +1);
```

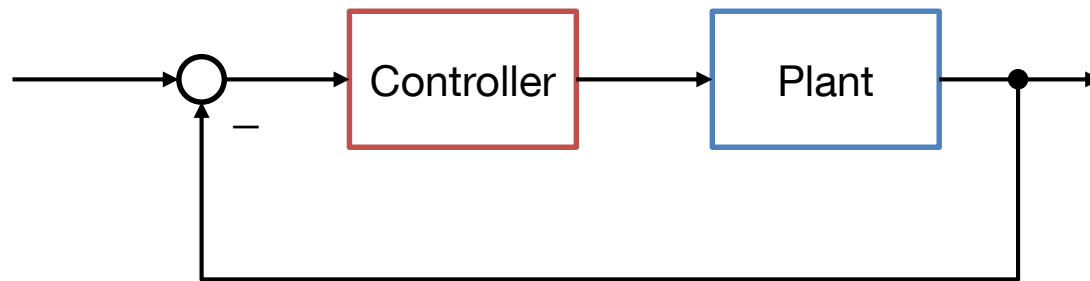
- In the case of MIMO systems, you can specify which inputs and outputs to connect.

```
sys = feedback(sys1, sys2, feedin, feedout);
```



## Model Interconnection

- Returning to the initial example...



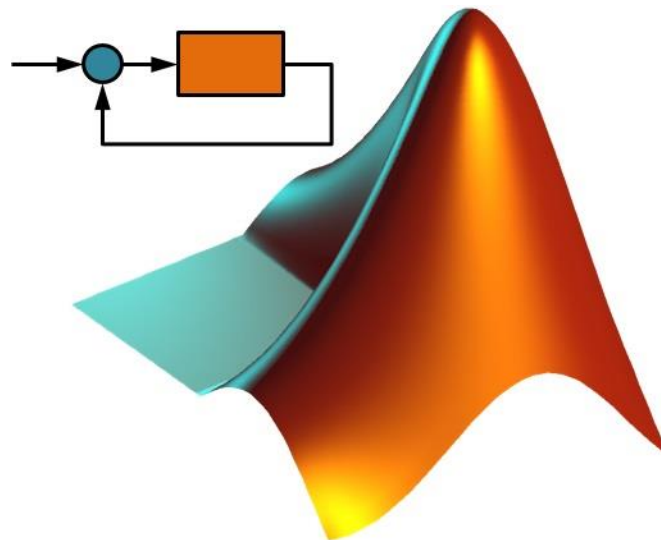
Other functions for  
model interconnection:

connect  
sumblk  
append  
blkdiag  
imp2exp  
inv  
lft

```
open_loop = series(controller, plant);  
closed_loop = feedback(open_loop, 1);
```

Note: the feedback  
system is a simple  
unitary gain.

## 4. Model Transformation





## Model Transformation – Type Conversion

- To convert the model type, simply apply the desired function of the new model type with the existing model as argument:

```
sys_tf    = tf(1, [1 1]);    % original model
sys_zpk   = zpk(sys_tf);    % conversion to Zero-Pole-Gain Model
sys_ss    = ss(sys_tf);     % conversion to State-Space Model
sys_pid   = pid(sys_tf);    % conversion to PID Model
sys_pids  = pidstd(sys_tf); % conversion to PID (standard form) Model
sys_tf2   = tf(sys_ss);     % conversion to transfer function
```

- Conversion to FRD model requires the desired frequencies as second argument:

```
sys_frd = frd(sys_tf, logspace(-1, 3));
```

- MIMO models can be assembled from SISO models using matrix notation.  
You can also access one particular SISO transfer of a MIMO model.

```
sys_tf_MIMO = [tf(1,[1 1]), 1; tf([1 2],[2 1]), tf(5,[1 3 1])];
sys_tf_SISO21 = sys_tf_MIMO(2,1); % From input 1 to output 2: tf([1 2],[2 1])
```

## Model Transformation – Continuous-Discrete Conversion

- There are various functions available for conversions between continuous-time and discrete-time. For instance, to discretize a continuous-time model:

```
sys_discrete = c2d(sys_continuous, sample_time);
```

- To create a continuous-time model from a discrete-time model:

```
sys_continuous = d2c(sys_discrete)
```

- To resample a discrete-time model:

```
sys_rs = d2d(sys, new_sample_time)
```

There are various options for all these functions, which may help improve the result in any particular case.

- To upsample a discrete-time model, i.e., to multiply the sampling rate by a factor  $L$ :

```
sys_us = upsample(sys, L)
```

Both `d2d` and `upsample` can be used to decrease sample time. Note that `upsample` provides better match in both time and frequency domain, but increases model order to do so. Moreover, it is only applicable if the new sampling rate is an integer multiple of the old sampling rate.

## Model Transformation – Simplification

- Models can also be simplified. To eliminate the states of a state-space model that do not affect the input-output response, apply structural pole-zero cancellations:

```
msys = sminreal(sys);
```

- A minimal realization or pole-zero cancellation of any type of dynamic model can be obtained as follows. The resulting system is of minimal order and has the same response characteristics as the original.

```
msys = minreal(sys);  
msys = minreal(sys, tolerance);
```

Default tolerance is `sqrt(eps)`. An increased tolerance forces additional cancellations.

- Specific states of a state-space model can be eliminated as follows:

```
rsys = modred(sys, elim);
```

Here, `elim` can be a vector of indices or a logical vector. This function is usually used in combination with `balreal` (see next slide).

## Model Transformation – Simplification

- `balreal` computes a balanced realization. Hankel singular values `g` are returned as well.

```
sys = zpk([-10 -20.01], [-5 -9.9 -20.1], 1);  
[bsys, g] = balreal(sys);
```

- Note that `balreal` works with state-space models only, so the ZPK object here is automatically transformed to an SS object first.
- The system above contains two near-cancelling pole-zero pairs. This is also reflected by small values in the vector `g`.

```
g'  
  
ans =  
  
    0.1006    0.0001    0.0000
```

Other functions for  
simplification:  
`balred`  
`hsvd`  
`hsvplot`

- Small values of `g` indicate that the corresponding states can be removed for simplification.

```
rsys = modred(sysb, g<0.001);
```

## Model Transformation – State-Coordinate Transform

- The function `canon` transforms linear models into canonical state-space models. The modal form is constituted of the system modes, whereas the companion form explicitly contains the characteristic polynomial coefficients.

Modal form of a system with eigenvalues  $(\lambda_1, \sigma \pm j\omega, \lambda_2)$ :

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \sigma & \omega & 0 \\ 0 & -\omega & \sigma & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix}$$

Companion form of a system with char. polynomial  $p = s^n + a_1s^{n-1} + \dots + a_{n-1}s + a_n$ :

$$A = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 & -a_n \\ 1 & 0 & 0 & \dots & 0 & -a_{n-1} \\ 0 & 1 & 0 & \dots & \vdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots \\ 0 & \vdots & \ddots & 1 & 0 & -a_2 \\ 0 & \dots & \dots & 0 & 1 & -a_1 \end{bmatrix}$$

```
msys = canon(sys, 'modal');
csys = canon(sys, 'companion');
```

- The state transformation matrix `T` can also be obtained and applied.

```
[csys, T] = canon(sys, type);
csys_alt = ss2ss(sys, T);    % Note: csys_alt = csys
```

Other functions for state-coordinate transform:

- `prescale`
- `ss2ss`
- `xperm`

## Model Transformation – Modal Decomposition

- Models can be decomposed based on the characteristics of the modes:
  - Stable-unstable decomposition using `stablesep`
  - Slow-fast decomposition using `freqsep`
  - Region-based decomposition (N regions) using `modsep`

```
[sys_stable, sys_unstable] = stablesep(sys);  
[sys_slow, sys_fast]      = freqsep(sys, f_cut);  
[H, H0]                   = modsep(sys, N, REGIONFCN);
```

- `REGIONFCN` specifies the regions of interest.  
For instance, to decompose a system into two systems `H1`, `H2`, which have their poles inside and outside the unit disk, respectively:

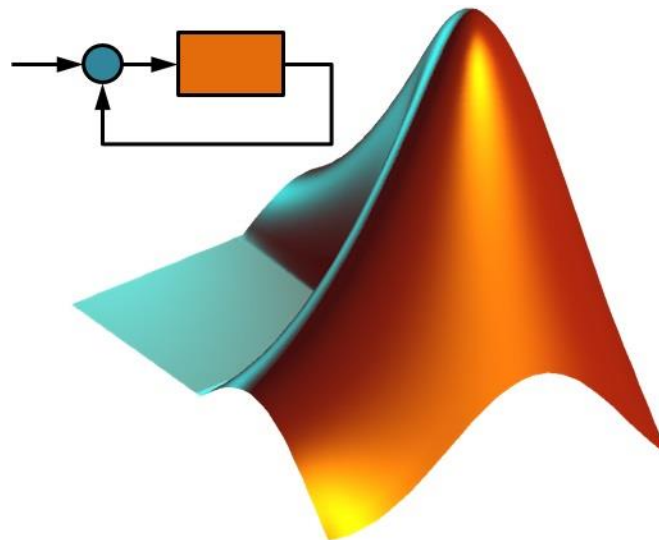
Note:

$\text{sys\_stable} + \text{sys\_unstable} = \text{sys}$   
 $\text{sys\_slow} + \text{sys\_fast} = \text{sys}$

```
function r = udsep(p)  
if abs(p)<1, r = 1;  
else      r = 2;  
end
```

```
[H, H0] = modsep(sys, 2, @udpsep);
```

## 5. Linear Analysis



## Linear Analysis – Stability

- After linear dynamic models have been created, connected and possibly transformed, it is time to analyze them. Poles and zeros of a system can be found as follows:

```
vector_of_poles = pole(sys);  
vector_of_zeros = zero(sys);
```

- The function `zero` can also return the overall gain:

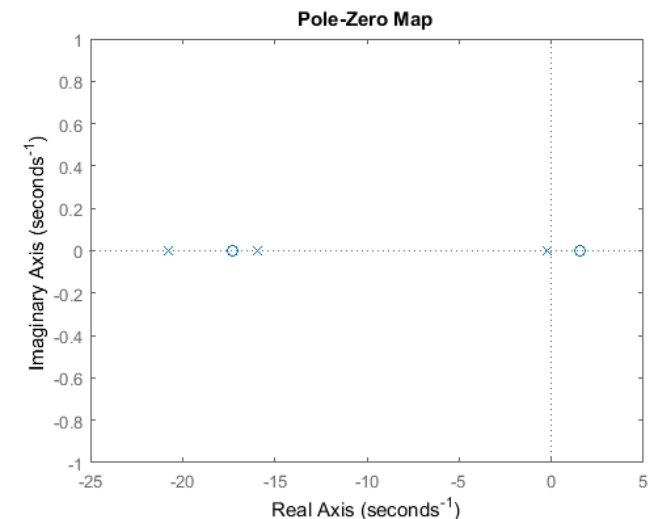
```
[vector_of_zeros, overall_gain] = zero(sys);
```

- To visualize both the poles and the zeros of a system, one of the following functions can be used:

```
pzplot(sys)  
pzmap(sys)
```

Multiple different types of systems can be handed over and `pzmap` can return poles and zeros.

```
pzplot(sys1, sys2, sys3)  
[poles, zeros] = pzmap(sys)
```





## Linear Analysis – Stability

- More detailed information about the poles is provided by the function `damp`.

```
damp(sys) % displays a table of natural frequencies, damping ratios and time constants
[omega, zeta] = damp(sys); % returns natural frequencies and damping ratios
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-1.20e+00	1.00e+00	1.20e+00	8.33e-01
-2.16e+00 + 8.90e-01i	9.25e-01	2.34e+00	4.63e-01
-2.16e+00 - 8.90e-01i	9.25e-01	2.34e+00	4.63e-01

- Stability margins can be determined as follows:

```
[gain_margin, phase_margin, omega_GM, omega_PM] = margin(sys)
```

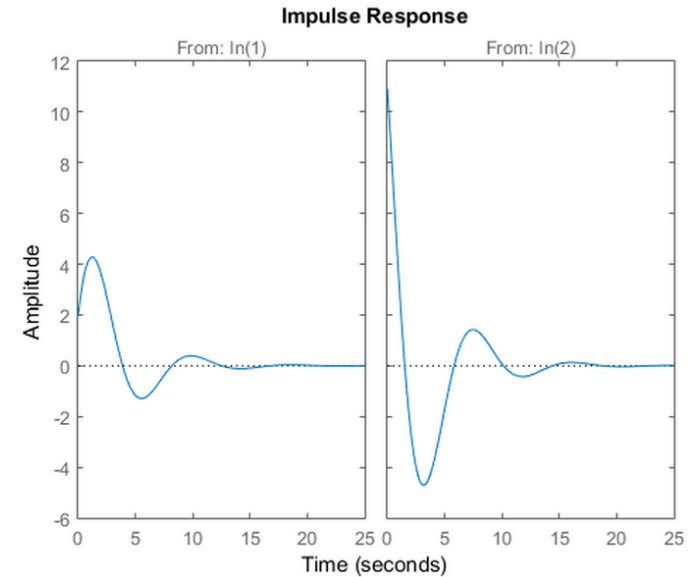
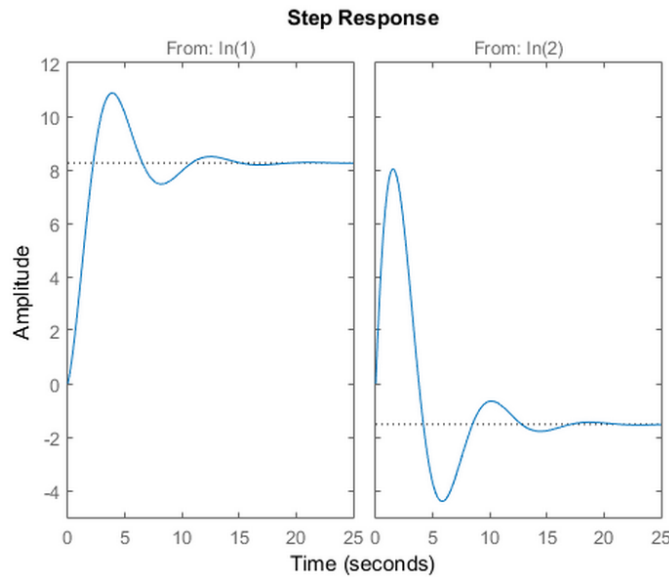
`omega_GM` and `omega_PM` are the frequencies where the gain / phase margin is measured.

- Calling `margin` without output arguments creates a Bode plot (see slides below) with margins indicated.

## Linear Analysis – Time Domain

- Step and impulse response of linear systems can be easily visualized:

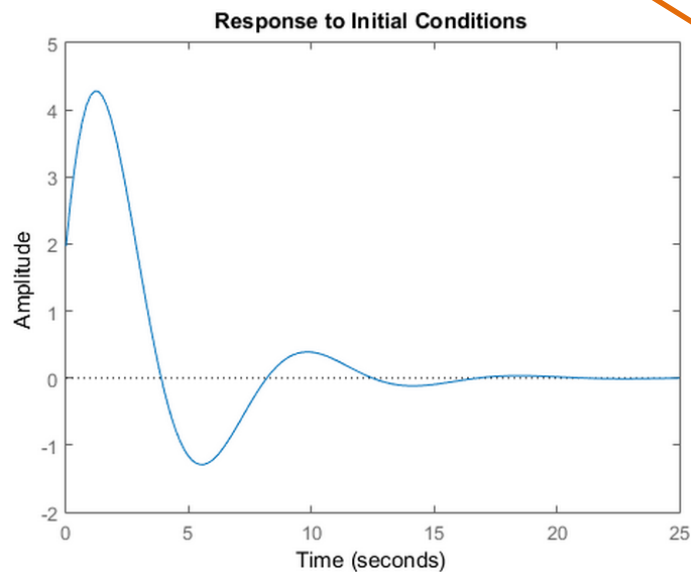
```
a = [-0.5572, -0.7814; 0.7814, 0];  
b = [1, -1; 0, 2]; c = [1.9691, 6.4493];  
sys = ss(a,b,c,0);  
step(sys)  
impz(sys)
```



## Linear Analysis – Time Domain

- The response to initial conditions can be investigated as follows.

```
a = [-0.5572, -0.7814; 0.7814, 0];  
c = [1.9691, 6.4493];  
sys = ss(a, [0;0], c, 0);  
x0 = [1; 0];  
initial(sys, x0)
```

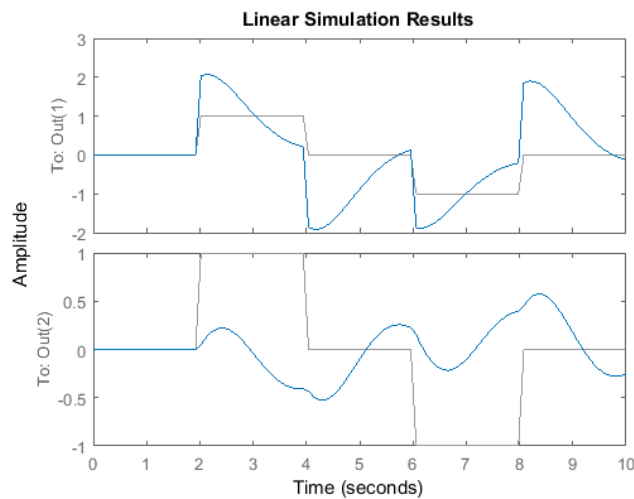


Note: system inputs are irrelevant for the response to initial conditions. Therefore, the input matrix can be set to zero here. However, the dimension of the input matrix has to match

## Linear Analysis – Time Domain

- Apart from the common step-, impulse- and initial condition response analysis, the Control System Toolbox also provides a function that returns the time response to arbitrary inputs.

```
sys = [tf([2 5 1],[1 2 3]); tf([1 -1],[1 1 5])];  
t = linspace(0,10,100);  
L = length(t)/5;  
u = [zeros(1,L), ones(1,L), zeros(1,L), -ones(1,L), zeros(1,L)];  
lsim(sys,u,t)
```



- The system outputs, simulations time stamps (and states in the case of state-space models) can be retrieved as follows:

```
y = lsim(sys,u,t);  
[y,t,x] = lsim(sys,u,t);
```

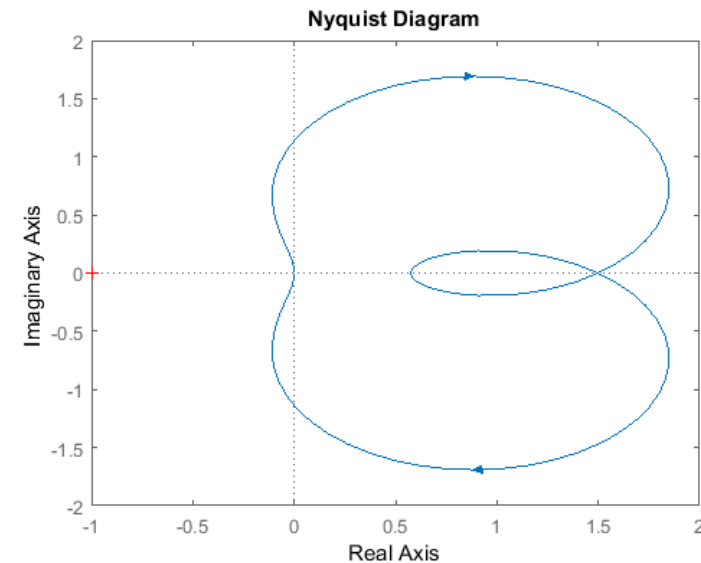
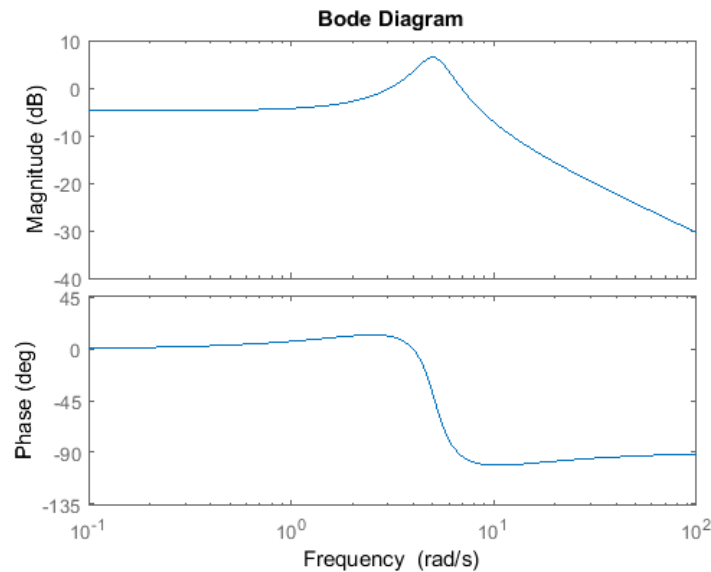
- Initial conditions of state-space models can be specified as a fourth argument:

```
lsim(sys,u,t,x0);
```

## Linear Analysis – Frequency Domain

- In the frequency domain, models can be analyzed using Bode or Nyquist diagrams.

```
sys = tf([3, 15], [1, 2, 26]);  
bode(sys)  
nyquist(sys)
```



## Linear Analysis – Frequency Domain

- A Nichols chart with its grid lines can be created as follows:

```
sys = tf([3, 15], [1, 2, 26]);  
nichols(sys)  
ngrid
```

- The low-frequency gain of a system, i.e., the transfer function value at  $s = 0$ , can be determined as follows.

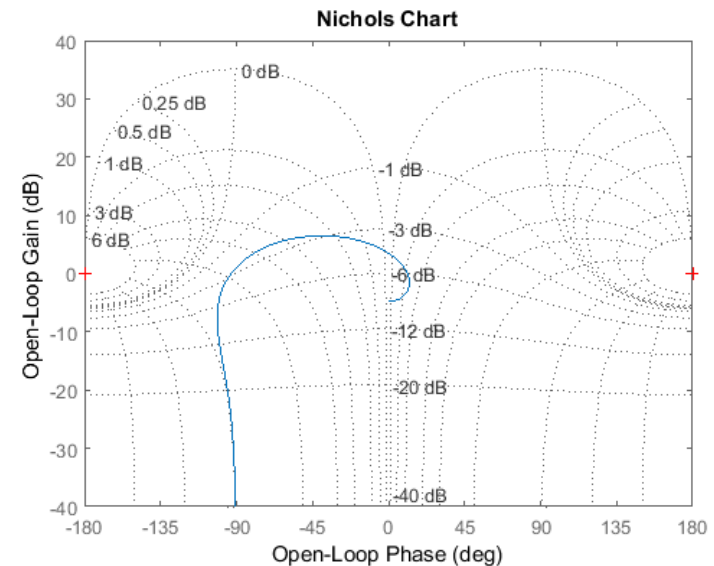
```
k = dcgain(sys);
```

- To find all frequencies where a system has a certain principal gain  $g$

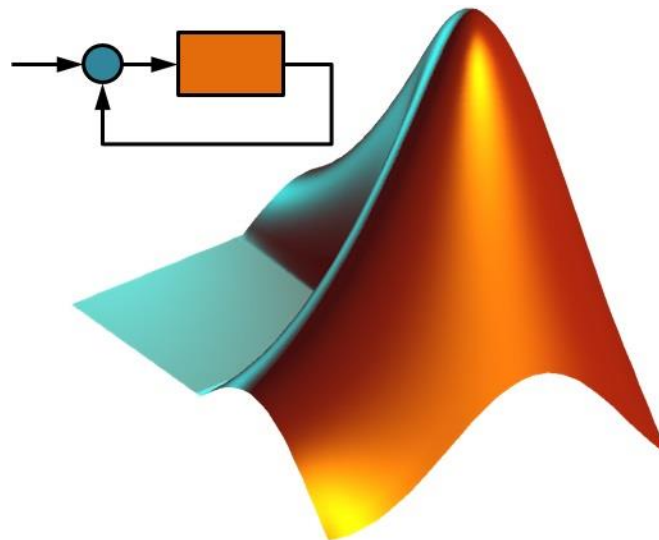
```
omega_c = getGainCrossover(sys, g);
```

- The peak gain of a system is given by the following function.

```
g_peak = getPeakGain(sys);
```



## 6. Control Design



## Control Design – PID Controller Tuning

- The Control System Toolbox provides an automatic PID tuning algorithm:

```
controller = pidtune(sys, 'pi');
```

- The second argument specifies the controller type (p, i, pi, pd, pdf, pid or pidf). When handing over an existing controller as second argument, a controller of the same type and form is generated.
- The algorithm tries to balance performance (response time) and robustness (stability margins). You can specify a target **crossover frequency** or hand over additional options:

```
controller = pidtune(sys, 'p', omega_c_target);  
controller = pidtune(sys, 'p', options);
```

- Options need to be created using the following function.

```
options = pidtuneOptions('PhaseMargin', 45, 'DesignFocus', 'reference-tracking');
```

- More information on the resulting closed-loop system can be retrieved as follows.

```
[controller, info] = pidtune(sys, 'pid');
```

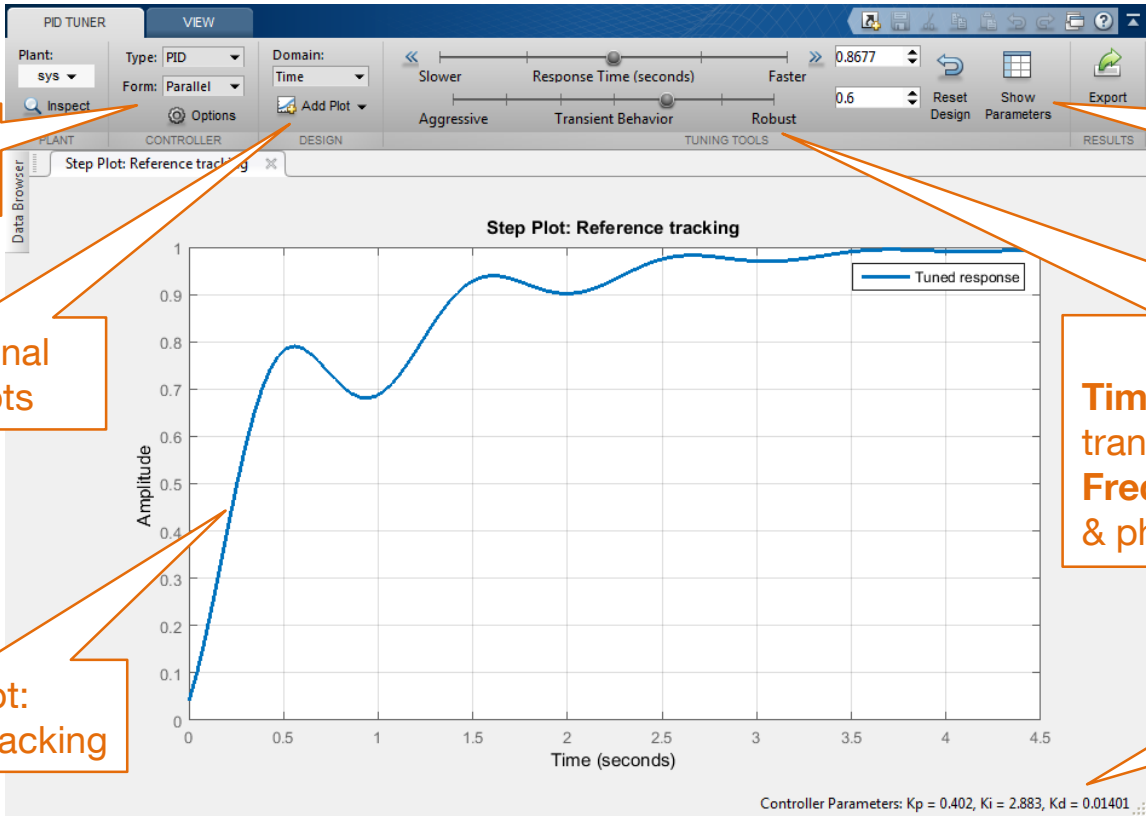
Design focus is either:  
balanced (default)  
reference-tracking  
disturbance-rejection



# Control Design – PID Controller Tuning

- PID controllers can also be tuned using a graphical user interface (GUI)

```
pidTuner(sys, 'pid')
```



Controller type and form

Add additional analysis plots

Analysis plot: reference tracking

New window with all relevant parameters (incl. margins)

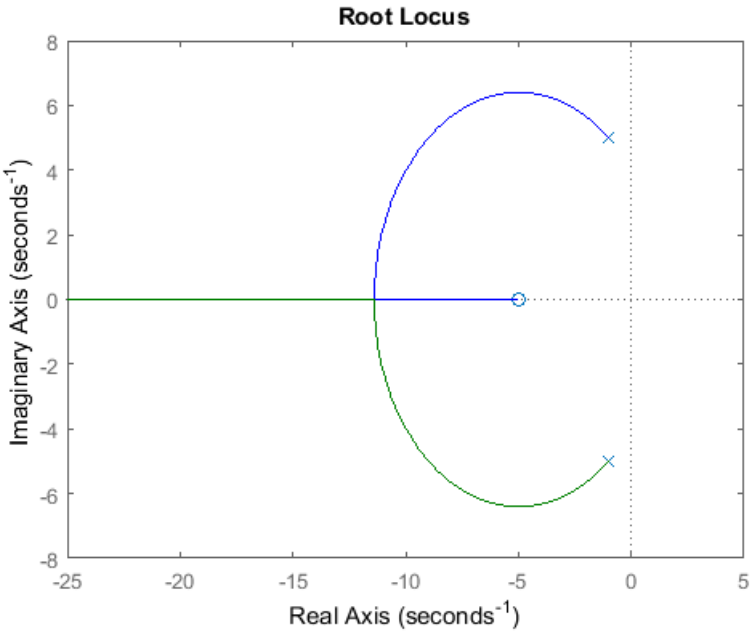
Sliders  
**Time domain:** response time & transient behavior  
**Frequency domain:** bandwidth & phase margin

Current controller parameters

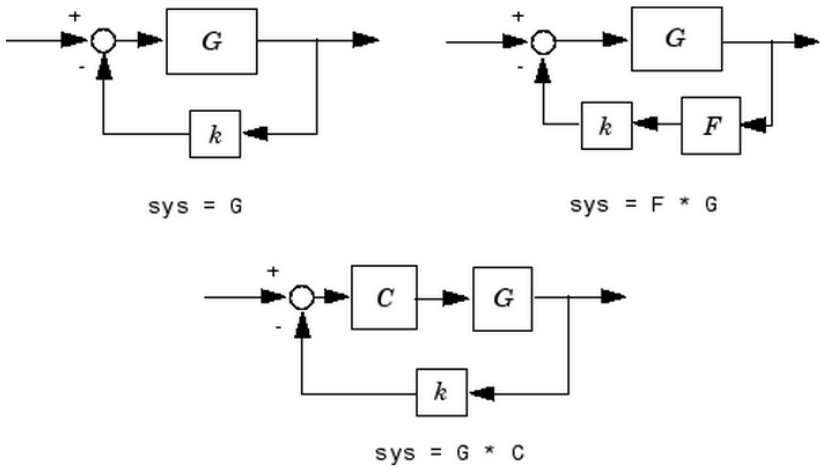
# Control Design – SISO Feedback Loops: Root Locus

- The following root locus function may be considered an analysis tool, but can certainly be useful for control design.

```
rlocus(sys)
```



This function can be applied to any of the following **negative** feedback loops:



# Control Design – SISO Feedback Loops: SISO Design Tool

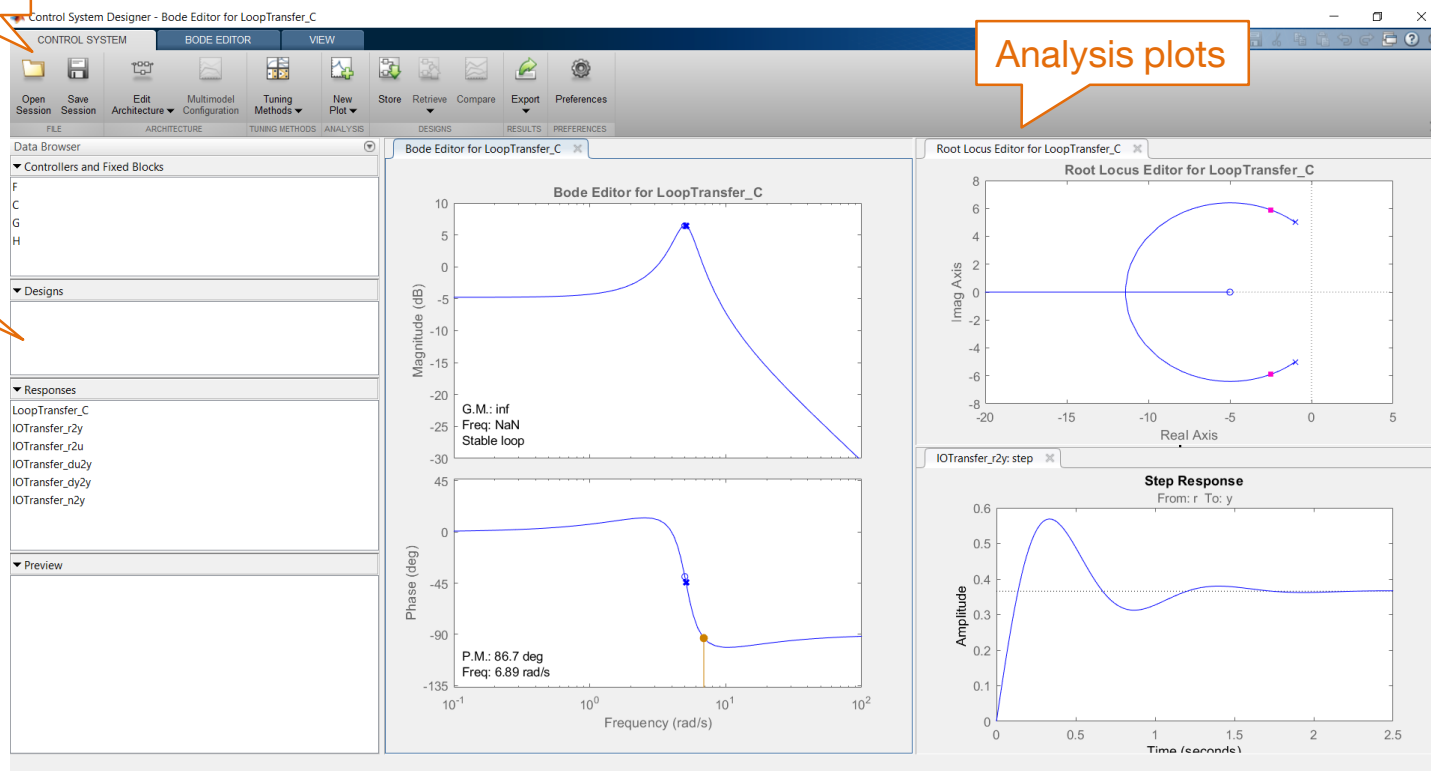
- A comfortable GUI for the design of various controllers is the SISO Design Tool:

```
controlSystemDesigner(sys)
```

Toolstrip

Analysis plots

Data Browser



# Control Design – SISO Feedback Loops: SISO Design Tool

Click here to change the control architecture...

Edit Architecture

Multimodel Configuration

Tuning Methods

New Plot

Store

Retrieve

Compare

Export

Preferences

Edit Architecture - Configuration 1

Select Control Architecture:

Specify feedback sign(s)

Identifier	Block Name	Value
C	C	<1x1 zpk>
F	F	<1x1 zpk>
G	G	<1x1 tf>
H	H	<1x1 tf>

OK Cancel Help

Choose an architecture!

# Control Design – SISO Feedback Loops: SISO Design Tool

The screenshot shows the SISO Design Tool interface with several key components and annotations:

- Compensator Editor:** A central window titled "Compensator Editor" with a "Compensator" dropdown set to "C" and a value of "1". It has tabs for "Pole-Zero" and "Parameter".
- Gain:** An annotation points to the "1" in the compensator value field.
- Additional dynamics show up here:** An annotation points to the "Dynamics" section in the "Pole-Zero" tab.
- Choose the controller component to edit (C, F, ...) and right click "Open Selection":** An annotation points to the "C" in the compensator dropdown.
- Right click here to add or delete poles/zeros:** An annotation points to the "Add Pole or Zero" button in the "Dynamics" section.
- When a pole or zero is selected in the window "Dynamics," dialog windows appear here, which allow to edit the selected pole or zero.** An annotation points to the "Edit Selected Dynamics" dialog box, which lists options: Real Pole, Complex Pole, Integrator, Real Zero, Complex Zero, Differentiator, Lead, Lag, and Notch.

## Control Design – Pole Placement, LQR, LQG, Kalman Filter

- The Control System Toolbox also comprises functions for more advanced control design. The pole placement gain for a state feedback  $u = -Kx$  can be computed as follows.

```
K = place(A,B,p); % A and B are system and input matrix, p is the vector of poles.
```

- A Linear-Quadratic Regulator (LQR) consists of a state feedback gain  $K$  that minimizes  $J(u) = \int_0^\infty (x^T Qx + u^T Ru + 2x^T Nu)dt$ . The `lqr` function returns the optimal gain  $K$ , the solution to the Riccati equation  $S$  and the closed-loop eigenvalues  $e$ .

```
[K,S,e] = lqr(sys, Q, R, N);
```

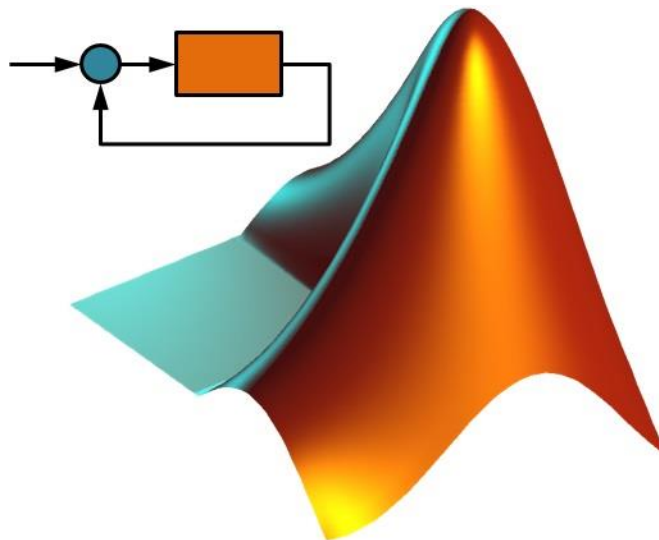
- A Linear-Quadratic-Gaussian (LQG) regulator minimizes a different cost function and can be applied to systems affected by process and measurement noise. With the weighting matrices  $QXU$  and  $QWV$  given, the `lqg` function returns this regulator.

```
regulator = lqg(sys, QXU, QWV);
```

- Similarly, a Kalman filter or state estimator can be designed.

```
[k_est,L,P] = kalman(sys, Qn, Rn, Nn)
```

# 7. Matrix Computations



## Matrix Computations

- For state-space models, the controllability and observability matrices can be found by

```
M_ctr = ctrb(sys);  
M_obs = obsv(sys);
```

- The  $H_2$  norm of a linear system is returned by

```
N_H2 = norm(sys);
```

whereas the following command computes the  $H_\infty$  norm

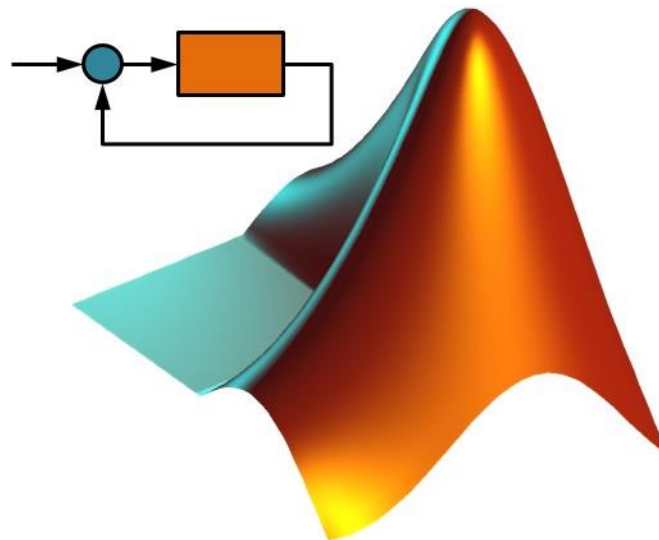
```
N_Hinf = norm(sys,inf);
```

- The Lyapunov equation  $AX + XA^T + Q = 0$  can be solved using `lyap`.

```
X = lyap(A,Q)
```



## 8. List of Commands



# List of Commands

Command	Explanation	Slide #	Command	Explanation	Slide #
tf	Transfer function	7	isstable	Check if system is stable	16
zpk	Zero-pole-gain model	7	isproper	Check if system is proper	16
ss	State-space model	7	issiso	Check if system is SISO	16
dss	Descriptor state-space model	7	hasdelay	Check if system has delays	16
filt	Digital filter model	7	order	Determine system order	16
frd	Freq. response data model	7	series	Series connection	19
pade	Padé approximation	12	parallel	Parallel connection	20
absorbDelay	Replace delays with poles	12	feedback	Feedback connection	21
realp	Real tunable parameter	13	connect	Block diagram connection	-
genss	Generalized SS model	13	sumblk	Summing-junction	-
genfrd	Generalized FRD model	14	append	Group models	-
pid	PID controller	15	blkdiag	Block-diagonal concatenat°	-
pidstd	PID controller (standard form)	15	im2exp	Implicit to explicit	-
isct	Check if system is continuous	16	inv	Invert models	-
isdt	Check if system is discrete	16	lft	Generalized feedback con.	-

# List of Commands

Command	Explanation	Slide #
c2d	Continuous to discrete conv.	25
d2c	Discrete to continuous conv.	25
d2d	Resample	25
upsample	Upsample	25
sminreal	Structural minimal realization	26
minreal	Minimal realization	26
modred	Model order reduction	26
balreal	Balanced Realization	27
balred	Model order reduction	-
hsvd	Hankel singular values	-
hsvplot	Hankel singular value plot	-
canon	Canonical realization	28
prescale	Optimal scaling	-
ss2ss	State coordinate transform	28
xperm	Reorder states	-

Command	Explanation	Slide #
stablesep	Stable/unstable decompos.	29
freqsep	Slow/fast decomposition	29
modsep	Custom mode separation	29
pole	System poles	31
zero	System zeros	31
pzplot	Pole-zero plot	31
pzmap	Pole-zero map	31
damp	System pole characteristics	32
margin	Stability margins	32
step	Step response plot	33
impulse	Impulse response plot	33
initial	Initial value response plot	34
lsim	Linear simulation	35

## List of Commands

Command	Explanation	Slide #
bode	Create Bode diagram	36
nyquist	Create Nyquist diagram	36
nichols	Create Nichols chart	37
ngrid	Plot Nichols grid lines	37
dcgain	Compute DC gain	37
getGainCrossover	Get crossover frequencies	37
getPeakGain	Get peak gain	37
pidtune	Tune PID controller	39
pidtuneOptions	Options for pidtune	39
pidTuner	PID tune GUI	40
rlocus	Plot root locus	41
controlSystemDesigner	Open SISO Design Tool	42
place	Pole placement	45

Command	Explanation	Slide #
lqr	Design LQR regulator	45
lqg	Design LQG regulator	45
kalman	Design Kalman filter	45
ctrb	Controllability matrix	47
obsv	Observability matrix	47
norm	System norm	47
lyap	Solve Lyapunov equation	47