deeplearning.ai

Optimization Algorithms

Mini-batch gradient descent

Batch vs. mini-batch gradient descent

X, Y $\qquad$ $X^{\{t\}}, Y^{\{t\}}$

Vectorization allows you to efficiently compute on m examples.

$$X = [\underbrace{x^{(1)}\ x^{(2)}\ x^{(3)} \cdots x^{(1000)}}_{X^{\{1\}}\ (n_x, 1000)} \mid \underbrace{x^{(1001)} \cdots x^{(2000)}}_{X^{\{2\}}\ (n_x, 1000)} \mid \cdots \mid \underbrace{\cdots x^{(m)}}_{X^{\{5,000\}}\ (n_x, 1000)}]$$

(n_x, m)

$$Y = [\underbrace{y^{(1)}\ y^{(2)}\ y^{(3)} \cdots y^{(1000)}}_{Y^{\{1\}}\ (1, 1000)} \mid \underbrace{y^{(1001)} \cdots y^{(2000)}}_{Y^{\{2\}}\ (1, 1000)} \mid \cdots \mid \underbrace{\cdots y^{(m)}}_{Y^{\{5,000\}}\ (1, 1000)}]$$

$(1, m)$

What if $m = 5,000,000$?

5,000 mini-batches of 1,000 each

Mini-batch t: $X^{\{t\}}, Y^{\{t\}}$

$x^{(i)}$

$z^{[l]}$

$X^{\{t\}}, Y^{\{t\}}$.

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Mini-batch gradient descent

repeat {

for $t = 1, \ldots, 5000$ {

Forward prop on $X^{\{t\}}$.

$$\left.\begin{aligned} Z^{[1]} &= W^{[1]} X^{\{t\}} + b^{[1]} \\ A^{[1]} &= g^{[1]}(Z^{[1]}) \\ &\vdots \\ A^{[L]} &= g^{[L]}(Z^{[L]}) \end{aligned}\right\} \text{Vectorized implementation (1000 examples)}$$

Compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{l} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \cdot 1000} \sum_{l} \|W^{[l]}\|_F^2$.

from $X^{\{t\}}, Y^{\{t\}}$.

Backprop to compute gradients wrt $J^{\{t\}}$ (using $(X^{\{t\}}, Y^{\{t\}})$)

$W^{[l]} := W^{[l]} - \alpha dW^{[l]}$, $b^{[l]} := b^{[l]} - \alpha db^{[l]}$

}

}

1 step of gradient descent using $X^{\{t\}}, Y^{\{t\}}$. (as if $m = 1000$)

X, Y

"1 epoch" – pass through training set.

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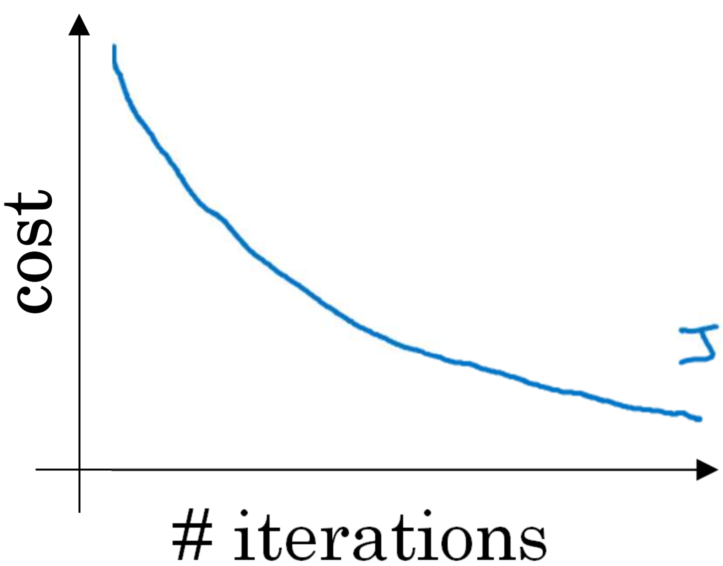
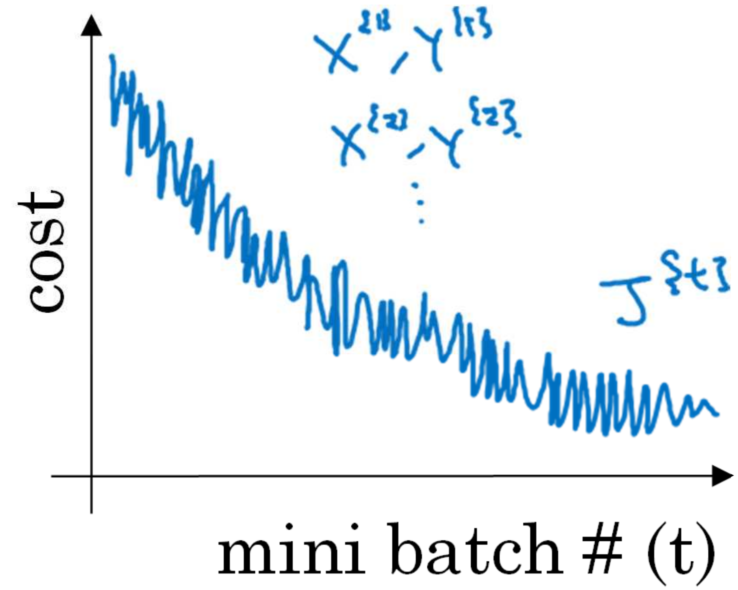
Training with mini batch gradient descent

Batch gradient descent



Mini-batch gradient descent



Plot $J^{\{t\}}$ computed using $\underline{X^{\{t\}}, Y^{\{t\}}}$

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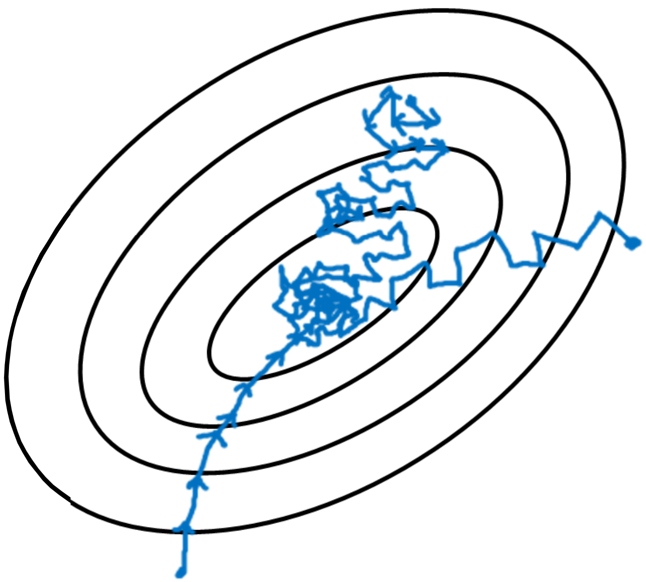
Choosing your mini-batch size

→ If mini-batch size $= m$: Batch gradient descent. $(X^{\{1\}}, Y^{\{1\}}) = (X, Y)$.

→ If mini-batch size $= 1$: Stochastic gradient descent. Every example is it own mini-batch.
$(X^{\{1\}}, Y^{\{1\}}) = (x^{(1)}, y^{(1)}) \ldots (x^{(2)}, y^{(2)})$

In practice: Somewhere in-between $\underline{1}$ and $\underline{m}$



Stochastic gradient descent

↓

Lose speedup from vectorization

In-between (mini-batch size not too big/small)

↓

Fastest learning.

- Vectorization. (~ 1000)
- Make progress without processing entire training set.

Batch gradient descent (mini-batch size $= m$)

↓

Too long per iteration

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Choosing your mini-batch size

If small trng set: Use batch gradient descent.
$(m \leq 2000)$

Typical mini-batch sizes:

$\rightarrow \underbrace{\underset{2^6}{64}, \underset{2^7}{128}, \underset{2^8}{256}, \underset{2^9}{512}}$ $\qquad \underset{2^{10}}{1024}$

Make sure mini-batch fit in CPU/GPU memory.
$X^{\{t\}}, Y^{\{t\}}$

Temperature in London

$\theta_1 = 40°F$ 4°C ←

$\theta_2 = 49°F$ 9°C

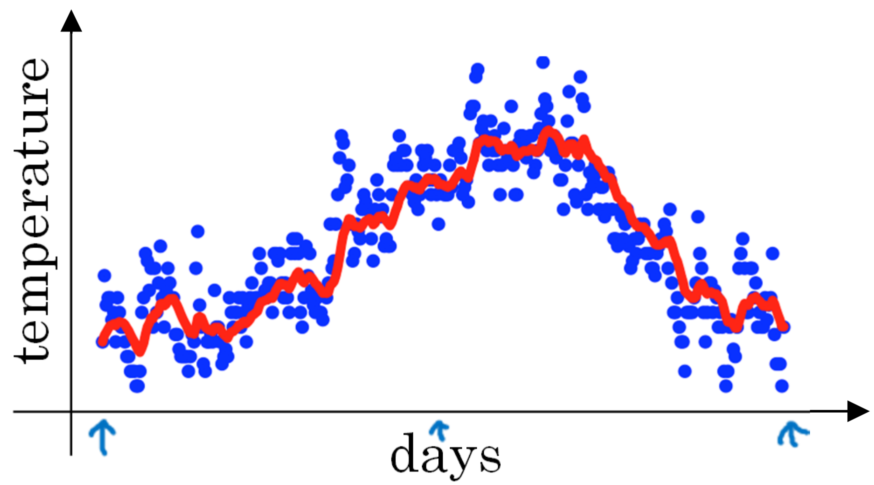
$\theta_3 = 45°F$ ⋮

⋮

$\theta_{180} = 60°F$ 15°C

$\theta_{181} = 56°F$ ⋮

⋮



$V_0 = 0$

$V_1 = 0.9 V_0 + 0.1 \theta_1$

$V_2 = 0.9 V_1 + 0.1 \theta_2$

$V_3 = 0.9 V_2 + 0.1 \theta_3$

⋮

$$\boxed{V_t = 0.9 V_{t-1} + 0.1 \theta_t}$$

Exponentially weighted averages

moving

$\underline{V_t} = \underline{\beta}\, \underline{V_{t-1}} + \underline{(1-\beta)}\, \underline{\theta_t} \leftarrow$

$\beta = 0.9$: ≈ 10 days' temperature.

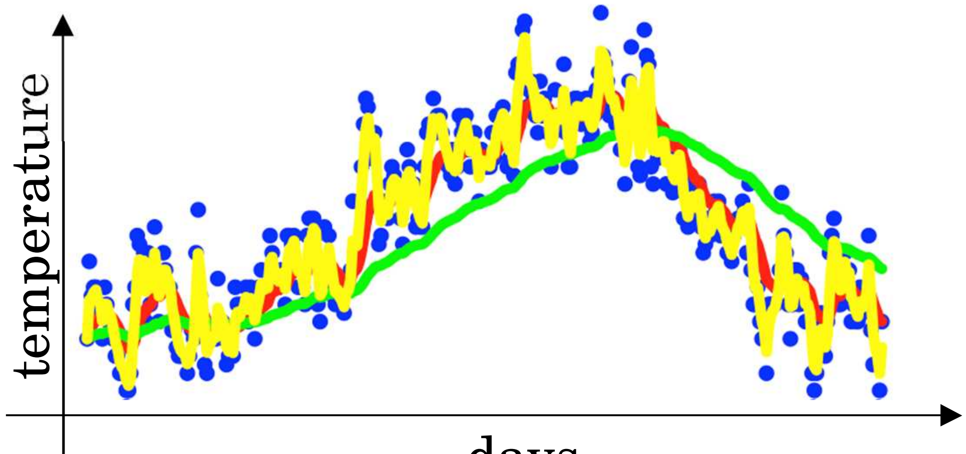
$\underline{\beta = 0.98}$: ≈ 50 days

$\beta = 0.5$: ≈ 2 days

V_t as approximately average over

$\rightarrow \approx \frac{1}{1-\beta}$ days' temperature.

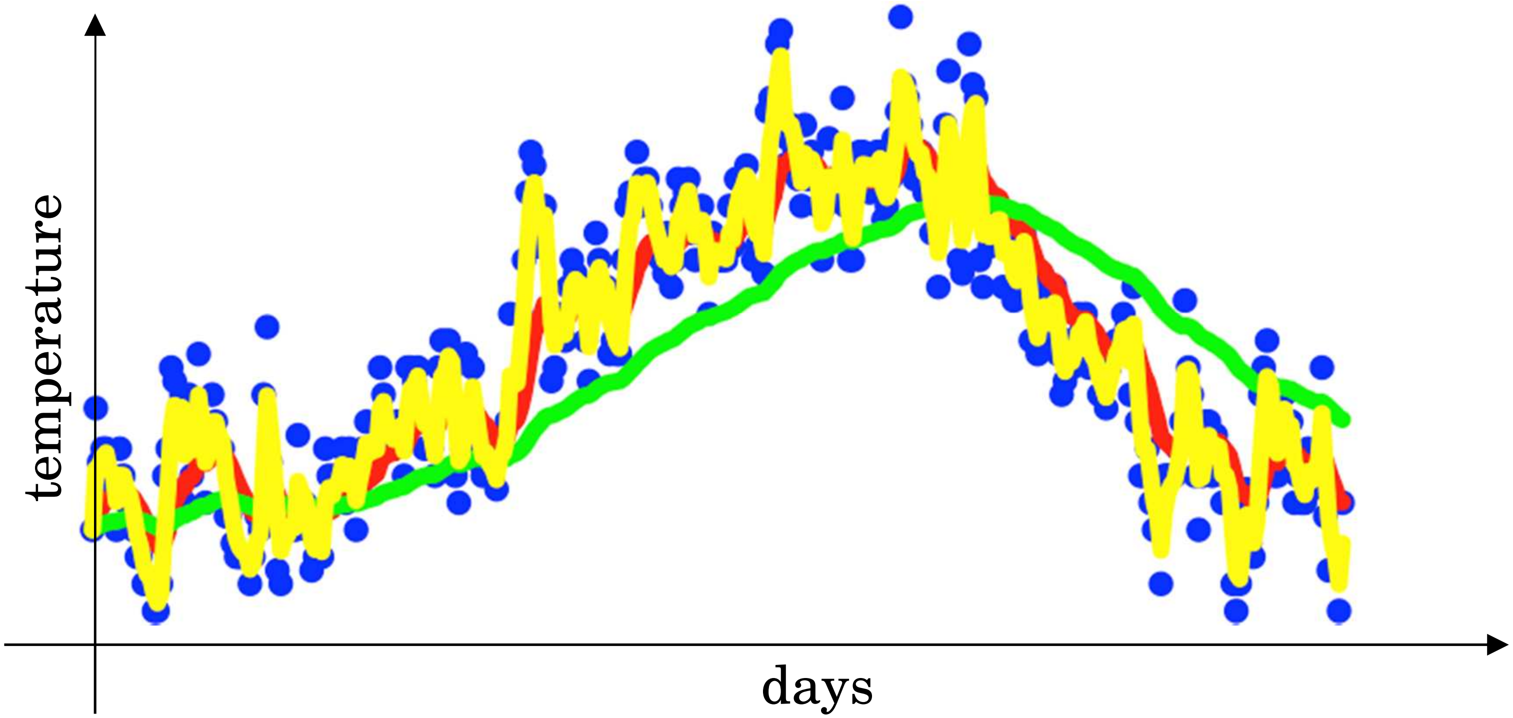
$\frac{1}{1-0.98} = 50$



Exponentially weighted averages

$$\boxed{v_t = \beta v_{t-1} + (1-\beta)\theta_t}$$

$\beta = 0.9$ 0.98 0.5



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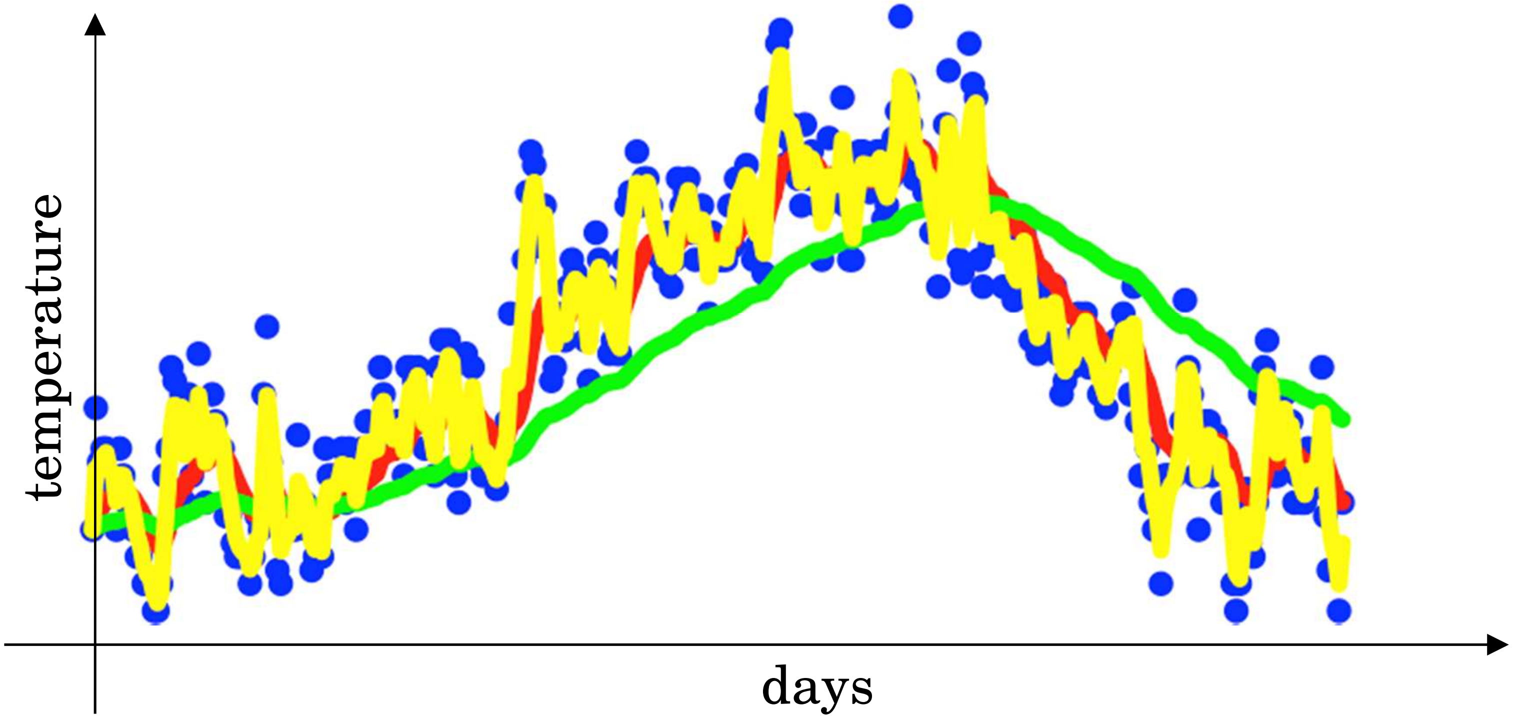
Exponentially weighted averages

$$\boxed{v_t = \beta v_{t-1} + (1-\beta)\theta_t}$$

$\beta = 0.9$ $\quad$ 0.98 $\quad$ 0.5



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Implementing exponentially weighted averages

$v_0 = 0$

$v_1 = \beta v_0 + (1-\beta)\,\theta_1$

$v_2 = \beta v_1 + (1-\beta)\,\theta_2$

$v_3 = \beta v_2 + (1-\beta)\,\theta_3$

...

$V_\theta := 0$

$V_\theta := \beta v + (1-\beta)\,\theta_1$

$V_\theta := \beta v + (1-\beta)\,\theta_2$

$\vdots$

$\rightarrow V_\theta = 0$

Repeat {

Get next θ_t

$V_\theta := \beta V_\theta + (1-\beta)\theta_t \leftarrow$

}

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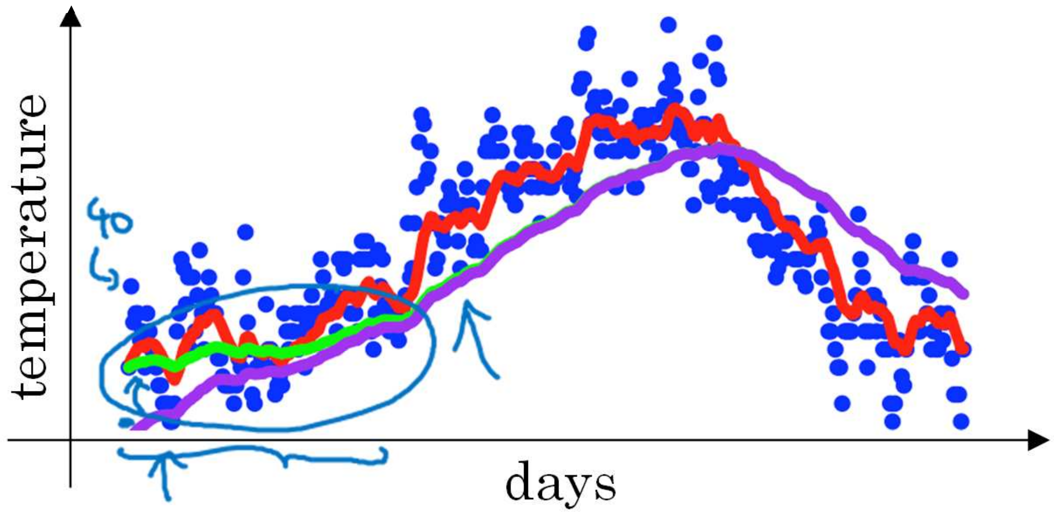
Bias correction



$\beta = 0.98$

$\rightarrow v_t = \beta v_{t-1} + (1-\beta)\theta_t$

$V_0 = 0$

$V_1 = \cancel{0.98 V_0} + 0.02\,\theta_1$

$V_2 = 0.98\,V_1 + 0.02\,\theta_2$

$= 0.98 \times 0.02 \times \theta_1 + 0.02\,\theta_2$

$= 0.0196\,\theta_1 + 0.02\,\theta_2$

$\frac{V_t}{1-\beta^t}$

$t=2: \quad 1-\beta^t = 1-(0.98)^2 = 0.0396$

$\frac{V_2}{0.0396} = \frac{0.0196\,\theta_1 + 0.02\,\theta_2}{0.0396}$

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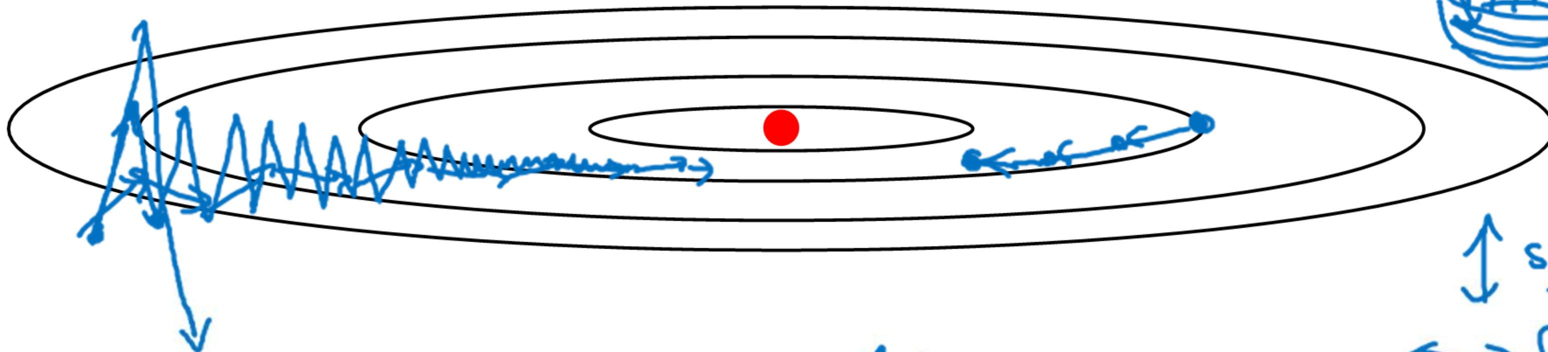
Gradient descent example



$\updownarrow$ slower learning

$\leftrightarrow$ faster learning.

Momentum:

On iteration t:

Compute dW, db on current mini-batch.

$V_{dW} = \beta V_{dW} + (1-\beta) dW$ "$V_\theta = \beta V_\theta + (1-\beta)\theta_t$"

$V_{db} = \beta V_{db} + (1-\beta) db$

friction → β ← velocity ← acceleration

$W := W - \alpha V_{dW}$, $b := b - \alpha V_{db}$

Implementation details

$v_{dW}=0$, $v_{db}=0$

On iteration t:

Compute dW, db on the current mini-batch

$v_{dW} = \beta v_{dW} + (1-\beta) dW$

$v_{db} = \beta v_{db} + (1-\beta) db$

$W = W - \alpha v_{dW}, \ b = b - \alpha v_{db}$

$v_{dW} = \beta v_{dW} + dW$ ←

$\cancel{\dfrac{v_{dW}}{1-\beta^t}}$

Hyperparameters: α, β $\qquad \beta = 0.9$

average over last ≈10 gradients

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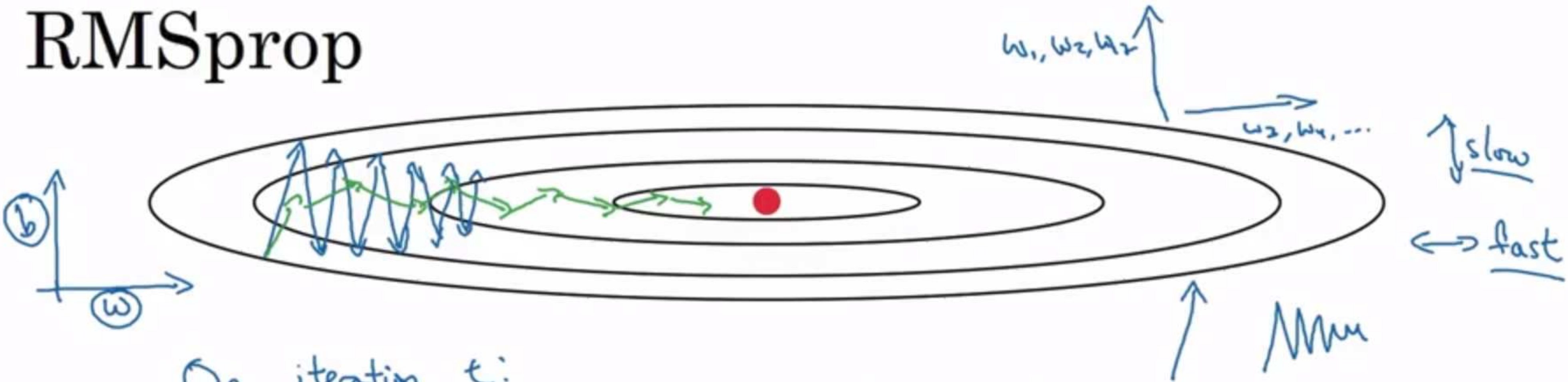
RMSprop



On iteration t:

Compute dW, db on current mini-batch

element-wise

$S_{dW} = \beta_2 S_{dW} + (1-\beta_2)\, dW^2$ ← small

$S_{db} = \beta_2 S_{db} + (1-\beta_2)\, db^2$ ← large

$W := W - \alpha \frac{dW}{\sqrt{S_{dW}+\varepsilon}}$ $\qquad$ $b := b - \alpha \frac{db}{\sqrt{S_{db}+\varepsilon}}$

$\varepsilon = 10^{-8}$

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Adam optimization algorithm

$V_{dw} = 0, \; S_{dw} = 0. \quad V_{db} = 0, \; S_{db} = 0$

On iteration t:

Compute dW, db using current mini-batch

$V_{dw} = \beta_1 V_{dw} + (1-\beta_1) dW, \quad V_{db} = \beta_1 V_{db} + (1-\beta_1) db \quad \leftarrow$ "momentum" β_1

$S_{dw} = \beta_2 S_{dw} + (1-\beta_2) dW^2, \quad S_{db} = \beta_2 S_{db} + (1-\beta_2) db \quad \leftarrow$ "RMSprop" β_2

$$V_{dw}^{corrected} = V_{dw} / (1-\beta_1^t), \quad V_{db}^{corrected} = V_{db} / (1-\beta_1^t)$$

$$S_{dw}^{corrected} = S_{dw} / (1-\beta_2^t), \quad S_{db}^{corrected} = S_{db} / (1-\beta_2^t)$$

$$W := W - \alpha \frac{V_{dw}^{corrected}}{\sqrt{S_{dw}^{corrected}} + \varepsilon} \qquad b := b - \alpha \frac{V_{db}^{corrected}}{\sqrt{S_{db}^{corrected}} + \varepsilon}$$

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Hyperparameters choice:

$\rightarrow \alpha$: needs to be tune

$\rightarrow \beta_1$: 0.9 $\longrightarrow$ ($\underline{dw}$)

$\rightarrow \beta_2$: 0.999 $\longrightarrow$ ($\underline{dw^2}$)

$\rightarrow \varepsilon$: 10^{-8}

Adam: Adaptive moment estimation

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Learning rate decay

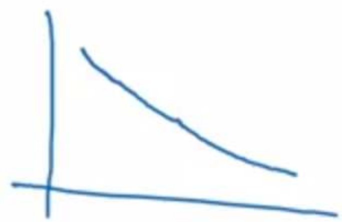
1 epoch = 1 pass through data.

$$\alpha = \underbrace{\frac{1}{1 + \text{decay\_rate} * \text{epoch\_num}}\,\alpha_0}$$

$\alpha_0 = 0.2$

decay_rate = 1

Epoch	α
1	0.1
2	0.67
3	0.5
4	0.4
⋮	⋮



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Other learning rate decay methods

formula

$\alpha = 0.95^{\text{epoch-num}} \cdot \alpha_0$ — exponentially decay.

$\alpha = \dfrac{k}{\sqrt{\text{epoch-num}}} \cdot \alpha_0$ or $\dfrac{k}{\sqrt{t}} \cdot \alpha_0$

discrete staircase

Manual decay.

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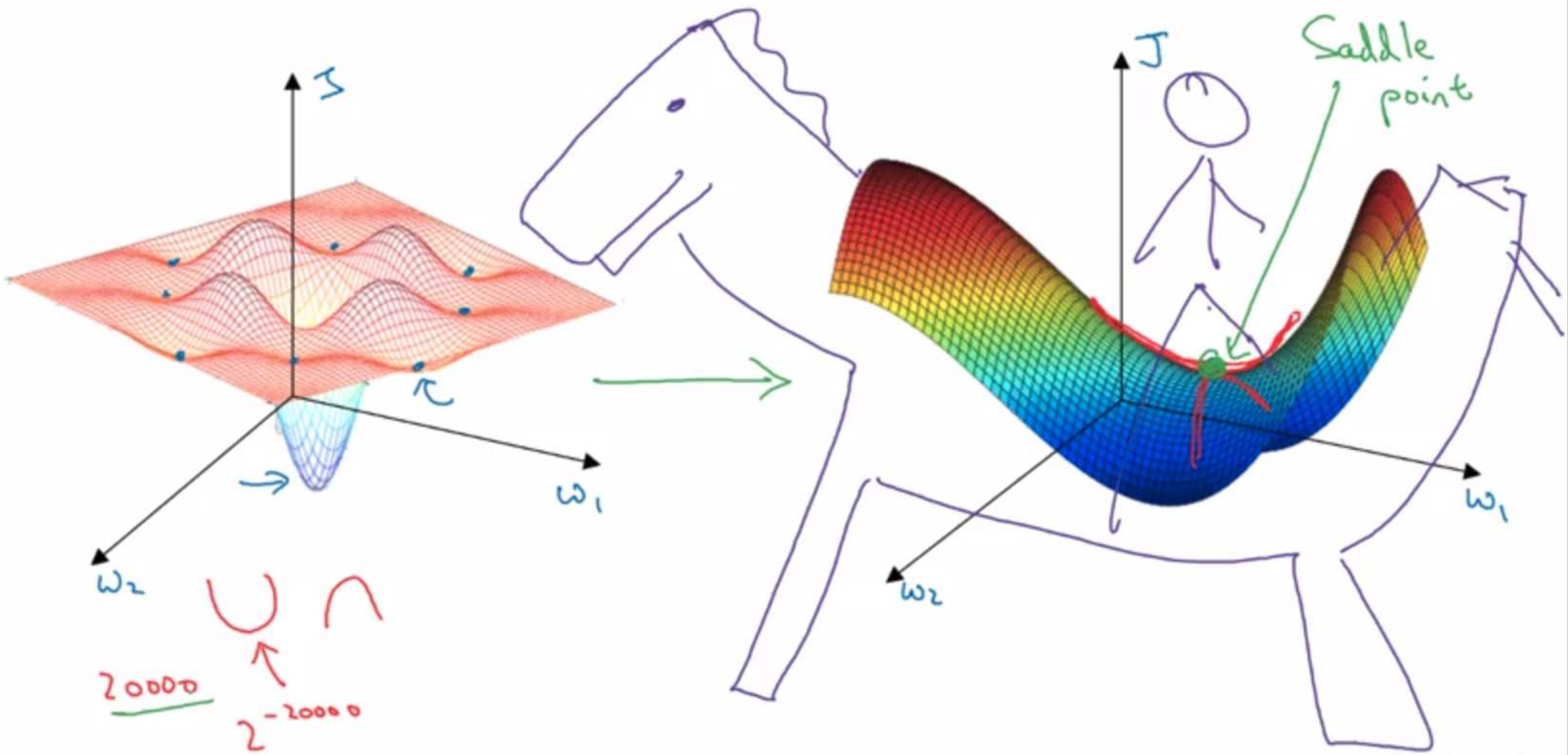
Local optima in neural networks



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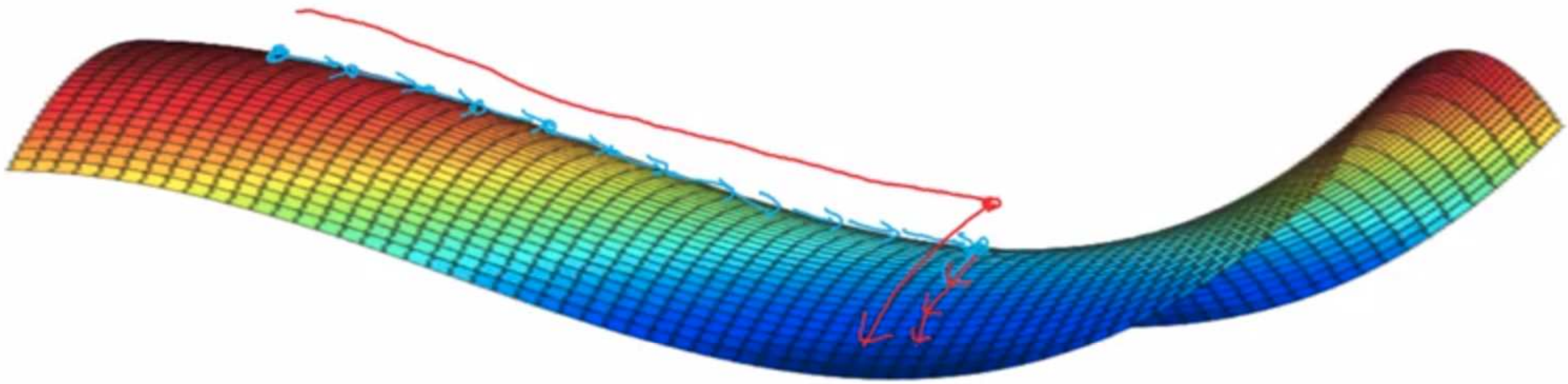
Problem of plateaus



- Unlikely to get stuck in a bad local optima
- Plateaus can make learning slow

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