Please read the following instructions carefully:

- There are **eight problems** in this exam.
- There is **one bonus** problem.
- Solve the **bonus** problem only at the end.
- You have **90 minutes** to complete the exam
- The point distribution is given in the table below.
- Please write each solution on a separate page.
- You **must have your camera on** during the exam.
- This is a **closed book, closed notes exam**. You must not consult any resource while attempting the exam.
- Upload your work to Gradescope.
- Submitting the exam implies you abide by the honor pledge stated below:

I pledge on my honor that I have not given or received any unauthorized assistance on this quiz/examination

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	10	10	10	10	10	10	0	80

1. (a) (5 points) Compute the following limit:

$$\lim_{n\to\infty} \frac{\sqrt{n}-2}{\sqrt{n}+2}$$

   (b) (5 points) Compute the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{2^{n+2}}{7^{n+3}}.$$

2. (10 points) Determine whether the following series converges. State any tests you use.

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2+1}.$$

3. (10 points) Determine whether the series

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{n3^{n+1}}$$

diverges, converges conditionally, or converges absolutely. State any tests that you use.

4. (10 points) Determine whether the following series converges. State any tests you use.

$$\sum_{n=0}^{\infty} \frac{2^n(n!)^2}{(2n)!}.$$

5. (10 points) Determine the radius of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{n^2x^n}{4^n(n^2+1)}$$

6. (10 points) Find the radius of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{2\cdot 5\cdot 8\cdots(3n-1)(x-2)^n}{2\cdot 4\cdot 6\cdots 2n}$$

7. (10 points) Compute the power series of the function $f(x) = x^2\cos(5x)$ about $x = 0$.

8. (10 points) Express the following integral as a power series:

$$\int_0^1 e^{-x^2}\,dx.$$

9. (5 points (bonus)) In class, we argued that it is a non-trivial question to determine whether the Taylor series about the point $x = a$ associated to a function, $f(x)$, converges to $f(a)$. As an extreme example, consider the function:

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Show that the power series of $f(x)$ about $x = 0$ is identically zero. The function, however, is non-zero for $x \neq 0$. Hence, the Taylor series of this infinitely differentiable function does not converge to $f(x)$ for any $x \neq 0$.

Note: Yeesh! What's going on? Is Math broken? Well, not quite. The functions we've dealt with in class are quite special: they're *smooth* (infinitely differentiable) and *analytic* (admit power series converging to the function). The function above is only smooth, but not analytic. The study of calculus of *analytic* functions is the area of mathematics called complex analysis[1]. Real analysis (a.k.a calculus) is usually more messy, and deals only with smooth functions at best.

[1]This really is a misnomer. There's nothing complex about the subject. Analytic functions behave much more nicely than just smooth functions. Why? Since they admit power series representations, they're like *polynomials of infinite degree so to speak*. And we love polynomials, don't we?