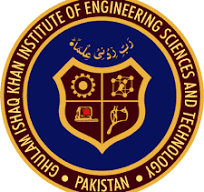
**Ghulam Ishaq Khan Institute of Engineering Sciences and Technology**

**Faculty of Computer Science and Engineering**

**CS342 – Numerical Analysis Project**

**Submitted To: Sir Aamir Shehzad**

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# Bisection Method

## ****Method Overview****

The bisection method is a root-finding algorithm that:

* Requires a continuous function that changes sign over an interval [a, b]
* Repeatedly bisects the interval and selects the subinterval containing the root
* Guarantees convergence to a root if the function is continuous
* Uses a tolerance of 1e-6 and maximum 100 iterations as stopping criteria
* Calculates error as the difference between consecutive approximations

## Explanation of the Code

The implementation consists of three main functions:

**bisectionMethod:**

* Takes function, interval bounds [a, b], tolerance, and max iterations
* Returns root approximation, iteration count, and iteration data
* Checks for sign change at interval endpoints
* Updates interval based on function value at midpoint

**printResults:**

* Shows current interval bounds [a, b] and their function values
* Displays midpoint (x) and its function value
* Calculates error between consecutive approximations
* Formats output in a clear tabular form

**plotResults:**

* Error vs Iterations plot showing convergence
* Function curve plot with root location

## Tables of Results

**Stopping Tolerance: 10-6**

**f(x) = x3- 4x + 1**

**Interval: [0,1]**

| **Iteration** | **a** | **f(a)** | **b** | **f(b)** | **x** | **f(x)** | **Error** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0.000000 | 1.000000 | 1.000000 | -2.000000 | 0.500000 | -0.875000 | inf |
| 2 | 0.000000 | 1.000000 | 0.500000 | -0.875000 | 0.250000 | 0.015625 | 0.250000 |
| 3 | 0.250000 | 0.015625 | 0.500000 | -0.875000 | 0.375000 | -0.447266 | 0.125000 |
| 4 | 0.250000 | 0.015625 | 0.375000 | -0.447266 | 0.312500 | -0.219482 | 0.062500 |
| 5 | 0.250000 | 0.015625 | 0.312500 | -0.219482 | 0.281250 | -0.102753 | 0.031250 |
| 6 | 0.250000 | 0.015625 | 0.281250 | -0.102753 | 0.265625 | -0.043758 | 0.015625 |
| 7 | 0.250000 | 0.015625 | 0.265625 | -0.043758 | 0.257812 | -0.014114 | 0.007812 |
| 8 | 0.250000 | 0.015625 | 0.257812 | -0.014114 | 0.253906 | 0.000744 | 0.003906 |
| 9 | 0.253906 | 0.000744 | 0.257812 | -0.014114 | 0.255859 | -0.006688 | 0.001953 |
| 10 | 0.253906 | 0.000744 | 0.255859 | -0.006688 | 0.254883 | -0.002973 | 0.000977 |
| 11 | 0.253906 | 0.000744 | 0.254883 | -0.002973 | 0.254395 | -0.001115 | 0.000488 |
| 12 | 0.253906 | 0.000744 | 0.254395 | -0.001115 | 0.254150 | -0.000185 | 0.000244 |
| 13 | 0.253906 | 0.000744 | 0.254150 | -0.000185 | 0.254028 | 0.000279 | 0.000122 |
| 14 | 0.254028 | 0.000279 | 0.254150 | -0.000185 | 0.254089 | 0.000047 | 0.000061 |
| 15 | 0.254089 | 0.000047 | 0.254150 | -0.000185 | 0.254120 | -0.000069 | 0.000031 |
| 16 | 0.254089 | 0.000047 | 0.254120 | -0.000069 | 0.254105 | -0.000011 | 0.000015 |
| 17 | 0.254089 | 0.000047 | 0.254105 | -0.000011 | 0.254097 | 0.000018 | 0.000008 |
| 18 | 0.254097 | 0.000018 | 0.254105 | -0.000011 | 0.254101 | 0.000003 | 0.000004 |
| 19 | 0.254101 | 0.000003 | 0.254105 | -0.000011 | 0.254103 | -0.000004 | 0.000002 |
| 20 | 0.254101 | 0.000003 | 0.254103 | -0.000004 | 0.254102 | -0.000000 | 0.000001 |

**f(x) =**

**Interval: [0,10]**

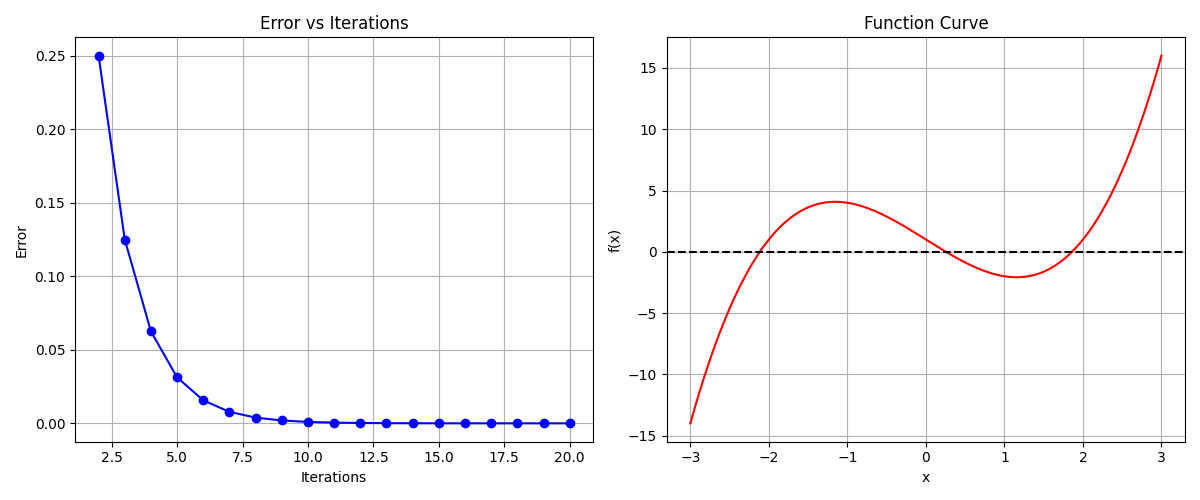
| **Iteration** | **a** | **f(a)** | **b** | **f(b)** | **x** | **f(x)** | **Error** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0.000000 | -2.000000 | 10.000000 | 1.162278 | 5.000000 | 0.236068 | inf |
| 2 | 0.000000 | -2.000000 | 5.000000 | 0.236068 | 2.500000 | -0.418861 | 2.500000 |
| 3 | 2.500000 | -0.418861 | 5.000000 | 0.236068 | 3.750000 | -0.063508 | 1.250000 |
| 4 | 3.750000 | -0.063508 | 5.000000 | 0.236068 | 4.375000 | 0.091650 | 0.625000 |
| 5 | 3.750000 | -0.063508 | 4.375000 | 0.091650 | 4.062500 | 0.015564 | 0.312500 |
| 6 | 3.750000 | -0.063508 | 4.062500 | 0.015564 | 3.906250 | -0.023576 | 0.156250 |
| 7 | 3.906250 | -0.023576 | 4.062500 | 0.015564 | 3.984375 | -0.003910 | 0.078125 |
| 8 | 3.984375 | -0.003910 | 4.062500 | 0.015564 | 4.023438 | 0.005851 | 0.039062 |
| 9 | 3.984375 | -0.003910 | 4.023438 | 0.005851 | 4.003906 | 0.000976 | 0.019531 |
| 10 | 3.984375 | -0.003910 | 4.003906 | 0.000976 | 3.994141 | -0.001465 | 0.009766 |
| 11 | 3.994141 | -0.001465 | 4.003906 | 0.000976 | 3.999023 | -0.000244 | 0.004883 |
| 12 | 3.999023 | -0.000244 | 4.003906 | 0.000976 | 4.001465 | 0.000366 | 0.002441 |
| 13 | 3.999023 | -0.000244 | 4.001465 | 0.000366 | 4.000244 | 0.000061 | 0.001221 |
| 14 | 3.999023 | -0.000244 | 4.000244 | 0.000061 | 3.999634 | -0.000092 | 0.000610 |
| 15 | 3.999634 | -0.000092 | 4.000244 | 0.000061 | 3.999939 | -0.000015 | 0.000305 |
| 16 | 3.999939 | -0.000015 | 4.000244 | 0.000061 | 4.000092 | 0.000023 | 0.000153 |
| 17 | 3.999939 | -0.000015 | 4.000092 | 0.000023 | 4.000015 | 0.000004 | 0.000076 |
| 18 | 3.999939 | -0.000015 | 4.000015 | 0.000004 | 3.999977 | -0.000006 | 0.000038 |
| 19 | 3.999977 | -0.000006 | 4.000015 | 0.000004 | 3.999996 | -0.000001 | 0.000019 |
| 20 | 3.999996 | -0.000001 | 4.000015 | 0.000004 | 4.000006 | 0.000001 | 0.000010 |
| 21 | 3.999996 | -0.000001 | 4.000006 | 0.000001 | 4.000001 | 0.000000 | 0.000005 |
| 22 | 3.999996 | -0.000001 | 4.000001 | 0.000000 | 3.999999 | -0.000000 | 0.000002 |
| 23 | 3.999999 | -0.000000 | 4.000001 | 0.000000 | 4.000000 | -0.000000 | 0.000001 |
| 24 | 4.000000 | -0.000000 | 4.000001 | 0.000000 | 4.000000 | 0.000000 | 0.000001 |

**f(x) = cos(x) - x**

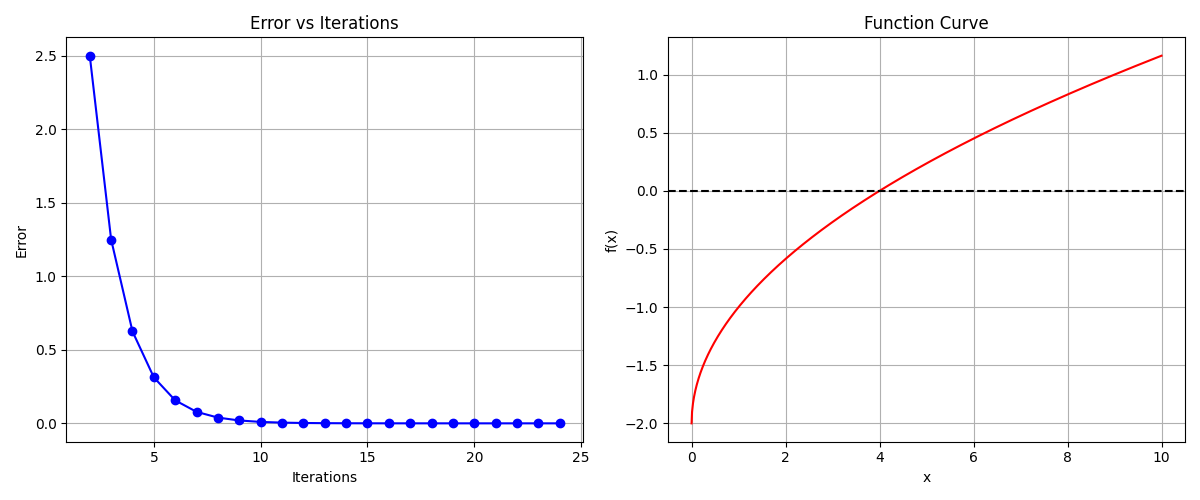
**Interval: [0,1]**

| **Iteration** | **a** | **f(a)** | **b** | **f(b)** | **x** | **f(x)** | **Error** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 0.000000 | 1.000000 | 1.000000 | -0.459698 | 0.500000 | 0.377583 | inf |
| 2 | 0.500000 | 0.377583 | 1.000000 | -0.459698 | 0.750000 | -0.018311 | 0.250000 |
| 3 | 0.500000 | 0.377583 | 0.750000 | -0.018311 | 0.625000 | 0.185963 | 0.125000 |
| 4 | 0.625000 | 0.185963 | 0.750000 | -0.018311 | 0.687500 | 0.085335 | 0.062500 |
| 5 | 0.687500 | 0.085335 | 0.750000 | -0.018311 | 0.718750 | 0.033879 | 0.031250 |
| 6 | 0.718750 | 0.033879 | 0.750000 | -0.018311 | 0.734375 | 0.007875 | 0.015625 |
| 7 | 0.734375 | 0.007875 | 0.750000 | -0.018311 | 0.742188 | -0.005196 | 0.007812 |
| 8 | 0.734375 | 0.007875 | 0.742188 | -0.005196 | 0.738281 | 0.001345 | 0.003906 |
| 9 | 0.738281 | 0.001345 | 0.742188 | -0.005196 | 0.740234 | -0.001924 | 0.001953 |
| 10 | 0.738281 | 0.001345 | 0.740234 | -0.001924 | 0.739258 | -0.000289 | 0.000977 |
| 11 | 0.738281 | 0.001345 | 0.739258 | -0.000289 | 0.738770 | 0.000528 | 0.000488 |
| 12 | 0.738770 | 0.000528 | 0.739258 | -0.000289 | 0.739014 | 0.000120 | 0.000244 |
| 13 | 0.739014 | 0.000120 | 0.739258 | -0.000289 | 0.739136 | -0.000085 | 0.000122 |
| 14 | 0.739014 | 0.000120 | 0.739136 | -0.000085 | 0.739075 | 0.000017 | 0.000061 |
| 15 | 0.739075 | 0.000017 | 0.739136 | -0.000085 | 0.739105 | -0.000034 | 0.000031 |
| 16 | 0.739075 | 0.000017 | 0.739105 | -0.000034 | 0.739090 | -0.000008 | 0.000015 |
| 17 | 0.739075 | 0.000017 | 0.739090 | -0.000008 | 0.739082 | 0.000005 | 0.000008 |
| 18 | 0.739082 | 0.000005 | 0.739090 | -0.000008 | 0.739086 | -0.000002 | 0.000004 |
| 19 | 0.739082 | 0.000005 | 0.739086 | -0.000002 | 0.739084 | 0.000001 | 0.000002 |
| 20 | 0.739084 | 0.000001 | 0.739086 | -0.000002 | 0.739085 | -0.000000 | 0.000001 |

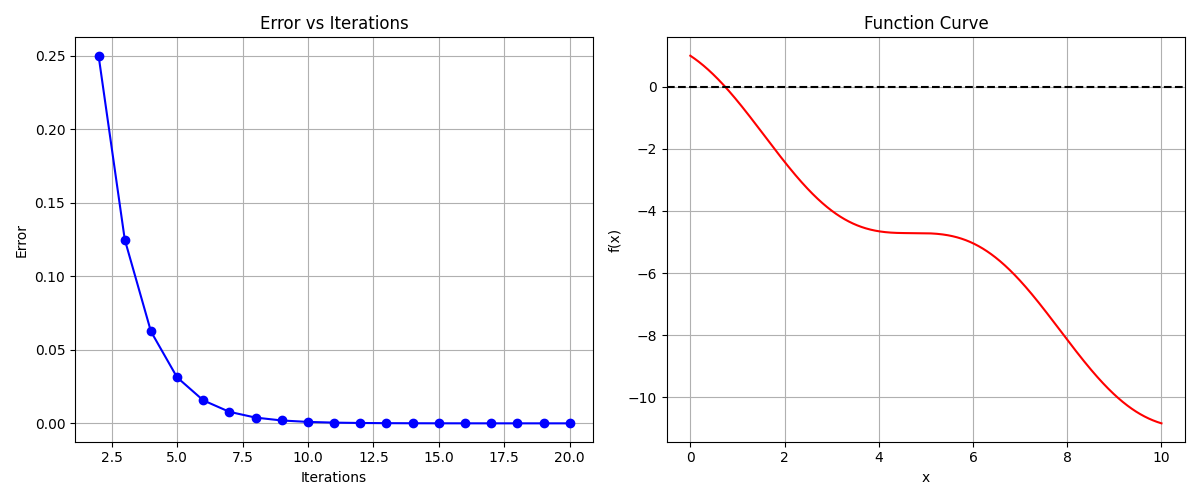
## Graphs

**f(x) = x3- 4x + 1**

**f(x) =**

****

**f(x) = cos(x) - x**

****

## Interpretation of Results

**Convergence:**

* Method converges in 20 iterations
* Error decreases by approximately half each iteration
* Final error < tolerance (1e-6)

**Accuracy:**

* Root found: x ≈ 0.254102
* Final f(x) ≈ 0.000001
* Accurate to 6 decimal places

**Performance:**

* Reliable convergence
* Consistent error reduction
* Efficient use of iterations
* No unnecessary computations

**Observations:**

* Initial interval [0, 1] contains one root
* Function changes sign at endpoints
* Method successfully narrows down to root
* Error decreases monotonically