Assignment 2

Junaid Ahmad Bhat

January 14, 2021

Question

If A and B are square matrices of the same order such that AB=BA, then prove by induction that $AB^n = B^nA$.
Further, prove that $(AB)^n = A^nB^n$ for all nϵN.

Solution

A and B are square matrices of the same order such that AB=BA

To prove: P(n):$AB^n = B^nA$,nϵN

For n=1, we have:

P(1):$AB = BA$ [Given]

$AB^1 = B^1A$

i,e the result is true for n=1.

Let the result be true for n=k.

P(k):$AB^k = B^kA$ (1)

Now, we prove that the result is rue for n=k+1.

$AB^{k+1} = AB^k.B = (B^kA)B$ (By(1))

$$= B^k(AB) \qquad \text{(By Associative law)}$$

$$= B^k(BA) \qquad \text{(AB=BA (given))}$$

$$= (B^kB)A \qquad \text{(By Associative law)}$$

$$= B^{k+1}A$$

i,e the result is true for n=k+1.

Thus, by the principle of mathematical induction , we have $AB^n = B^nA, n\epsilon N$.

Now, we prove that $(AB^n) = A^nB^n$ for all $n\epsilon N$

For n=1, we have:

$(AB)^1 = A^1B^1 = AB$

i,e the result is true for n=1.

Let it be true for n=k.

$$(AB)^k = A^kB^k \qquad\qquad (2)$$

Now we prove that the result is true for n=k+1.

$$(AB)^{k+1} = (AB)^k.(AB) = (A^kB^k)(AB). \qquad \text{(By (2))}$$

$$= A^k(B^k\ A)B \qquad \text{(By Associative law)}$$

$$= A^k(AB^k)B \qquad \text{(AB}^n\text{=B}^n\text{A, n}\epsilon\text{N)}$$

$$= (A^kA).(B^kB) \qquad \text{(By Associative law)}$$

$$= A^{k+1}B^{k+1}$$

i,e the result is true for n=k+1.

Therefore, by the principle of mathematical induction , we have $(AB)^n = A^nB^n$,

for all natural numbers.

Question

Let A=$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$,show that $(aI + bA)^n = a^n I + n a^{n-1} bA$,where I is tha identity matrix of order 2 and nϵN.

Solution

Using principle of mathematical induction
For n=1,we have

$P(1){:}(aI{+}bA){=}aI{+}a^0 bA{=}aI{+}bA$

i,e result is true for n=1

Let it be true for n=k

i,e $P(k){:}(aI{+}bA)^k{=}a^k I{+}k a^{k-1} bA$

Now,we have to prove the result is true for n=k+1.

$(aI{+}bA)^{k+1}{=}(aI{+}bA)^k (aI{+}bA)$

$\qquad\qquad {=}(a^k I{+}k a^{k-1} bA)(aI{+}bA)$

$\qquad\qquad {=}a^{k+1}I{+}k a^k bAI{+}a^k bIA{+}ka^{\,k-1} b^2 A^2$

$\qquad\qquad {=}a^{k+1}I{+}(k{+}1)a^k bA{+}k a^{k-1} b^2 A^2 \qquad\qquad (1)$

Now, A^2=$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$=0

Therfore,eqn(1) becomes
$(aI{+}bA)^{k+1}{=}a^{k+1}I{+}(k{+}1)a^k bA$

Hence proved result is true for n=k+1

Therefore, by principle of mathematical induction we have
$(aI + bA)^n = a^n I{+}n a^{n-1} bA$