Assignment 3

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Question

Find the inverse and QR decomposition of the A= $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$.

Solution

To find inverse of a given matrix,first we need check value of determinant of given matrix.If determinant is not equal to zero it exists.

So,check $|A|$;

$|A|$=3*2-5*1=1

i,e $|A| \neq 0$.

Therefore, A is non-singular.Hence its inverse exists

Now,

A^{-1}=$\frac{AdjA}{|A|}$—

$$A^{-1}= \frac{Adj\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}}{\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}} = \frac{1}{1}\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

Therefore, A^{-1}=$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

QR Decomposition

Now to find QR decomposition of given matrix,let's first have a look at its definition:
The QR decomposition (also called the QR factorization) of a matrix is a decomposition of the matrix into an orthogonal matrix and a triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as A = QR,where Q is an orthogonal matrix and R is an upper triangular matrix. If A is nonsingular, then this factorization is unique.

Using Gram-Schmidt method of decompostion

Given A= $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

its column matrices are:

a=$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$,b=$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

A=QR (in decomposition representation)

where Q=(q1,q2) q1,q2 being column matrices itself.

let's find q1 and q2:
we know,

$$q1=\frac{\mathbf{a}}{length\ of\ \mathbf{a}}$$

$$q1=\frac{1}{\sqrt{3^2+5^2}}*\begin{pmatrix} 3 \\ 5 \end{pmatrix}=\begin{pmatrix} \frac{3}{\sqrt{34}} \\ \frac{5}{\sqrt{34}} \end{pmatrix}$$

$$q2=\frac{q2'}{length\ of\ \mathbf{q2'}}$$ (q2' being prime q2)

q2'=b-(b•q1)q1 (dot product between b and q1) q2'= $\begin{pmatrix}1\\2\end{pmatrix}$ - $\left(\begin{pmatrix}1\\2\end{pmatrix}\bullet\begin{pmatrix}\frac{3}{\sqrt{34}}\\\frac{5}{\sqrt{34}}\end{pmatrix}\right)\begin{pmatrix}\frac{3}{\sqrt{34}}\\\frac{5}{\sqrt{34}}\end{pmatrix}$

q2'= $\begin{pmatrix}1\\2\end{pmatrix}$ - $\frac{13}{\sqrt{34}}\begin{pmatrix}\frac{3}{\sqrt{34}}\\\frac{5}{\sqrt{34}}\end{pmatrix}$

q2'= $\begin{pmatrix}1\\2\end{pmatrix}$ - $\begin{pmatrix}\frac{39}{34}\\\frac{65}{34}\end{pmatrix}$

q2'= $\begin{pmatrix}\dfrac{-5}{34}\\\dfrac{3}{34}\end{pmatrix}$

AS q2= $\dfrac{1}{length\ of\ q2'}\begin{pmatrix}\dfrac{-5}{34}\\\dfrac{3}{34}\end{pmatrix}$

length of q2'= $\sqrt{(-5/34)^2+(3/34)^2}=\dfrac{1}{\sqrt{34}}$

Therefore q2= $\begin{pmatrix}\dfrac{-5}{\sqrt{34}}\\\dfrac{3}{\sqrt{34}}\end{pmatrix}$

So,we have Q= $\begin{pmatrix}\dfrac{3}{\sqrt{34}} & \dfrac{-5}{\sqrt{34}}\\\dfrac{5}{\sqrt{34}} & \dfrac{3}{\sqrt{34}}\end{pmatrix}$

And we know ,R= $Q^T.A$

So,R=$\begin{pmatrix} \frac{3}{\sqrt{34}} & \frac{5}{\sqrt{34}} \\ \frac{-5}{\sqrt{34}} & \frac{3}{\sqrt{34}} \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

R=$\begin{pmatrix} \frac{9}{\sqrt{34}} + \frac{25}{\sqrt{34}} & \frac{3}{\sqrt{34}} + \frac{10}{\sqrt{34}} \\ \frac{-15}{\sqrt{34}} + \frac{15}{\sqrt{34}} & \frac{-5}{\sqrt{34}} + \frac{6}{\sqrt{34}} \end{pmatrix}$ (matrix multiplication)

R=$\begin{pmatrix} \frac{34}{\sqrt{34}} & \frac{13}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{34}} \end{pmatrix}$

Therefore, $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{34}} & \frac{-5}{\sqrt{34}} \\ \frac{5}{\sqrt{34}} & \frac{3}{\sqrt{34}} \end{pmatrix} \begin{pmatrix} \frac{34}{\sqrt{34}} & \frac{13}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{34}} \end{pmatrix}$

Hence decomposed

Question

Find the inverse and QR decomposition of the A= $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$
.

Solution

First check $|A|$;

$|A|$=4*4-3*5=1

i,e $|A| \neq 0$.

Therefore, A is non-singular.Hence its inverse exists

Now,

A^{-1}=$\frac{AdjA}{|A|}$—

$$A^{-1}= \frac{Adj\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}}{\begin{vmatrix} 4 & 5 \\ 3 & 4 \end{vmatrix}}$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}$$

Therefore,

$$A^{-1}=\begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}$$

Now,to find its QR decomposition:

Using Gram-Schmidt method of decompostion

Given A= $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

its column matrices are:

a=$\begin{pmatrix}4\\3\end{pmatrix}$,b=$\begin{pmatrix}5\\4\end{pmatrix}$

A=QR (in decomposition representation)

where Q=(q1,q2) q1,q2 being column matrices itself.

let's find q1 and q2:
we know,

q1=$\dfrac{\mathbf{a}}{length\ of\ \mathbf{a}}$

q1=$\dfrac{1}{\sqrt{4^2+3^2}}*\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}\frac{4}{5}\\\frac{3}{5}\end{pmatrix}$

q2=$\dfrac{q2'}{length\ of\ \mathbf{q2'}}$ (q2' being prime q2)

q2'=b-(b●q1)q1 (dot product between b and q1)

q2'=$\begin{pmatrix}5\\4\end{pmatrix}$ - $\left(\begin{pmatrix}5\\4\end{pmatrix}\bullet\begin{pmatrix}\frac{4}{5}\\\frac{3}{5}\end{pmatrix}\right)\begin{pmatrix}\frac{4}{5}\\\frac{3}{5}\end{pmatrix}$

q2'=$\begin{pmatrix}5\\4\end{pmatrix}$ - $\dfrac{32}{5}\begin{pmatrix}\frac{4}{5}\\\frac{3}{5}\end{pmatrix}$

q2'=$\begin{pmatrix}5\\4\end{pmatrix}$ - $\begin{pmatrix}\frac{128}{25}\\\frac{96}{25}\end{pmatrix}$

q2'= $\begin{pmatrix} \frac{-3}{25} \\ \frac{4}{25} \end{pmatrix}$

AS q2=$\frac{1}{length\ of\ q2'}$ $\begin{pmatrix} \frac{-3}{25} \\ \frac{4}{25} \end{pmatrix}$

length of q2'=$\sqrt{(-3/25)^2 + (4/25)^2}$=$\frac{1}{5}$

Therefore q2=$\begin{pmatrix} \frac{-3}{5} \\ \frac{4}{5} \end{pmatrix}$

So,we have Q=$\begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$

And we know ,R=Q^T.A

So,R=$\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{-3}{5} & \frac{4}{5} \end{pmatrix}$ $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

R=$\begin{pmatrix} \frac{16}{5} + \frac{9}{5} & \frac{20}{5} + \frac{12}{5} \\ \frac{-12}{5} + \frac{12}{5} & \frac{-15}{5} + \frac{16}{5} \end{pmatrix}$ (matrix multiplication)

R=$\begin{pmatrix} \frac{25}{5} & \frac{32}{5} \\ 0 & \frac{1}{5} \end{pmatrix}$

Therefore, $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 5 & \frac{32}{5} \\ 0 & \frac{1}{5} \end{pmatrix}$

Hence decomposed