

Assignment 5

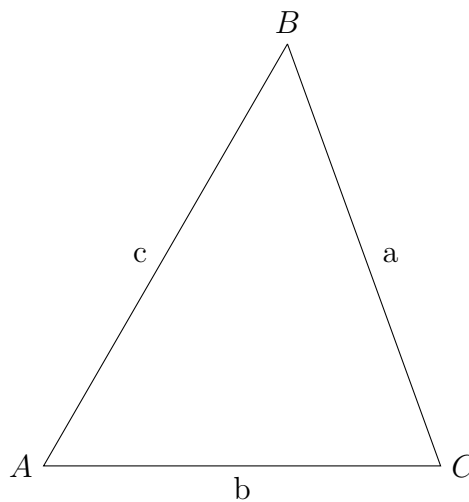
Junaid Ahmad Bhat

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Question:

If $AC = 7$, $\angle A = 60^\circ$ and $\angle B = 50^\circ$, can you draw the triangle?

Solution:



Rough Sketch

To draw the Triangle with given conditions, Let's first find the value of $\angle C$.

Using property, Sum of angles of triangle,

We have,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$60^\circ + 50^\circ + \angle C = 180^\circ \quad (\text{given})$$

Therefore, $\angle C = 70^\circ$.

To draw the triangle, we need know the coordinates of all vertices.

Assuming starting vertex as A with coordinates as (0,0)

Therefore,
vertex C has coordinates as (7,0) (as given AC=7)

Coordinates of vertex B:

Actually vertex B is intersection point of lines AB and CB, so let's find it.

Eqn of line AB: $y = mx + c$; (m=slope of line and c is constt.)

$$y = 1.732x + c \quad (\text{as given } \angle A = 60^\circ \Rightarrow m = \tan(60^\circ))$$

$$\text{value of } c = 0 - 1.732(0) = 0 \quad (\text{as line AB passes through } (0,0))$$

Therefore,

$$\text{line AB: } y = 1.732x. \quad (1)$$

Similarly,

Eqn of line CB: $y = mx + c$; (m=slope of line and c is constt.)

$$y = -2.727x + c \quad (\text{as } \angle C = 70^\circ \Rightarrow m = \tan(110^\circ))$$

$$\text{value of } c = 0 + 2.747(7) = 19.23 \quad (\text{as line CB passes through } (7,0))$$

Therefore,

$$\text{line CB: } y = -2.747x + 19.23. \quad (2)$$

Finding Intersection point of line AB and CB

Substuting,eqn(1) in eqn(2),we get

$$x = \frac{19.23}{(1.73 + 2.727)}$$
$$\Rightarrow x = 4.292. \quad (3)$$

Substuting,eqn(3) in eqn(1),we get

$$y = 1.73(4.292)$$
$$\Rightarrow y = 7.44. \quad (4)$$

Therefore,coordinates of vertex B are (4.292,7.44)

Below figure 1 is possible triangle we can draw with coordinates as A(0,0), B(4.292,7.44) and C(7,0).

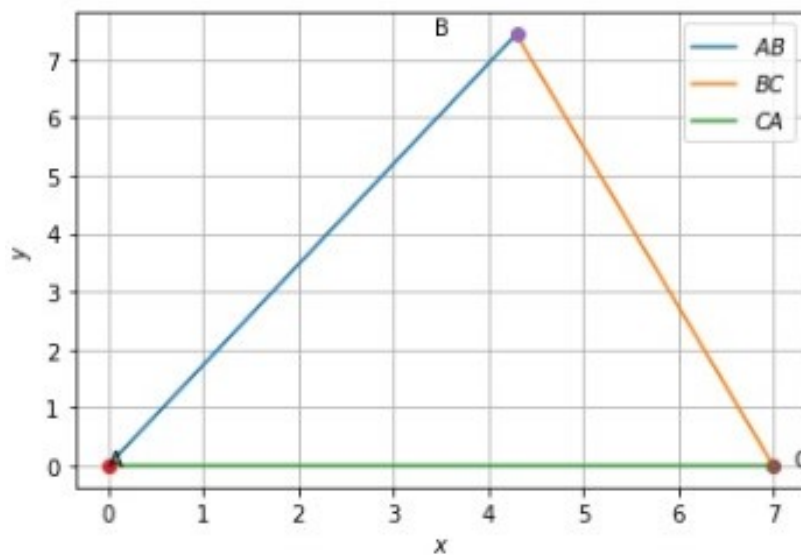
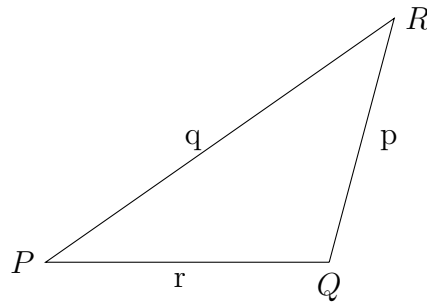


Figure 1: Figure using python

Question:

Construct $\triangle PQR$ if $PQ = 5$, $\angle Q = 105^\circ$ and $\angle R = 40^\circ$.

Solution:

Rough Sketch

To draw the Triangle with given conditions, Let's first find the value of $\angle P$.

Using property, Sum of angles of triangle,

We have,

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\angle P + 105^\circ + 40^\circ = 180^\circ \quad (\text{given})$$

Therefore, $\angle P = 35^\circ$.

To draw the triangle, we need know the coordinates of all vertices.

Assuming starting vertex as P with coordinates as (0,0)

Therefore,

vertex Q has coordinates as (5,0) (as given $PQ=5$)

Coordinates of vertex R:

Actually vertex R is intersection point of lines PR and QR,so let's find it.

Eqn of line PR: $y=mx+c$; (m=slope of line and c is constt.)

$$y=0.7x+c \quad (\text{as } \angle P=35^\circ \Rightarrow m=\tan(35^\circ))$$

$$\text{value of } c=0-0.7(0)=0 \quad (\text{as line PR passes through } (0,0))$$

Therefore,

$$\text{line PR: } y=0.7x. \quad (1)$$

Similarly,

Eqn of line QR: $y=mx+c$; (m=slope of line and c is constt.)

$$y=3.73x+c \quad (\text{as } \angle Q=105^\circ \Rightarrow m=\tan(75^\circ))$$

$$\text{value of } c=0-3.73(5)=-18.65 \quad (\text{as line QR passes through } (5,0))$$

Therefore,

$$\text{line QR: } y=3.73x-18.65. \quad (2)$$

Finding Intersection point of line PR and QR

Substituting,eqn(1) in eqn(2),we get

$$x = \frac{18.65}{(3.73 - 0.7)}$$

$$\Rightarrow x=6.15. \quad (3)$$

Substituting,eqn(3) in eqn(1),we get

$$y=0.7(6.15)$$

$$\Rightarrow y=4.3. \quad (4)$$

Therefore,coordinates of vertex R are (6.15,4.3)

Below figure 2 is given triangle with coordinates as $P(0,0)$, $Q(7,0)$ and $R(6.15,4.3)$.

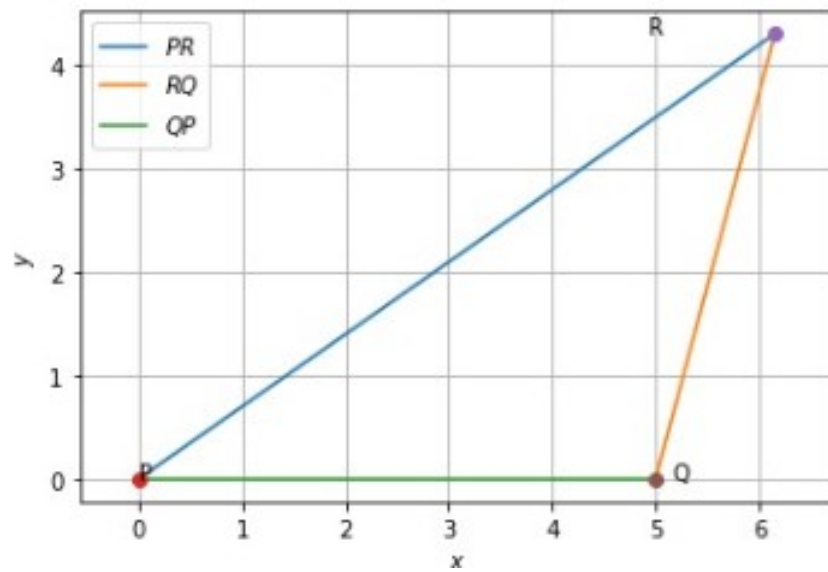


Figure 2: Figure using python

Note: m (i.e slope) = $\tan(\text{angle measured anti-clock wise from x-axis})$