Assignment 8

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Question

Express the matrix
$$B = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$
as the sum of a symmetric and a skew symmetric matrix.

Solution

We can write given matrix B as below:

B = $\frac{2B}{2}$ + (B'- B') (B' being transpose of B)

B = $\frac{B+B'}{2} + \frac{B-B'}{2}$

we have,
$$B' = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

Let X = $\frac{B+B'}{2}$; Y = $\frac{B-B'}{2}$

Therefore, B = X + Y (1)

let's evaluate X:

2X = B + B'

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix}$$

and

$$X' = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix} = X$$

i,e X is symmetric matrix (as X'= X)

Similarly, evaluating Y

2Y = B - B'

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{pmatrix}$$

and

$$Y' = \begin{pmatrix} 0 & 1/2 & 5/2 \\ -1/2 & 0 & -3 \\ -5/2 & 3 & 0 \end{pmatrix} = -Y$$

i,e Y is skew symmetric matrix (as Y'= -Y)

As B = X(symmetric) + Y(skew symmetric) (by eqn (1))

$$\Rightarrow \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{pmatrix}$$

Hence,it is decomposed/expressed as a sum of symmetric and skew symmetric matrix.