

Assignment 8

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January 25, 2021

Question

Express the matrix

$$B = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$$

as the sum of a symmetric and a skew symmetric matrix.

Solution

We can write given matrix B as below:

$$B = \frac{2B}{2} + \frac{B' - B'}{2} \quad (B' \text{ being transpose of } B)$$
$$\Rightarrow B = \frac{B + B'}{2} + \frac{B - B'}{2}$$

we have,

$$B' = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

$$\text{Let } X = \frac{B + B'}{2} ; Y = \frac{B - B'}{2}$$

$$\text{Therefore, } B = X + Y \quad (1)$$

let's evaluate X:

$$2X = B + B'$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix}$$

and

$$X' = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix} = X$$

i.e X is symmetric matrix (as $X' = X$)

Similarly, evaluating Y

$$2Y = B - B'$$

$$= \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{pmatrix}$$

and

$$Y' = \begin{pmatrix} 0 & 1/2 & 5/2 \\ -1/2 & 0 & -3 \\ -5/2 & 3 & 0 \end{pmatrix} = -Y$$

i.e Y is skew symmetric matrix (as $Y' = -Y$)

As $B = X(\text{symmetric}) + Y(\text{skew symmetric})$ (by eqn (1))

$$\Rightarrow \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{pmatrix}$$

Hence, it is decomposed/expressed as a sum of symmetric and skew symmetric matrix.