

## LESSON 5: BINOMIAL DISTRIBUTION

Q. 2 coin flips

$$\# \text{ HEADS} = \# \text{ TAILS}$$

? 2

H H

H T ]

T H ]

T T

Q 4 coin flips

$$\# \text{ HEADS} = \# \text{ TAILS}$$

? 6

H H H H

H H H T

H H T H

H H T T

H T H H

H T H T

H T T H

H T T T

T H H H

T H H T

T H T H

T H T T

T T H H

T T H T

T T T H

T T T T

Q 2 5 coin flips

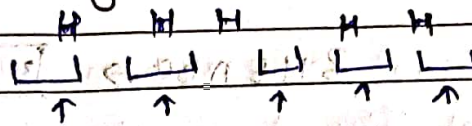
# HEADS = # TAILS

? 0

→ Answer is 0 because if we use odd no. of coins then one must be larger than the other.

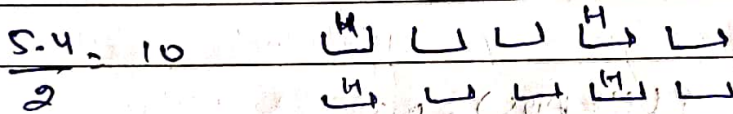
Q 5 coin flip

# Exactly 1 H → 5



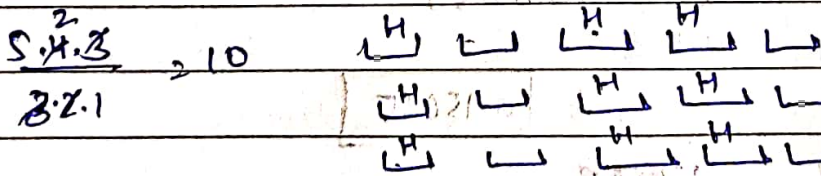
Q 5 coin flips

# Exactly 2 H → 10



Q

5 coin flips 3 H → 10



Q

10 coin flips 4 H → 210

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$$



Date.....

Q. 10 coin flips 5 HEADS  $\rightarrow$  [252]

$$\frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{8 \times 4 \times 2 \times 2 \times 1} = 252$$

Binomial Distribution formula

$$= \frac{n!}{k!(n-k)!}$$

Q. 125 coins 3 HEADS  $\rightarrow$  [317,750]

$$\frac{125 \times 124 \times 123}{3 \times 2 \times 1} = 317,750$$

2. coin flips -

Probabilities

$$P(\text{HEADS}) = 0.5$$

Q. - FLIP coins 5 times

$$P(\# \text{ HEADS}) = 1$$

$$0.15625$$

$$5 \times 4 \times 3 \times 2 \times 1 = 120 \times 0.5 = 60.0$$

$$\frac{5!}{1!4!} = \frac{120}{24} = 5$$

$$\frac{5!}{1!4!} \times \frac{1}{2^5} = 5 \times \frac{1}{32} = 0.15625$$

Q Flip coin 5 times  $P(\# \text{HEADS}) = 3 \Rightarrow \boxed{0.3125}$

$$\frac{5!}{3! \times 2!} \times 10 \Rightarrow \frac{10}{32} = 0.3125$$

2. Loaded coins

Q Flip coin 3 times  $P(H) = 0.8$   
 $P(\# \text{HEADS} \neq 1) \quad P(T) = 0.2$

↓

$$\boxed{0.096}$$

H H H

H H T

H T H

H T T

T H H

T H T

T T H

T T T

$$0.8 \times 0.2 \times 0.2 = 0.032$$

$$0.2 \times 0.8 \times 0.2$$

$$0.2 \times 0.2 \times 0.8$$

$$0.2 \times 0.2 \times 0.2$$

$$0.2 \times 0.2 \times 0.2$$

$$0.2 \times 0.2 \times 0.2$$

Q Flip coin 5 times  $P(H) = 0.8$

$$P(\# \text{HEADS} \neq 1) \Rightarrow \boxed{0.4096}$$

$$\frac{5!}{4!1!} \times (0.8)^4 \times (0.2)^1$$

$$5 \times (0.8)^4 \times (0.2)^1 = \boxed{0.4096}$$

Q Flip coin 5 times  $P(H) = 0.8$

$$P(\# \text{HEADS}) = 3$$

$$\frac{5!}{3!2!} \times 10 \times (0.8)^3 \times (0.2)^2 = \boxed{0.2048}$$



Q. Flip coin 12 times  
 $P(H) = 0.8$   
 $P(\# \text{ HEADS} = 9) \Rightarrow \boxed{0.236}$

$$\frac{12!}{9! 3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$$

$$220 \cdot (0.8)^9 \cdot (0.2)^3 = \boxed{0.236}$$

## 2. summary

1. The Binomial Distribution helps us determine the probability of a string of independent 'coin flip like events'.

$$P(X=x) = \frac{n!}{x!(n-x)!} \times p^x (1-p)^{n-x}$$

$n \rightarrow$  number of events.

$x \rightarrow$  number of successes.

$p \rightarrow$  probability of success.