13/05/2020 Date...............

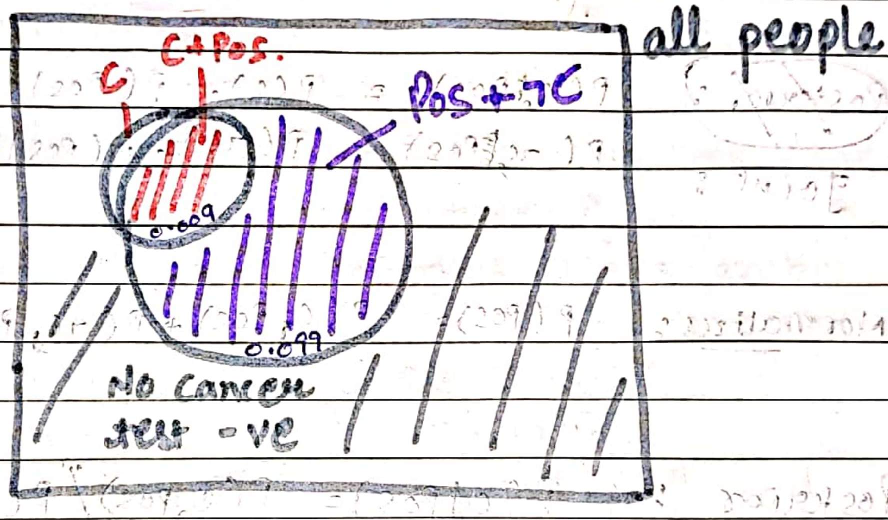
LESSON 7 : Bayes Rule

Example :

$P(C) = 0.01$

TEST : 90% it is positive if you have C. ← SENSITIVITY
90% it is negative if you don't have C. ← SPECITIVITY

Ques :- TEST = POSITIVE
PROBABILITY of HAVING CANCER



The Question being asked is this: 1% of the population has cancer. Given that there is a 90% chance that you will test positive if you have cancer & that there is a 90% chance you will test -ve if you don't have cancer, what is the probability that you have cancer if you test positive?

- [ ] 90%
- [ ] 8%
- [ ] 1%

Date.......

Bayes Rule

Prior Probability $\cdot$ Test evidence $\rightarrow$ Posterior Probability

Prior: $P(C) = 0.01$ $\quad P(\neg C) = 0.99$

$P(Pos|C) = 0.9 = 90\%$ $\quad P(Pos|\neg C) = 0.1$

$P(Neg|\neg C) = 0.9$

~~Posterior~~ Joint: $P(C, Pos) = P(C) \cdot P(Pos|C) = \boxed{0.009}$

$P(\neg C, Pos) = P(\neg C) \cdot P(Pos|\neg C) = \boxed{0.099}$

Normalizer: $P(Pos) = P(C, Pos) + P(\neg C, Pos) = \boxed{0.108}$

Posterior: $\left.\begin{array}{l} P(C|Pos) = P(C, Pos) / P(Pos) = \boxed{0.083} \\ P(\neg C|Pos) = P(\neg C, Pos) / P(Pos) = \boxed{0.9167} \end{array}\right\} \boxed{1}$

14/05/2020

If test is pve:

$$
\begin{tikzcd}
 & P(C) \arrow[dl, "\text{mult. } P(Pos|C)"'] \arrow[dr, "\text{mult. } P(Pos|\neg C)"] & & \\
P(C, Pos) \arrow[rr, dash] \arrow[d, "\text{divide by } P(Pos)"'] & & P(\neg C, Pos) \arrow[r, "\text{add}"] \arrow[d, "\text{divide by } P(Pos)"] & P(Pos) \\
P(C|Pos) & & P(\neg C|Pos) &
\end{tikzcd}
$$

Date...............

If test is -ve

$$
\begin{tikzcd}
 & P(C) \arrow[dl, "\text{mult. by } P(Neg|C)"'] \arrow[dr, "\text{mult. by } P(Neg|\neg C)"] & & \\
P(C, Neg) \arrow[rr, dash] \arrow[d, "\text{divide by } P(Neg)"'] & & P(\neg C, Neg) \arrow[r, "\text{add}"] \arrow[d, "\text{divide by } P(Neg)"] & P(Neg) \\
P(C|Neg) & & P(\neg C|Neg) &
\end{tikzcd}
$$

Q. In previous question we have asked what is the probability that you have cancer if you test +ve. Now, in this question what is the probability that you have cancer if you test negative.

$P(C) = 0.01$ $\qquad P(\neg C) = \boxed{0.99}$

$P(Pos|C) = 0.9$ $\qquad P(Neg|C) = \boxed{0.1}$

$P(Neg|C) = 0.9$ $\qquad P(Pos|\neg C) = \boxed{0.1}$

Test = Negative

$P(C, Neg) = \boxed{0.001}$

$P(\neg C, Neg) = \boxed{0.891}$

$P(Neg) = \boxed{0.892}$

$P(C|Neg) = \boxed{0.0011}$

$P(\neg C|Neg) = \boxed{0.9989}$

Quiz: Disease Test 1

$P(C) = 0.1$ $P(\neg C) = \boxed{0.9}$

$P(Pos|C) = 0.9$ $P(Neg|C) = \boxed{0.1}$

$P(Neg|\neg C) = 0.5$ $P(\cancel{Neg}\ Pos|\neg C) = \boxed{0.5}$

Test = Neg

$P(C, Neg) = \boxed{0.01}$

$P(\neg C, Neg) = \boxed{0.45}$

$P(Neg) = \boxed{0.46}$

$P(C|Neg) = \boxed{0.0217}$ $\propto \frac{P(C, Neg)}{P(Neg)}$

$P(\neg C|Neg) = \boxed{0.9783}$ $\approx \frac{P(\neg C, Neg)}{P(Neg)}$

Test = Pos

$\cancel{P(C|Pos)} = \boxed{\cancel{0.9783}}$

$\cancel{P(\neg C|Pos)} = \boxed{\cancel{0.0217}}$

$P(C, Pos) = \boxed{0.09}$

$P(\neg C, Pos) = \boxed{0.45}$

$P(Pos) = \boxed{0.54}$

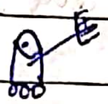
$P(C|Pos) = \boxed{0.1667}$

$P(\neg C|Pos) = \boxed{0.8333}$

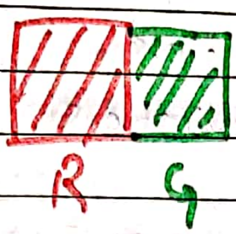
Date..............

Quiz 2: Robot sensing

Example:



~ $P(Red) = P(Green) = 0.5$



$P(\text{see } R \mid \text{at } R) = 0.8$

$P(\text{see } G \mid \text{at } G) = 0.8$

$P(\neg\text{see } R \mid \text{at } R) = \boxed{0.2}$

$P(\neg\text{see } G \mid \text{at } G) = \boxed{0.2}$

Sees Red:

Posterior Probabilities: ?

$P(\text{at } R \mid \text{see } R) = \boxed{0.8}$

$P(\text{at } G \mid \text{see } R) = \boxed{0.2}$

$P(\text{at } R, \text{see } R) = 0.4$

$P(\text{at } G, \text{see } R) = 0.1$

Normalize: $P(\text{see } R) = 0.5$

$P(\text{at } R \mid \text{see } R) = 0.8$

$P(\text{at } G \mid \text{see } R) = 0.2$

Posterior Probabilities finding

Step-by-step walkthrough

Let's start with what we know:

Prior Probabilities

The robot is perfectly ignorant about where it is, so prior probabilities are as follows:

$P(\text{at red}) = 0.5$

$P(\text{at green}) = 0.5$

Conditional Probabilities

The robot's sensor are not perfect. Just because the robot sees red does not mean the robot is at red.

$$P(\text{see red} \mid \text{at red}) = 0.8$$
$$P(\text{see green} \mid \text{at green}) = 0.8$$

Posterior Probabilities

From these prior & conditional probabilities we are asked to calculate the following posterior probabilities after the robot sees red:

1. $P(\text{at red} \mid \text{see red})$
2. $P(\text{at green} \mid \text{see red})$

and as a reminder, Bayes rule lookes like this:

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

or if we want to use our "versions" of A & B (for posterior #1)...

$$P(\text{at red} \mid \text{see red}) = \frac{P(\text{see red} \mid \text{at red}) \cdot P(\text{at red})}{P(\text{see red})}$$

Now, we can read two of those terms from what we already know about our prior & conditional probabilities which means we can rewrite this as...

Date...............

$$P(\text{at red} \mid \text{see red}) = \frac{0.8 \times 0.5}{P(\text{see red})}$$

But we still have one unknown! what was the probability that we could see red? The answer is 0.5 and there are two ways I can convince myself of that. The first is intuitive & the second is mathematical.

why is P(see red) 0.5?

Argument 1: Intuitive

→ Of course it's 0.5! what else could it be? The robot had a 50% belief that it was in red & 50% belief that it was in green.

→ Sure, it's sensors are unreliability is symmetric & not biased towards mistakenly seeing either color.

→ So whatever the probability of seeing red is, that will also be the probability of seeing green. Since these two colors are the only possible colors the probability must be 50% for each!

Argument 2: Mathematical (Law of total Probability)

→ There are exactly two situations where the robot would see red.

1. when the robot is in a red square & its sensors work correctly.
2. when the robot is in a green square & it's sensors makes a mistake.

Date...............

→ I just need to add up these two probabilities to get the total probability of seeing red.

$$P(\text{see red}) = P(\text{at red}) \cdot P(\text{see red} \mid \text{at red}) + P(\text{at green}) \cdot P(\text{see red} \mid \text{at green})$$

→ I can read these quantities from above!

$$P(\text{see red}) = 0.5 \times 0.8 + 0.5 \times 0.2$$
$$P(\text{see red}) = 0.4 + 0.1$$
$$P(\text{see red}) = 0.5$$

Example 2

$P(R) = 0$
$P(G) = 1$
$P(\text{see } R \mid \text{at } R) = 0.8$
$P(\text{see } G \mid \text{at } G) = 0.8$

See: Red

Posterior Probabilities:

$P(\text{at } R \mid \text{see } R) = \boxed{0}$
$P(\text{at } G \mid \text{see } R) = \boxed{1}$

$$P(\text{at } R \mid \text{see } R) = \frac{P(\text{see } R \mid \text{at } R) \times P(\text{at } R)}{P(\text{see } R)}$$

$$P(\text{see } R) = P(\text{at Red}) \cdot P(\text{see } R \mid \text{at } R) + P(\text{at } G) \times P(\text{see } R \mid \text{at } G)$$
$$= 0 \times 0.8 + 1 \times 0.2$$
$$= 0.2$$

$P(\text{at } R \mid \text{see } R) = (0.8 \times 0)/0.2 = 0$
$P(\text{at } G \mid \text{see } R) = (0.2 \times 1)/0.2 = 1$

Date..............

Example 3

$P(R) = 0.5$

$P(G) = 0.5$

$P(\text{see } R \mid \text{at } R) = 0.8$

$P(\text{see } G \mid \text{at } G) = 0.5$

sees Red :

Posterior Probabilities

$P(\text{at } R \mid \text{see } R) =$

$P(\text{at } G \mid \text{see } R) =$

$P(\text{see } R) = P(\text{Red}) \cdot P(\text{see } R \mid \text{at } R) + P(\text{at } G) \cdot P(\text{see } R \mid \text{at } G)$

$= 0.5 \times 0.8 + 0.5 \times 0.5$

$= 0.4 + 0.25$

$= 0.65$

$$P(\text{at } R \mid \text{see } R) = \frac{P(\text{see } R \mid \text{at } R) \times P(R)}{P(\text{see } R)}$$

$$= \frac{0.8 \times 0.5}{0.65}$$

$$= 0.6153$$

$$P(\text{at } G \mid \text{see } R) = \frac{P(\text{see } R \mid \text{at } G) \times P(G)}{P(\text{see } R)}$$

$$= \frac{0.5 \times 0.5}{0.65}$$

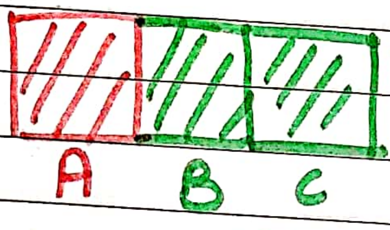
$$= 0.3847$$

Date..............

Example 4

Robot sees R

$P(A) = P(B) = P(C) = \frac{1}{3} = 0.333$



$P(R|A) = 0.9$
$P(G|B) = 0.9$
$P(G|C) = 0.9$

$P(A, R) = \boxed{0.3}$
$P(B, R) = \boxed{0.033}$
$P(C, R) = \boxed{0.033}$
Normalizer : $P(R) = \boxed{0.366}$

Posterior $P(A|R) = \boxed{0.818} \approx P(A,R) / P(R)$
$P(B|R) = \boxed{0.091} \approx P(B,R) / P(R)$
$P(C|R) = \boxed{0.091} \approx P(C,R) / P(R)$
$\boxed{\Sigma = 1}$

~~$P(A|R) = \frac{P(R) \times P(R|A)}{P(A)} = \frac{0.366 \times 0.9}{0.333} = 0.9891$~~

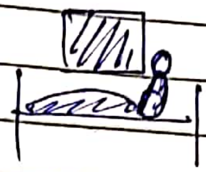
~~$P(B|R) = \frac{P(R) \times P(R|B)}{P(B)} = \frac{0.366 \times 0.1}{0.333} = 0.1099$~~

Date..............

SEBESTIAN AT HOME :

$P(gone) = 0.6$

$P(home) = 0.4$



$P(rain|home) = 0.01$

$P(rain|gone) = 0.3$

$P(home|rain) = \boxed{0.0217}$

Posterior Probability

$$P(home|rain) = \frac{P(rain|home) \times P(home)}{P(rain)}$$

$$= \frac{0.01 \times 0.4}{P(rain)}$$

$$= \frac{0.01 \times 0.4}{0.184}$$

$$= 0.0217$$

$$P(rain) = P(gone) . P(rain|gone) + P(home) \times P(rain|home)$$

$$= 0.6 \times 0.3 + 0.4 \times 0.01$$

$$= 0.18 + 0.004$$

$$= 0.184$$