7. Testing hypothesis about a population mean
one sample t-test :

→ Setting up a test for population mean

Cartwheel Study

- 25 team members/colleagues (all adults) asked to perform a cartwheel.
- variable : Cartwheel Distance (in inches)

→ Research Question

- Is the average cartwheel distance for adults more than 80 inches?

→ Population : All Adults.

→ Parameter of Interest : Population mean cartwheel distance μ.

Date..............

→ Main Goal :

Perform a one-sample test regarding the value for the mean cartwheel distance for the population of all such adults.

→ Step1 : Define the NULL & Alternative hypothesis

NULL : Population mean CW distance (μ) is 80 inches.
Alternative: Population mean is greater than (>) 80 inches.

More compact notation :

- $H_0 : \mu = 80$
  $H_a : \mu > 80$
  where μ → represents the population mean cartwheel distance (inches) for all adults.

→ significance Level = 5%

Standard significance level is 5%

→ Step2 : Examine Results, Check Assumptions, Summarize data via Test Statistic.

Assumptions :

- Sample of CW Distance measurements considered a simple random sample
- Normal distribution for CW distances in population (not as critical given large sample size, but still graph our data.

Date..............

mean of 25 sample data = 82.48 inches

* Is sample mean of 82.48 inches significantly greater than hypothesized mean of 80 inches?

↓ To find this, we will find standard error

Standard error of the sample mean $= \frac{\sigma}{\sqrt{n}}$

→ Test statistic

Assuming sampling distribution of sample mean is normal,

$$t = \frac{\text{best estimate} - \text{null value}}{\text{estimated std. error}} = \frac{\bar{x} - 80}{\frac{s}{\sqrt{n}}}$$

$$= \frac{82.48 - 80}{\frac{15.06}{\sqrt{25}}} = \frac{2.48}{3.012} = \boxed{0.82}$$

Our sample mean is only 0.82 (estimated) standard errors above null value of 80 inches

Scanned with CamScanner

Date..............

→ Step 3 : Determine p-value

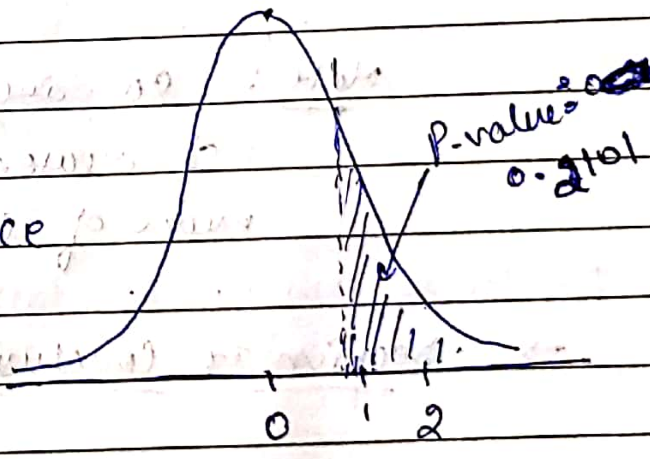
- If null hypothesis was true, would a test statistic value of only $t = 0.82$ be unusual enough to reject the null.

- p-value = Probability of seeing test statistic of 0.82 or more extreme assuming the null hypothesis is true.

- If null hypothesis was true, t test statistic follows a → Student t distribution with degrees of freedom $n-1 = 25-1 = 24$

- Since, we have a one tailed test to the right
  → More extreme measured to the right (upper tail)

- p-value = 0.21



If population mean CW distance was really was 80 inches, then observing a sample mean of 82.48 inches (i.e., a t statistic of 0.82) or larger is quite likely.

Date...............

→ Step 4 : Make a decision about the null.

Since our P-values is much bigger than 0.05 significance level, weak evidence against the NULL.

↳ we fail to reject the null!

So, Based on estimated mean (82.48 inches), we cannot support the population mean of CW distance is greater than 80 inches.

→ 90% confidence Interval Estimate

$$\bar{x} \pm t^{*} \left(\frac{s}{\sqrt{n}}\right)$$

(77.33 inches, 87.63 inches)

Note: 80 inches is IN confidence interval of reasonable values for population mean of CW distance

→ Decision or Conclusion

for the population of interest (all adults) ~ regardless of assumptions made & inference approach used → There is not sufficient evidence to support that the population mean CW distance is more than 80 inches.

Scanned with CamScanner

Date..............

21/05/2020

t- Difference in means for ~~[illegible]~~ Paired Data

→ Testing a population mean difference

Home Renovations

- 20 homes remodeling their kitchens, requesting cabinet quotes from 2 suppliers.

→ Research Question

Is there an average difference in cabinet quotes from these two suppliers?

→ variable : Difference in cabinet quotes (Supplier A - Supplier B)

→ Populations : All houses

→ Parameter of Interest :

Population mean difference of cabinet quotes μ_d (Supplier A - Supplier B).

→ Test for a significant mean difference in cabinet quotes at the 5% significance level.

→ Hypotheses:

$H_0 : \mu_d = 0$

$H_a : \mu_d \neq 0$

→ Significance level

$\alpha = 0.05$

Date...............

Cabinet Data

Supplier A	Supplier B	Difference
\$ 280	\$225	\$55
\$ 560	\$470	\$ 90
\$ 425	\$ 420	\$ 5
\$ 389	\$ 375	\$ 14
\$ 568	\$ 574	-\$ 6
\$ 651	\$ 595	\$ 56

→ **Assumptions**

- We need to assume that we have a random sample of differences, i.e., a random sample of mouses.

- We also need the population of differences to be normally distribute. We can get around this assumption if we have a large sample size (about 25+).

→ **Summarize the Data**

n = 20 observations

Minimum = - \$30

Maximum = \$90

Median = \$13.50

Mean = \$17.30

Standard Deviation = \$28.49

Scanned with CamScanner

Date..............

→ Test Statistic

Assuming the sample distribution of the sample mean difference is normal,

$$t = \frac{\text{best estimate} - \text{hypothesized estimate}}{\text{estimated standard errors of estimate}}$$

$$t = \frac{\bar{x}_d - 0}{s_d/\sqrt{n}}$$

$n = 20$

$\bar{x}_d = \$17.30$

$SD = \$28.49$

$$t = \frac{17.30 - 0}{\frac{28.49}{\sqrt{20}}}$$

$$t = \frac{17.30}{6.37}$$

$$\boxed{t = 2.72}$$

How you interpret a test statistic?

↓

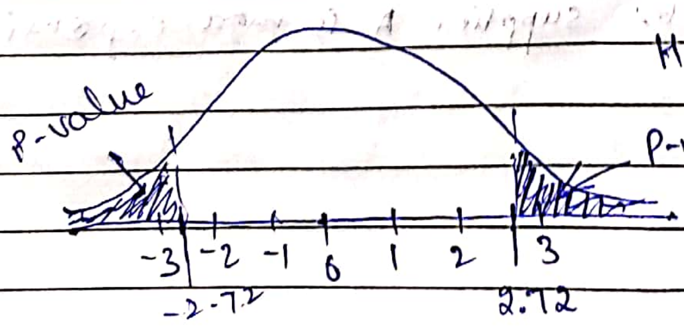
Our observed mean difference is 2.72 (estimated) standard errors above our null value of 0.

→ Test Statistic Distribution & P-value

$t(19)$ ← degree of freedom = no. of observation − 1

$= 20 - 1$

$= 19$



$H_a: \mu_a \neq 0$

P-val = 0.014

- Two side shaded bcoz H_a is $\neq 0$.

Date...............

→ Decision & Conclusion

$$p\text{-val} = 0.014 < 0.05 = \alpha$$

↓

reject null hypothesis

↳ have evidence against mean difference in cabinet quotes is 0.

- Formally based on our sample and our p-value, we reject the null hypothesis. We conclude that the mean difference of cabinet quote prices for Supplier A less (-) B is significantly different from 0.

→ 95% Confidence Interval

- Now, if we want to get a sense of in what way it's different from zero, we might look at a 95% Confidence Interval.

C.I. = ($3.97, $30.63)

↳ Note 0 is NOT in our range of reasonable values for mean difference in cabinet prices.

↳ This indicates supplier A is more expensive than supplier B.

Date...............

→ Wilcoxon Signed Rank Test

If normality doesn't hold, we can use the Wilcoxon Signed Rank Test to test for the median.

p-val = 0.020

↳ Again, we reject H_0 and conclude that the median difference in cabinet quotes, Supplier. A less B, is different from 0.

2. Difference in Means for independent groups (Two sample t-test)

→ Testing for a difference in Population Means (for Independent Groups).

→ Research Question

Considering Mexican-American Adults (ages 18-29) living in the united states, do males have a significantly higher mean Body mass Index than females?

→ Population: Mexican-American adults (age 18-29) in the U.S.

→ Parameter of Interest ($\mu_1 - \mu_2$): Body Mass Index or BMI (kg/m^2)

Date..............

→ Task : Perform an independent sample t-test regarding the value for the difference in mean BMI b/w males & females.

→ Steps to perform a Hypothesis Test

1. Define null & alternative hypothesis.
2. Examine data, check assumptions, & calculate test statistic
3. Determine corresponding p-value.
4. Make a decision about null hypothesis.

→ Step1 : Define Hypothesis

NULL : H_0 : There is no difference in mean BMI

Alternative : H_1 : There is a significant difference in mean BMI.

$H_0 : \mu_1 = \mu_2$ (or $\mu_1 - \mu_2 = 0$)

$H_1 : \mu_1 \neq \mu_2$ (or $\mu_1 - \mu_2 \neq 0$)

→ Significance level: 5%

→ Step2 : Check Assumptions

- Samples are considered simple random samples.
- Samples are independent from one another.
- Both populations of responses are approximately normal (or sample sizes are both 'large' enough)

Date......... ...

→ Calculate Test statistic

$H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 \neq 0$

Best Estimate : $\bar{x}_1 - \bar{x}_2 = 23.57 - 22.83 = 0.74$

↳ Is our sample mean difference of $0.74 \, kg/m^2$ significantly different than 0?

↓ we will find out using

Test statistic.

Test statistic :

- It is a measure of how far our sample statistic is from our hypothesized population parameter, in terms of estimated standard errors.

- The further away our sample statistic is, the less confident we'll be in our null hypothesized value!

$$t = \frac{\text{bestestimate} - \text{null value}}{\text{estimated standard error}}$$

- To calculate estimated standard error there are 2 approaches:

1) Pooled Approach: The variance of the two populations are assumed to be equal ($\sigma_1^2 = \sigma_2^2$)

2) Unpooled Approach: The assumption of equal variance is dropped.

Pooled Approach:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

or

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Unpooled Approach:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Note: Because the IQR & standard deviations are similar, the pooled approach will be used.

- $$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(23.57 - 22.83)}{\sqrt{\frac{(257) 6.24^2 + (238) 6.43^2}{495}} \sqrt{\frac{1}{258} + \frac{1}{239}}}$$

$$t = \frac{0.74}{0.0898 * 6.332}$$

$$\boxed{t = 1.30}$$

Scanned with CamScanner

Date...............

↳ So, what this "t" is telling us?

↓

It tells us that, our difference in sample means is only 1.30 (estimated) standard errors above the null difference of 0 kg/m^2.

→ Step 3 : Determine p-value

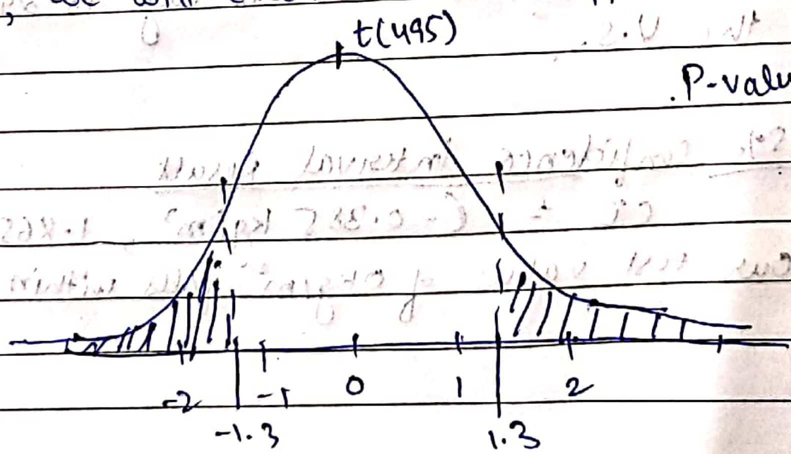
$t = 1.30$

- If the null hypothesis ($\mu_1 - \mu_2 = 0$) were true, would a test statistic value of 1.30 be unusual enough to reject the null?

- p-value : assuming the null hypothesis is true, it is the probability of observing a test statistic of 1.30 or more extreme.

- Using a $t(df)$ distribution, where, $df = n_1 + n_2 - 2$ (degree of freedom)

- Our alternative hypothesis is two-sided ($\mu_1 - \mu_2 \neq 0$) So, we will check both the upper & lower tail.



P-value : 0.1942

Date..............

p-value = 0.19

- If the difference in population mean BMI b/w males & females was really 0 kg/m², then observing a difference in sample means of 0.74 kg/m² (i.e., a t-statistic of 0.130) or more extreme is fairly likely.

- Since, our p-value is larger than the 0.05 significance level, which means there is a weak evidence against the null.

- Thus, we fail to reject the NULL!

→ Step 4: Make a Decision

Based on our estimated difference in sample means, we cannot support that there is a significante difference b/w the population mean BMI for males & the population mean BMI for females for the population of all Mexican-Americans adults (age 18-29) living in the U.S.

→ 95% Confidence Interval Result

- CI = (−0.385 kg/m², 1.865 kg/m²)
- Our test value of 0 kg/m² falls within our interval.