Fibonacci sequence

By

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Introduction:

The Fibonacci sequence is a series of numbers where a number is found by adding up the two numbers before it. Starting with 0 and 1, the sequence goes 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, **. . .** Written as a rule, the expression is $X_n = X_{n-1} + X_{n-2}$.

Explanation:

[1] Fibonacci sequence is a type of sequence, defined by recurrence relation.

RECURSION:

[2] The process of defining an object in terms of smaller versions of itself is called recursion. A recursive definition has two parts:

1. **BASE:**

An initial simple definition which cannot be expressed in terms of smaller versions of itself.

2. **RECURSION**:

The part of definition which can be expressed in terms of smaller versions of itself.

THE FIBONACCI SEQUENCE:

[3] The Fibonacci sequence, f_0, f_1, f_2 , . . . , is defined by the initial conditions $f_0 = 0$, $f_1 = 1$, and the recurrence relation

$$f_n = f_{n-1} + f_{n-2}$$

for n = 2, 3, 4, ...

$$F_2 = F_1 + F_0 = 1 + 1 = 2$$

$$F_3 = F_2 + F_1 = 2 + 1 = 3$$

$$F_4 = F_3 + F_2 = 3 + 2 = 5$$

$$F_5 = F_4 + F_3 = 5 + 3 = 8$$

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FIBONACCI RECURRENCE RELATION:

A recurrence relation for a sequence $a_0, a_1, a_2, \ldots,$ is a formula that relates each term a_k to certain of its predecessors $a_{k-1}, a_{k-2}, \ \ldots \ , a_{k-i},$

where i is a fixed integer and k is any integer greater than or equal to i. The initial conditions for such a recurrence relation specify the values of

$a_0, a_1, a_2, \ldots, a_{i-1}$.

EXAMPLE 1:

Find the first four terms of the following recursively defined sequence.

$b_1 = 2$
$b_k = b_{k-1} + 2 \cdot k$, for all integers $k \geq 2$

SOLUTION:

$b_1 = 2$ (given in base step)
$b_2 = b_1 + 2 \cdot 2 = 2 + 4 = 6$
$b_3 = b_2 + 2 \cdot 3 = 6 + 6 = 12$
$b_4 = b_3 + 2 \cdot 4 = 12 + 8 = 20$

EXAMPLE 2:
Find the first five terms of recursively defined sequence given below.
$t_0 = -1$, $t_1 = 1$
$t_k = t_{k-1} + 2 \cdot t_{k-2}$, for all integers $k \geq 2$

SOLUTION:
$t_0 = -1$, (given in base step)
$t_1 = 1$ (given in base step)
$t_2 = t_1 + 2 \cdot t_0 = 1 + 2 \cdot (-1) = 1 - 2 = -1$
$t_3 = t_2 + 2 \cdot t_1 = -1 + 2 \cdot 1 = -1 + 2 = 1$
$t_4 = t_3 + 2 \cdot t_2 = 1 + 2 \cdot (-1) = 1 - 2 = -1$

Conclusion:

Hence, the Fibonacci sequence is an integer sequence defined by a simple linear recurrence relation.

References

[1] https://en.wikipedia.org/wiki/Fibonacci_number

[2] https://brilliant.org/wiki/fibonacci-series

[3] Rosen, K. H. (n.d.). *Discrete Mathematics and Its Applications* (Seventh Edition ed.).