Solutions

Question 1

(a)

$$[\omega] = \begin{bmatrix} 0 & -5 & 2 \\ 5 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

(b)

$$det(R) = 0$$

$$\therefore R \text{ is not a valid rotation matrix}$$

(c)

$$\hat{s} = \frac{\omega}{\|\omega\|} = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$$

$$h = \frac{\omega^T v}{\|\omega\|} = 0$$

$$q = \frac{\omega \times v}{\|\omega\|^2} = \begin{bmatrix} 1/3 \\ 1/3 \\ -1/3 \end{bmatrix}$$

$$\dot{\theta} = \|\omega\| = \sqrt{6}$$

(d)

$$^b v_r =^b v_0 +^b \omega \times \vec{or} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

Question 2

(a)

$$^O T_D = \ ^O T_B \cdot \ ^B T_D$$

$$^O V_D = \ ^O \dot{T}_D \cdot \ ^D T_D^{-1} = (^O \dot{T}_B^B T_D + \ ^O T_B \cdot^B \dot{T}_D)(^B T_D^{-1} \cdot \ ^O T_B^{-1})$$

$$\because \ {}^{B}\dot{T}_{D} = 0$$

$$\therefore \ {}^{B}V_{D} = [Ad_{{}^{O}T_{B}^{-1}}] \cdot {}^{o}V_{D} = [Ad_{{}^{O}T_{B}^{-1}}]({}^{O}\dot{T}_{B} \ {}^{B}T_{D})({}^{B}T_{D}^{-1} \cdot {}^{O}T_{B}^{-1})$$

(b)

$${}^{B}F_{D} = [Ad_{({}^{B}T_{D})^{-1}}]^{T} \ {}^{D}F = \begin{bmatrix} {}^{B}P_{D} \times {}^{B}R \cdot {}^{D}f \\ {}^{B}R \cdot {}^{D}f \end{bmatrix}$$

Question 3

(a)

$${}^{s}J \in R^{6\times 2}$$

(b)

$${}^{s}J_{a} = E(\theta){}^{s}J(\theta)$$

$$Where \ E(\theta) = [-[{}^{s}O_{B}] \mid I_{3\times 3}]$$

Question 4

(a)

$${}^{0}M = \begin{bmatrix} 0 & 0 & 1 & L_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$${}^{0}\bar{S}_{1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad {}^{0}\bar{S}_{2} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ -L_1 \end{bmatrix}$$

(c)

$$^0T_2(\theta_1, \theta_2) = e^{[^0\bar{S}_1]\theta_1} e^{[^0\bar{S}_2]\theta_2} M_2, \ where \ M_2 = \begin{bmatrix} 0 & 1 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$^0J(\theta) = [^0\bar{S}_1 \mid [Ad_{\hat{T}_1}]^0\bar{S}_2 \mid [Ad_{\hat{T}_1\hat{T}_2}]^0\bar{S}_3],$$

$$where \ \hat{T}_n = e^{[^0\bar{S}_n]\theta_n}$$

$$or$$

$$^0J(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -(L_1 + L_2) \\ 0 & -L_1 & 0 \end{bmatrix}$$

Question 5

(a)

$$^bF = \begin{bmatrix} ^bn \\ ^bf \end{bmatrix}$$

$$where \ \ ^bn = \vec{bc} \ \times \ ^bf = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$so \ \ ^bF = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

(b)

$$ {}^{b}F_J = -{}^{b}F = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} $$

(c)

$$ \tau = {}^{b}S_n^T \cdot {}^{b}F_J = 2 $$

(d)

$$ {}^{b}V_rod = {}^{b}S_{rod} \cdot \dot{\theta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^{b}A_{rod} = {}^{b}\mathring{V}_{rod} + {}^{b}V_b \times {}^{b}V_{rod} = {}^{b}\dot{S}_{rod} \cdot \dot{\theta} + {}^{b}S_{rod} \cdot \ddot{\theta} + [{}^{b}V_b \times] \cdot {}^{b}V_{rod} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} $$

(e)

$$ {}^{b}V_{rod} = {}^{b}V_{rod/wall} + {}^{b}X_a \cdot V_{wall} $$

$$ {}^{b}X_a = \begin{bmatrix} {}^{b}R_a & \\ [{}^{b}P_a] \cdot {}^{b}R_a & {}^{b}R_a \end{bmatrix}, \; {}^{b}R_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}, \; {}^{b}P_a = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} $$

$$ Then,\; {}^{b}V_{rod} = \begin{bmatrix} 1 \\ \sqrt{2}/2 \\ -\sqrt{2}/2 \\ \sqrt{2}/2 - 1 \\ \sqrt{2}/2 \\ 1 - \sqrt{2}/2 \end{bmatrix} $$