**MAE5009: Continuum Mechanics B**

**Assignment 06: Fluid Statics**

**Due December 17, 2021**

1. As shown below, a cylinder of diameter 122 mm and length 200 mm is placed inside a concentric long pipe of diameter 125 mm. An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of 1 m/s? Assume that the kinematic viscosity of oil is $3 \times 10^{-5}$ $m^2/s$ and the specific gravity is 0.9.


Solution:

As we can know, the cyliner is affected by pulling force and shear stress and the force is balance.

So,

$$F = \tau \cdot A \Rightarrow \tau = \frac{F}{A} = \mu \frac{U}{h} \Rightarrow \boxed{F = \mu \frac{AU}{h}}$$

$$A = \pi d \cdot l = 0.0244\pi \, m^2$$

$$\rho = s \cdot \rho_w = 900 \, kg/m^3$$

$$\nu = 3 \times 10^{-5} \, m^2/s$$

$$\mu = \rho \cdot \nu = 2.7 \times 10^{-2} \, kg/ms$$

$$h = \frac{d_p - d_c}{2} = 1.5 \times 10^{-3} \, m$$

$$F = \mu \frac{AU}{h} = 1.38 \, N$$

2. What is the water pressure on the sea bottom at a depth of 6500 m? The specific gravity of sea water is assumed to be 1.03.

Solution:

$P_b = \rho g h + P_0$

$\rho = S \cdot \rho_w = 1030\ kg/m^3$

$g = 9.8\ m/s^2$

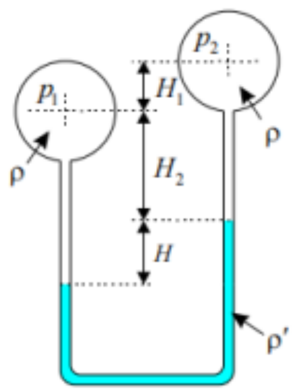
$h = 6500\ m$

$P_0 = 101325\ Pa$

$\Rightarrow$ So, $P_b = \rho g h + P_0$

$= 65.712325\ MPa$

3. Obtain the pressure difference $p_1 - p_2$:




Solution:

Let the pressure at the interface in the left be $P_0$, then:

In the left side: $P_0 = P_1 + \rho g (H_2 + H)$

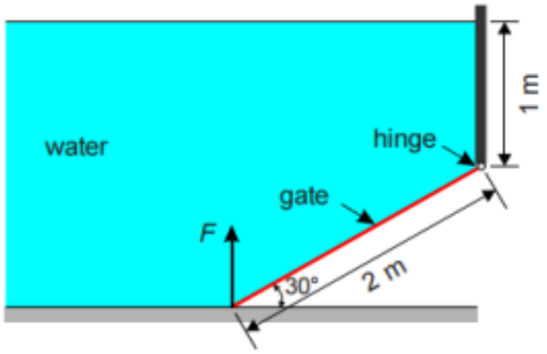
In the right side: $P_0 = P_2 + \rho g (H_1 + H_2) + \rho' g H$

So, $P_1 + \rho g (H_2 + H) = P_2 + \rho g (H_1 + H_2) + \rho' g H$

then, $P_1 - P_2 = \rho g (H_1 - H) + \rho' g H$

$= \rho g H_1 + \rho' g H - \rho g H$

4. A rectangle gate with width of 3 m is placed under the water, as shown below. The gate is hinged at the top. Determine the force $F$ needed to just lift the gate.




Solution:

Let the hinge point be origin, the Y axial be parallel with the gate and X axial on the paper.

The force of one element is: $dP = \rho g h dA = \rho g (1 + y\sin(30°)) dy \times 3m$

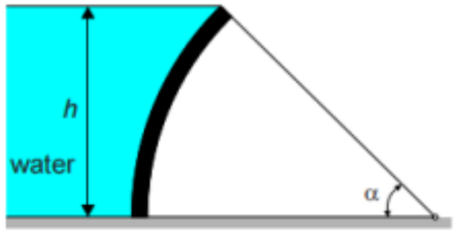
The overall force is: $P = \int_0^2 dP = 9\rho g = 88.2kN$

The force center D at: $P y_D = \int_A y dP = 10\rho g$. So, $y_D = \frac{10}{9} m$

According to the equilibrium of moment: $F\cos 30° \times 2m = P y_D$

So, $F = \frac{\sqrt{3}}{3} P y_D = 56.58 kN$

5. A circular shape water gate is shown as below, $\alpha = 45^\circ$, the water depth $h = 3.0$ m. determine the overall hydrostatic force acting on unit gate width and its direction.




Solution:

$P_x = \rho g h \frac{h}{2} \times 1m = 44100N$

The radius of this circle is $r = \frac{h}{\sin 45^\circ} = \sqrt{2}h$

So, $V_p = h(r - r\cos 45^\circ) - (\frac{1}{8}\pi r^2 - \frac{1}{2}hr\cos 45^\circ) \times 1m$

$= (\sqrt{2} - \frac{1}{2} - \frac{\pi}{4})h^2 \times 1m = 1.159 m^3$

$P_y = \sqrt{P_x^2 + P_y^2}$

The overall force: $P = \sqrt{P_x^2 + P_y^2} = 45540.03N$

The direction: $\theta = \tan^{-1}(\frac{P_y}{P_x}) = 14.45^\circ$