MAE5009: Continuum Mechanics B
Assignment 06: Fluid Statics
Due December 17, 2020

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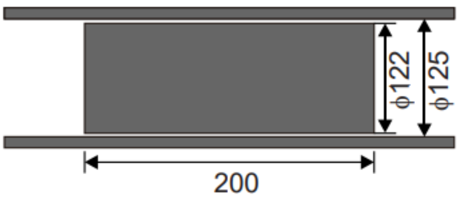
Contents

1. As shown below, a cylinder of diameter 122 mm and length 200 mm is placed inside a concentric long pipe of diameter 125 mm. An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of 1 m/s? Assume that the kinematic viscosity of oil is $3\times10^5 m^2/s$ and the specific gravity is 0.9.



Solution:

$$A = \pi dl = 0.0244\pi m^2, \rho_0 = s\rho_w = 900kg/m^3, \nu = 3\times10^5 m^2/s, \mu = \rho_0\nu = 2.7\times10^8 kg/ms$$

$$U = 1m/s, h = \frac{d_p - d_c}{2} = 1.5\times10^{-3}$$

$$\tau = \frac{F}{A} = \mu\frac{U}{h}, F = \mu\frac{AU}{h} = 1.38\times10^{10}N$$

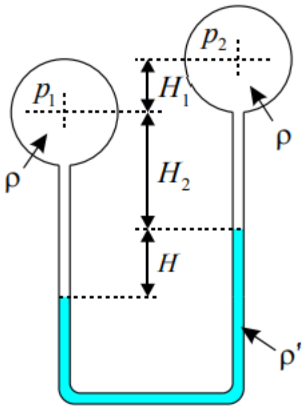
2. What is the water pressure on the sea bottom at a depth of 6500 m? The specific gravity of sea water is assumed to be 1.03.

Solution:

$$\rho = s\rho_w = 1030kg/m^3, g = 9.8m/s^2, h = 6500m, p_0 = 101325Pa$$

$$p_b = \rho gh + p_0 = 65712325Pa = 65.712325MPa$$

3. Obtain the pressure difference $P_1 - P_2$:



Solution:

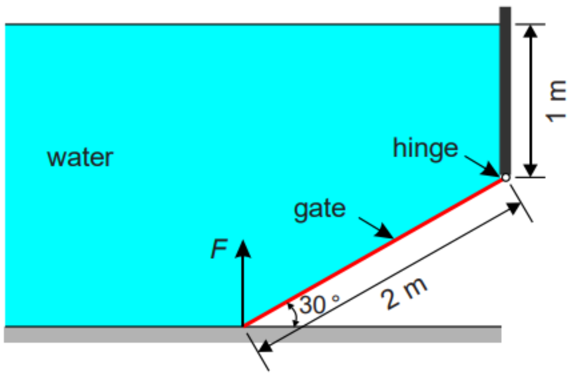
Let the pressure at the interface in the left be p_0, then:

In the left side: $p_0 = p_1 + \rho g(H_2 + H)$.

In the right side: $p_0 = p_2 + \rho g(H_1 + H_2) + \rho' gH$.

So, $p_1 + \rho g(H_2 + H) = p_2 + \rho g(H_1 + H_2) + \rho' gH$, then $p_1 - P_2 = -\rho gH + \rho gH_1 + \rho' gH$

4. A rectangle gate with width of 3 m is placed under the water, as shown below. The gate is hinged at the top. Determine the force F needed to just lift the gate.



Solution:

Let the hinge point be origin, the Y axial be parallel with the gate and X axial on the paper.

The force of one element is: $dP = \rho ghdA = \rho g(1 + ysin(30^{\circ}))dy \times 3m$.

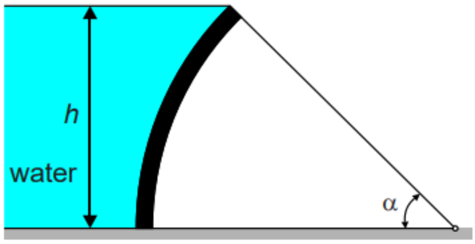
The overall force is: $P = \int_0^2 dP = 9\rho g = 88.2kN$.

The force center D at: $Py_D = \int_A y\,dP = 10\rho g$. So, $y_D = \frac{10}{9}m$

According to the equilibrium of moment: $Fcos(30^{\circ}) \times 2m = Py_D$.

So, $F = \frac{\sqrt{3}}{3}Py_D = 56.58kN$.

5. A circular shape water gate is shown as below, $\alpha = 45^{\circ}$, the water depth h = 3.0 m. determine the overall hydrostatic force acting on unit gate width and its direction.



Solution:

$$P_x = \rho g h \frac{h}{2} \times 1m = 44100N$$

The radius of this circle is $r = h/sin(45^{\circ}) = \sqrt{2}h$.
So, $V_p = (h(r - rcos(45^{\circ})) - (\frac{1}{8}\pi r^2 - \frac{1}{2}hrcos(45^{\circ}))) \times 1m = (\sqrt{2} - \frac{1}{2} - \frac{\pi}{4})h^2 \times 1m = 1.159m^3$.

$$P_y = \rho g V_p = 11361.52N$$

The overall force: $P = \sqrt{P_x^2 + P_y^2} = 45540.03N$.
The direction: $\theta = tan^{-1}(\frac{P_y}{P_x}) = 14.45^{\circ}$.