Part II

8-3

Consider a fourth-order tensor $\mathbb{A}$, with components A_{ijmn}, and a second-order tensor $\boldsymbol{B}$, with components B_{kl}:

$$C_{ijk} = w_{ij}u_k, \quad C'_{mnl} = A_{ijmn}w_{ij}B_{kl}u_k = A_{ijmn}B_{kl}w_{ij}u_k = A_{ijmn}B_{kl}C_{ijk},$$

so that the third-order tensor $C_{ijk} = w_{ij}u_k$ can be transformed into $C'_{mnl} = w'_{mn}u'_l$ through sixth-order tensor $A_{ijmn}B_{kl}$.

8-4

$$\begin{aligned}
(AB)_{jl} &= A_{jk}B_{kl} \\
(AB)_{jl,i} &= A_{jk,i}B_{kl} + A_{jk}B_{kl,i} \\
(AB)_{jl,ii} = (A_{jk,i}B_{kl} + A_{jk}B_{kl,i})_{,i} &= A_{jk,ii}B_{kl} + A_{jk,i}B_{kl,i} + A_{jk,i}B_{kl,i} + A_{jk}B_{kl,ii} \\
(AB)_{jl,ii} &= B_{kl}A_{jk,ii} + 2A_{jk,i}B_{kl,i} + A_{jk}B_{kl,ii} \\
(AB)_{,ii} &= AB_{,ii} + 2A_{,i}B_{,i} + BA_{,ii}.
\end{aligned}$$

8-11

Part I

$$\begin{aligned}
\int_V \phi_{,i}dV &= \int_S \mu_i\phi dS, \\
\int_V U_{i,i}dV &= \int_S \mu_iU_idS, \\
\int_V \varepsilon_{ijk}U_{j,i} &= \int_S \varepsilon_{ijk}\mu_jU_idS, \\
\int_V \mu_k\varepsilon_{ijk}U_{j,i} &= \int_S U_idL_i, \\
\int_V \phi_{,ii}dV &= \int_S \mu_i\phi_{,i}dS.
\end{aligned}$$

Part II

(a)

$$\begin{aligned}
\int_S \boldsymbol{E}\cdot d\boldsymbol{S} &= \frac{Q}{\varepsilon_0} \\
\int_V \nabla\cdot\boldsymbol{E}dV &= \int_V \frac{\rho}{\varepsilon_0}dV \\
\nabla\cdot\boldsymbol{E} &= \frac{\rho}{\varepsilon_0} \\
E_{i,i} &= \frac{\rho}{\varepsilon_0}
\end{aligned}$$

(b)

$$\begin{aligned}
\int_S \boldsymbol{B}\cdot d\boldsymbol{S} &= 0 \\
\int_V \nabla\cdot\boldsymbol{B}dV &= 0 \\
\nabla\cdot\boldsymbol{B} &= 0 \\
B_{i,i} &= 0
\end{aligned}$$

(c)

$$\int_L \boldsymbol{E}\cdot d\boldsymbol{l} = -\int_S \dot{\boldsymbol{B}}\cdot d\boldsymbol{S}$$
$$\int_S \boldsymbol{\mu}\cdot\nabla\times\boldsymbol{E}dS = -\int_S \dot{\boldsymbol{B}}\cdot d\boldsymbol{S}$$
$$\int_S \nabla\times\boldsymbol{E}\cdot d\boldsymbol{S} = -\int_S \dot{\boldsymbol{B}}\cdot d\boldsymbol{S}$$
$$\nabla\times\boldsymbol{E} = \dot{\boldsymbol{B}}$$
$$\varepsilon_{ijk}E_{j,i} = \dot{B}_k$$

(d)

$$\int_L \boldsymbol{B}\cdot d\boldsymbol{l} = -\int_S \mu_0(\boldsymbol{j}+\varepsilon_0\dot{\boldsymbol{E}})\cdot d\boldsymbol{S}$$
$$\int_S \boldsymbol{\mu}\cdot\nabla\times\boldsymbol{B}dS = -\int_S \mu_0(\boldsymbol{j}+\varepsilon_0\dot{\boldsymbol{E}})\cdot d\boldsymbol{S}$$
$$\int_S \nabla\times\boldsymbol{B}\cdot d\boldsymbol{S} = -\int_S \mu_0(\boldsymbol{j}+\varepsilon_0\dot{\boldsymbol{E}})\cdot d\boldsymbol{S}$$
$$\nabla\times\boldsymbol{B} = \mu_0(\boldsymbol{j}+\varepsilon_0\dot{\boldsymbol{E}})$$
$$\varepsilon_{ijk}B_{j,i} = \mu_0(j_k+\varepsilon_0\dot{E}_k)$$

8-2

Consider a second-order transformation tensor $\boldsymbol{a}$, with components a_{ij}:

$$A_{ijk} = u_i v_j w_k, \quad A'_{mnl} = a_{im}u_i a_{jn}v_j a_{kl}w_k = a_{im}a_{jn}a_{kl}u_i v_j w_k.$$

so that the third-order tensor A_{ijk} can be transformed into A'_{mnl} through sixth-order tensor $a_{im}a_{jn}a_{kl}$.

8-12

Consider the transformation between u_i and u'_j is

$$u_i = b_{ij}u'_j,$$

and the transformation between u'_j and u''_k is

$$u'_j = c_{jk}u''_k,$$

so that

$$u_i = b_{ij}u'_j = b_{ij}c_{jk}u''_k = a_{ik}u''_k, \quad a_{ik} = b_{ij}c_{jk}.$$

The value of u_i can be obtained by u''_k through a_{ik}, the same as the transformation through $b_{ij}c_{jk}$.

Question 5

$$\boldsymbol{AB} = A_{ij}B_{jk} = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix}$$
$$\boldsymbol{AB}^{\mathrm{T}} = A_{ij}B_{kj} = \begin{bmatrix} 46 & 28 & 10 \\ 118 & 73 & 28 \\ 190 & 118 & 46 \end{bmatrix}$$
$$\boldsymbol{A}:\boldsymbol{B}^{\mathrm{T}} = A_{ij}B_{ji} = \sum_{i,j=1}^{3} A_{ij}B_{ji} = 189$$

$$\boldsymbol{A} : \boldsymbol{B} = A_{ij}B_{ij} = \sum_{i,j=1}^{3} A_{ij}B_{ij} = 165$$

Question 6

$$\varepsilon_{ijk}\varepsilon_{klm} = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}), \quad \varepsilon_{ijk} = \varepsilon_{kij}.$$

Substitute $l = i$ and $m = j$, and all m and l are supposed to be converted into i and j, then

$$\varepsilon_{ijk}\varepsilon_{kij} = \varepsilon_{ijk}\varepsilon_{ijk} = \delta_{ii}\delta_{jj} - \delta_{ij}\delta_{ij} = 9 - 3 = 6.$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \varepsilon_{ijk}a_j(\varepsilon_{klm}b_lc_m) = \varepsilon_{ijk}\varepsilon_{klm}a_jb_lc_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})a_jb_lc_m = a_mb_ic_m - a_lb_lc_i = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot$$