南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

MAE5009
Continuum Mechanics B
Session 11: Fluid Dynamics Basics

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Introduction

- **Fluid kinematics**:
  - From geometrical viewpoint to describe fluid motion
  - Fluid field, velocity, acceleration, pathline, streamlines
  - Continuity equation – conservation of mass
- **Fluid dynamics**:
  - Based on Newton's second law
  - Euler's equation
  - Bernoulli's equation
  - Navier-Stokes equation

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Descriptions of fluid motion

- Fluid field:
  - Continuum that consists of infinite number of fluid particles
  - Fluid motion concerns with the motion of fluid particles in the whole fluid field
- Lagrangian description:
  - Focuses on describing the motion of each single particle

Coordinates:
$$\begin{cases} x = x(a,b,c;t) \\ y = y(a,b,c;t) \\ z = z(a,b,c;t) \end{cases}$$

- (a,b,c) are the initial coordinates of a specific fluid particle
- For a specific fluid particle, its position (x,y,z) is a function of only t

Velocity:
$$u_x = \frac{\partial x}{\partial t}, u_y = \frac{\partial y}{\partial t}, u_z = \frac{\partial z}{\partial t}$$

Acceleration:
$$a_x = \frac{\partial u_x}{\partial t}, a_y = \frac{\partial u_y}{\partial t}, a_z = \frac{\partial u_z}{\partial t}$$

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Descriptions of fluid motion

- Eulerian description:
  - Focuses on fluid flow at a general point (*x*,*y*,*z*) in the fluid field
  - Does not care much about which fluid particles cause the motion at the point in the fluid field
  - The velocity, acceleration, pressure and density fields are functions of four variables (*x*,*y*,*z*,*t*)

$$\mathbf{u}=\mathbf{u}(x,y,z,t) \qquad p=p(x,y,z,t)$$

$$\mathbf{a}=\mathbf{a}(x,y,z,t) \qquad \rho=\rho(x,y,z,t)$$

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Acceleration of fluid particles using Eulerian description

- Velocity of fluid field is a function of four variables (*x*,*y*,*z*,*t*)

$$\mathbf{u}=u_x(x,y,z,t)\mathbf{i}+u_y(x,y,z,t)\mathbf{j}+u_z(x,y,z,t)\mathbf{k}$$

Velocity field of a fluid:
- At a specific time, velocity changes with respect to space coordinates
- At a specific position, velocity changes with respect to time

- Fluid particle velocity is a function of four variables (*x*,*y*,*z*,*t*), and (*x*,*y*,*z*) are functions of *t*, then the acceleration of fluid particle is

$$a_x=\frac{du_x(x(t),y(t),z(t),t)}{dt}=\frac{\partial u_x}{\partial t}+\frac{\partial u_x}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial u_x}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial u_x}{\partial z}\frac{\partial z}{\partial t}$$

$$=\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}$$

$$a_y=\frac{du_y}{dt}=\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}$$

$$a_z=\frac{du_z}{dt}=\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}$$

$$\rightarrow \quad \mathbf{a}=\frac{d\mathbf{u}}{dt}=\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}$$

$$a_i=u_{i,t}+u_j u_{i,j}$$

where

$$\mathbf{u}\cdot\nabla=u_x\frac{\partial}{\partial x}+u_y\frac{\partial}{\partial y}+u_z\frac{\partial}{\partial z}$$

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Acceleration of fluid particles using Eulerian description

- Acceleration of fluid particles consists of two parts:

$$\mathbf{a}=\frac{d\mathbf{u}}{dt}=\frac{D\mathbf{u}}{Dt}=\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}$$

$\frac{\partial \mathbf{u}}{\partial t}$ local acceleration, time-induced particle velocity changing ratio at a specific point

$(\mathbf{u}\cdot\nabla)\mathbf{u}$ convective acceleration, location-induced particle velocity changing ratio

particle acceleration = local acceleration + convective acceleration

$\frac{D}{Dt}=\frac{\partial}{\partial t}+u_x\frac{\partial}{\partial x}+u_y\frac{\partial}{\partial y}+u_z\frac{\partial}{\partial z}$ material derivative

$\frac{\partial}{\partial t}$ local derivative $\quad (\mathbf{u}\cdot\nabla)$ convective derivative

particle derivative = local derivative + convective derivative

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Example

- Calculate the acceleration field of the following velocity field:

$$u_x = 2x, u_y = -2y, u_z = 0$$

$$\downarrow$$

$$a_x = \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z}$$
$$= 0 + (2x) \times 2 + (-2y) \times 0 + 0 \times 0 = 4x$$

$$a_y = \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z}$$
$$= 0 + (2x) \times 0 + (-2y) \times (-2) + 0 \times 0 = 4y$$

$$a_z = \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z}$$
$$= 0 + (2x) \times 0 + (-2y) \times 0 + 0 \times 0 = 0$$

$$\rightarrow \quad \mathbf{a} = 4x\mathbf{i} + 4y\mathbf{j}$$

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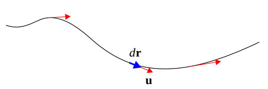
Fluid motion

- Path line:
  - The movement path of a fluid particle
  - Provides the history of the particle
  - Corresponding to Lagrangian approach
- Streamline:
  - A line in a flow to which all velocity vectors are tangent at a given time
  - Imaginary curve
  - Corresponding to Eulerian approach



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Streamline

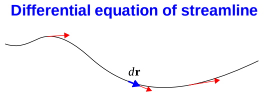
$$d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$$

$$d\mathbf{r} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ dx & dy & dz \\ u_x & u_y & u_z \end{vmatrix} = 0 \quad \rightarrow \quad \begin{cases} u_y dx - u_x dy = 0 \\ u_z dy - u_y dz = 0 \\ u_z dx - u_x dz = 0 \end{cases}$$

$$\downarrow$$

$$\frac{dx}{u_x(x,y,z,t)} = \frac{dy}{u_y(x,y,z,t)} = \frac{dz}{u_z(x,y,z,t)}$$

Differential equation of streamline



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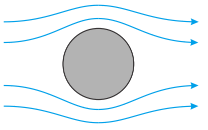
Streamline

- Streamline is about a specific time
- Path line is about a specific particle
- Streamline and path line are not necessarily overlapping
- Generally, streamlines will not intersect



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Example

- Determine the streamlines of the following velocity field:

$$u_x = -\frac{ay}{x^2+y^2},\ u_y = \frac{ax}{x^2+y^2},\ u_z = 0,\ a > 0$$

$$\frac{dx}{u_x} = \frac{dy}{u_y} \qquad \frac{dx}{-ay} = \frac{dy}{ax}$$

$$xdx + ydy = 0$$

Integrate WRT *x* & *y* $\quad x^2 + y^2 = C$

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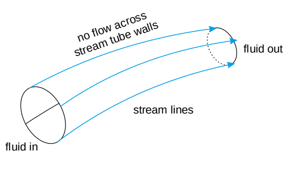
Stream tube

- By taking a given closed curve in a flow and drawing the streamlines passing all points on the curve, a tube can be formulated. This tube is called a stream tube
- Since fluid velocities are always parallel to the streamlines, fluid cannot flow in and out through the sides of stream tube



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Flow rate

- The amount of fluid that flows through a specific surface during a unit time span

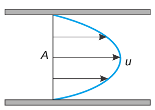
$$Q = \int_A u dA$$

A is the surface that is perpendicular to all streamlines, effective cross-section

generally,

$$Q = \int_A u_n dA = \int_A (\mathbf{u} \cdot \mathbf{n}) dA = \int_A u \cos(\mathbf{u}, \mathbf{n}) dA$$

n is the outer normal of the surface



Average velocity

$$\bar{u} = \frac{Q}{A}$$

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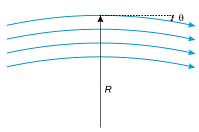
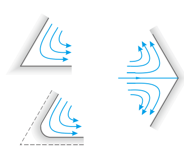
Gradually varied flow & rapidly varied flow

- **Gradually varied flow**:
  - θ is very small, or *R* is very large
  - The effective cross-section is nearly planar
  - The pressure distribution on the effective cross-section approximately follows that in fluid statics

$$z_1 + \frac{p_1}{\rho g} \approx z_2 + \frac{p_2}{\rho g}$$



gradually varied flow



rapidly varied flow

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Incompressible and compressible flows

- **Incompressible flow**:
  - The density of a fluid particle as it moves along is assumed to be constant

$$\frac{D\rho}{Dt} = 0 \quad \rightarrow \quad \frac{\partial \rho}{\partial t} = 0, \frac{\partial \rho}{\partial x} = 0, \frac{\partial \rho}{\partial y} = 0, \frac{\partial \rho}{\partial z} = 0$$

  - Liquid flows are assumed to be incompressible in most situations
  - Examples of incompressible airflows: air flow in conduits, around automobiles and small aircraft, and the takeoff and landing of commercial aircraft
- The Mach number M_a is used to determine if an air flow is compressible

$$M_a = \frac{u}{c}$$

u is the characteristic velocity and *c* is the speed of sound

If $M_a < 0.3$, we often assume the air flow to be incompressible

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Steady flow and unsteady flow

- **Steady flow**:
  - A flow whose flow state expressed by velocity, pressure, density, etc., at any position, does not change with time

$$\frac{\partial}{\partial t}=0$$

- **Unsteady flow**:
  - A flow whose flow state changes with time
  - Very slow unsteady flow can be approximated as steady flow

Steady flow can significantly reduce the complexity of differential equations of fluid flow

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One-, two- and three-dimensional flow

- **One-dimensional flow**:
  - All flow parameters depends on coordinates *x* and *t*

$$u=u(x,t)$$

  - Flow in a tube in terms of average velocity
  - Flow along streamlines
- **Two-dimensional flow**:
  - All flow parameters depends on coordinates *x, y* and *t*

$$u=u(x,y,t)$$

  - Flow between two parallel plates, if the flow states are the same on all planes parallel to the vertical cross-cut plane
- **Three-dimensional flow**:
  - All flow parameters depends on coordinates *x, y, z* and *t*

$$u=u(x,y,z,t)$$

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Gravitational and non-gravitational flow

- **Gravitational flow**:
  - If the gravity is considered in the fluid
  - Fluid flow with low velocity in which gravity is the main effect

- **Non-gravitational flow**:
  - Flow of gas, the gravity can be neglected

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Viscous and inviscid flow

- **Viscous flow**:
  - All fluids have viscosity and if the viscous effects cannot be neglected, it is a viscous flow
  - If velocity is very small, the viscosity-induced shear stress could be neglected, then the flow can be treated as **inviscid flow**
- **Inviscid flow (Ideal flow)**
  - No viscous force

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Spinning and non-spinning flow

- Spinning flow:
  - The fluid particles in the fluid field have spinning motion
  - i.e. $\boldsymbol{\omega} \neq 0$

$$\boldsymbol{\omega}(x,y,z,t)=\frac{1}{2}\nabla\times\mathbf{u}=\frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u_x & u_y & u_z\end{vmatrix}=\omega_x\mathbf{i}+\omega_y\mathbf{j}+\omega_z\mathbf{k}$$

where

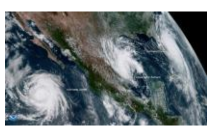
$$\omega_x=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right)$$

$$\omega_y=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right)$$

$$\omega_z=\frac{1}{2}\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)$$

$$\boldsymbol{\Omega}(x,y,z,t)=\nabla\times\mathbf{u}=2\boldsymbol{\omega}$$ vorticity





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Example

- Determine if the following velocity field gives a spinning flow:

$$u_x=-\frac{ay}{x^2+y^2},\ u_y=\frac{ax}{x^2+y^2},\ u_z=0,\ a>0$$

Streamline $\frac{dx}{u_x}=\frac{dy}{u_y} \rightarrow xdx+ydy=0 \rightarrow x^2+y^2=C$

Angular velocity

$$\omega_x=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right)=0$$

$$\omega_y=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right)=0$$

$$\omega_z=\frac{1}{2}\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)=\frac{a}{2}\left(\frac{y^2-x^2}{\left(x^2+y^2\right)^2}-\frac{y^2-x^2}{\left(x^2+y^2\right)^2}\right)=0$$

Although the fluid flows around a circle, it is not a spinning flow

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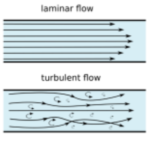
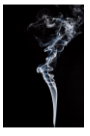
Laminar and turbulent flows

- A **viscous flow** is either a laminar flow or a turbulent flow
- **Turbulent flow**:
  - Mixing of fluid particles so that the motion of a given particle is random and highly irregular
  - Statistical averages are used to specify the velocity, the pressure, and other quantities
- **Laminar flow**:
  - There is negligible mixing of fluid particles
  - The motion is smooth and noiseless





turbulent flow

laminar flow

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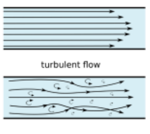
Laminar and turbulent flows

- Reynolds number:

$$Re = \frac{\rho u d}{\mu} = \frac{u d}{\nu}$$



turbulent flow

laminar flow

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Subsonic, transonic and supersonic flow

- Mach number is a measure of the relative speed

$$M_a = \frac{u}{c}$$ u is the characteristic velocity and c is the speed of sound

- $M_a < 1$: subsonic flow
- $M_a \approx 1$: transonic flow
- $1 < M_a \leq 5$: supersonic flow

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External and internal flow

- **External flow**:
  - Flow of a fluid over an object, such as in aerodynamics
- **Internal flow**:
  - Flow in pipe and channel, etc., where the fluid flows within a confining structure
  - Viscosity and the viscous shear stress with respect to the walls cannot be ignored

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Classification of fluid flow

Flow type		Based on
Compressible	Incompressible	Compressibility
Viscous	Inviscid	Viscosity
Unsteady	Steady	Time
Spinning	Non-spinning	Spinning & rotation
Turbulent	Laminar	Flow state
Gravitational	Non-gravitational	Gravity
3d, 2d, 1d		Dimension
Subsonic, transonic, supersonic		Velocity
External	Internal	Position

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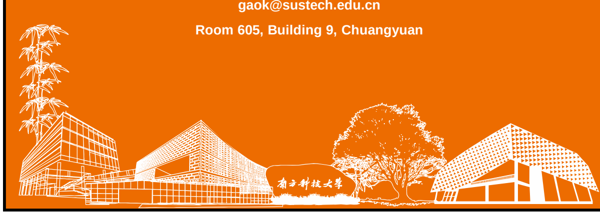

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