Homework 6

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1. $\Delta^2\phi = 24A+8B+24C=0$, that's $3A+B+3C=0$.

$\sigma_{xx} = \frac{\partial^2\phi}{\partial y^2} = 2Bx^2+12Cy^2$ $\quad \sigma_{yy} = \frac{\partial^2\phi}{\partial x^2} = 12Ax^2+2By^2$ $\quad \sigma_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = -4Bxy$

2. $\begin{cases} \frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + 0 = ky - ky + 0 = 0 \\ \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + 0 = 0+0+0=0 \end{cases}$

so, they satisfy the plane strain stress fomulation relations.

3. $\sigma_{xx} = \frac{\partial^2\phi}{\partial y^2} = 6Ay$, $\sigma_{yy} = \frac{\partial^2\phi}{\partial x^2} = 0$, $-\tau_{xy} = \frac{\partial^2\phi}{\partial x\partial y} = 0$.

So, it satisfies $\sigma_{yy}(x,\pm c)=0$, $\tau_{xy}(x,\pm c)=0$, $\tau_{xy}(\pm l,y)=0$,

$\int_{-c}^{c}\sigma_{xx}(\pm l,y)dy = \int_{-c}^{c} 6Ay\,dy = 0$, $\int_{-c}^{c}\sigma_{xx}(\pm l,y)y\,dy = \int_{-c}^{c} 6Ay^2dy = 4AC^3 = -M$.

So, $A = \frac{-M}{4C^3}$.

$$\begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\bar{v} & \\ -\bar{v} & 1 & \\ & & 2(1+\bar{v}) \end{bmatrix}\begin{bmatrix} 6Ay \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{6A}{E}y \\ \frac{-6A\bar{v}}{E}y \\ 0 \end{bmatrix}$$

Due to $e_{xx} = \frac{\partial u_x}{\partial x}$, $e_{yy} = \frac{\partial u_y}{\partial y}$, $e_{xy} = \frac{1}{2}\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right]$.

So, $u_x = \frac{6A}{E}xy + f(y) = -\frac{3M}{2EC^3}xy + f(y)$

$u_y = \frac{-3A\bar{v}}{E}y^2 + g(x) = \frac{3M\bar{v}}{4EC^3}y^2 + g(x)$

$\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = -\frac{3M}{2EC^3}x + f'(y) + g'(x) = 0$.

$-\frac{3M}{2EC^3}x + g'(x) = -f'(y) = w_0$, So $f(y) = -w_0y + u_0$.

$g'(x) = \frac{3M}{2EC^3}x - f'(y)$, So $g(x) = \frac{3M}{4EC^3}x^2 + w_0x + v_0$

So, $u_x = \frac{-3M}{2EC^3}xy - w_0y + u_0$, $u_y = \frac{3M\bar{v}}{4EC^3}y^2 + \frac{3M}{4EC^3}x^2 + w_0x + v_0$.

$u_x(-l,0) = u_0 = 0$, $u_y(l,0) = \frac{3M}{4EC^3}l^2 + w_0l + v_0 = 0$

$u_y(-l,0) = \frac{3M}{4EC^3}l^2 - w_0l + v_0 = 0$, $\Rightarrow$ $w_0 = 0$, $v_0 = \frac{-3M}{4EC^3}l^2$.

So, $u_x = \frac{-3M}{2EC^3}xy$, $u_y = \frac{3M\bar{v}}{4EC^3}y^2 + \frac{3M}{4EC^3}x^2 - \frac{3M}{4EC^3}l^2$.

4. In the plane stress case, when $\nu = 0.4$, $y=0$, $u_y = \frac{3M}{4Ec^3}(x^2 - l^2)$

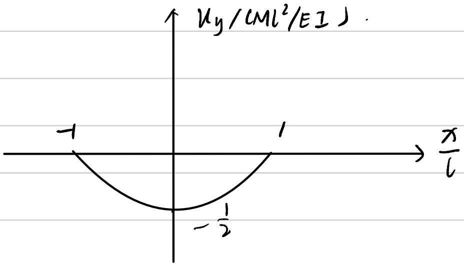
$u_y/(Ml^2/EI) = \frac{1}{2}[(\frac{x}{l})^2 - 1]$

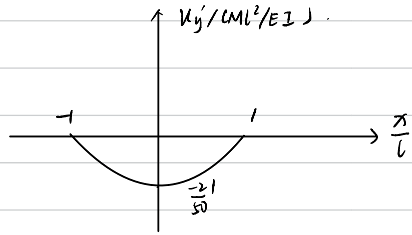
In the plane strain setting: $\bar{E} = \frac{E}{1-\nu^2}$, $\bar{\nu} = \frac{\nu}{1-\nu}$.

So $u_x' = \frac{-3(1-\nu^2)M}{2Ec^3} xy$, $u_y' = \frac{3(1-\nu^2)M}{4Ec^3}(\frac{\nu}{1-\nu}y^2 + x^2 - l^2)$

when $\nu = 0.4$, $y=0$, $u_y' = \frac{63M}{100Ec^3}(x^2 - l^2)$

$u_y'/(Ml^2/EI) = \frac{21}{50}[(\frac{x}{l})^2 - 1]$.





5. $e_{rr} = \frac{\partial u_r}{\partial r} = -\frac{A}{r^2}$, $e_{\theta\theta} = \frac{1}{r}(u_r + \frac{\partial u_\theta}{\partial \theta}) = \frac{1}{r}(\frac{A}{r} - B\sin\theta)$

$e_{r\theta} = \frac{1}{2}(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}) = \frac{1}{2}(0 + 0 - \frac{B\cos\theta}{r}) = \frac{B\cos\theta}{2r}$.

6. $\sigma_{rr} = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$, $\sigma_{\theta\theta} = \frac{\partial^2 \phi}{\partial r^2} = 0$, $\sigma_{r\theta} = -\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial \phi}{\partial \theta}) = \tau \cdot \frac{r_2^2}{r^2}$.

$e_{rr} = \frac{1}{E}(\sigma_{rr} - \nu\sigma_{\theta\theta}) = 0$, $e_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu\sigma_{rr}) = 0$, $e_{r\theta} = \frac{1+\nu}{E}\sigma_{r\theta} = \frac{1+\nu}{E}\cdot\tau\cdot\frac{r_2^2}{r^2}$

$u_r = 0$, $u_\theta = \frac{1+\nu}{E}\tau r_2^2(\frac{r}{r_1^2} - \frac{1}{r})$.