MAE5009: Continuum Mechanics B

Assignment 04: Formulation of Problems in Elasticity

Due November 2, 2021

1. Verify the following equations for plane strain problems with constant f_x and f_y:

   (a) $\frac{\partial}{\partial y}\nabla^2 u = \frac{\partial}{\partial x}\nabla^2 v$

   (b) $\frac{\partial}{\partial x}\nabla^2 u + \frac{\partial}{\partial y}\nabla^2 v = 0$

(c) $\nabla^4 u = \nabla^4 v = 0$, where $\nabla^4 = \nabla^2\left(\nabla^2\right)$.

2. A bar of constant mass density ρ hangs under its own weight and is supported by the uniform stress σ_0 as shown in the figure. Assume that the stresses σ_x, σ_y, τ_{xy}, τ_{xz} and τ_{yz} are all zero,

   (a) based on the above assumption, reduce 15 governing equations to seven equations in terms of σ_z, ε_x, ε_y, ε_z, u, v and w

(b) integrate the equilibrium equation to show that

$\sigma_z = \rho g z$

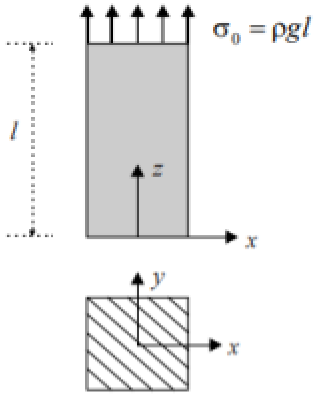
where g is the acceleration due to gravity. Also show that the prescribed boundary conditions are satisfied by this solution

(c) find ε_x, ε_y, ε_z from the generalized Hooke's law

(d) if the displacement and rotation components are zero at the point $(0,0,l)$, determine the displacement component u and v

(e) prove that

$$w = \frac{\rho g}{2E}\left(z^2 + \nu x^2 + \nu y^2 - l^2\right)$$



3. Express the boundary conditions for the following plate subjected to plate strain condition. The surface forces are functions of x and y only.