1. ① displacement is the change of coordinate that original point on a cube to the final place. Units: cm, mm.

② Stress is a inner force ~~that~~ of force produce on the unit surface. Unit: Mpa, pa

③ strain is the ~~unit~~ change of stress on the unit length, it reflect the change's of length. ~~Unit: None.~~ Have no unit.
regree.

④ normal stress is the stress produce ~~ver~~ vertial stress on ~~the~~ the a surface ~~cube~~. Unit: Mpa, pa

⑤ shear stress is the stress produce ~~parallel~~ parallel stress on the surface; it can ~~make~~ cause angel's change. Unit: pa, Mpa.

⑥ normal strain is the unit length change of the direction of normal stress.

⑦ shear strain is the unit length change of the direction of shear stress.

⑧ principal stress: ~~N~~ when shear stress vanish, the normal stress is ~~the~~ principal stress. Unit: pa. Mpa

⑨ ~~shear~~ principal strain: when shear strain vanish, the normal strain is principal strain.

2. (1) Equilibrium equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \quad ①$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y = 0 \quad ②$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \quad ③$$

physical meaning: The stress components satisfiedy the equilibrium of force on x, y, z, axes.

(2) In the absence of body forces, combine the conditions of question, the left of equation ① of (1) is

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = (6x+4y) + (-6x-4y) + 0 = 0$$

similarly, the left of ②:

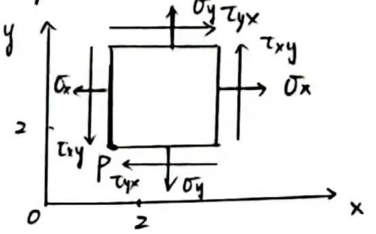
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = (-x-6y) + (x+6y) + 0 = 0$$

the left of ③:

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

To sum up, the stress components are satisfy the equilibrium equation, so the equilibrium exists.

(3) Based on the stress distribution, ~~when~~ the stress at point $P(2,2)$ can be calculate below and the stress components on an infinitesimal square can be draw below:



$\sigma_x|_{(2,2)} = -4$ Mpa.

$\sigma_y|_{(2,2)} = 24$ Mpa

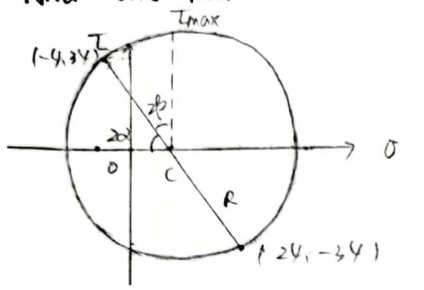
$\tau_{xy}|_{(2,2)} = \tau_{yx}|_{(2,2)} = -34$

so the stress matrix is:

$$\vec{\sigma} = \begin{pmatrix} -4 & -34 \\ -34 & 24 \end{pmatrix}$$

Basied on stress matrix, we can obtain two points $(-4, 34)$ and $(24, -34)$ on the Mohr's circle.

And the Mohr's circle can be draw below:



Center point: $C\left(\frac{\sigma_x+\sigma_y}{2}, 0\right) \Rightarrow (10, 0)$

radius: $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 37.6$

Thus the principal stress is

$\sigma_1 = x_c + R = 47.6$ Mpa

$\sigma_2 = -(R - x_c) = -27.6$ Mpa

~~And the~~ $\cancel{2\alpha = \arccos\left(\frac{R}{|\sigma_x|+x_c}\right) = \arccos\left(\frac{37.6}{14}\right)}$

And $2\alpha = \arccos\left(\frac{|\sigma_x|+x_c}{R}\right) = \arccos\left(\frac{14}{37.6}\right) = \arccos(0.3723)$

$\therefore \alpha = \frac{1}{2}\arccos(0.3723)$

α is the angle from ~~infinite~~ oraginal to ~~final~~ prıcipal stresses's direction.

(4) Basied on the Mohr's circle of (3)

$\tau_{max} = R = 37.6$ Mpa

At this situation, normal stress can be catulate below:

$\sigma_x' = \sigma_y' = \frac{\sigma_x+\sigma_y}{2} = 10$ Mpa.

And the direction of maximum shear sthress is

$2\beta = \cancel{\arccos} -\arcsin\left(\frac{|\sigma_x|+x_c}{R}\right) = -\arcsin(0.3723)$

$\therefore \beta = -\frac{1}{2}\arcsin(0.3723)$

β is the angle between maximum shear stress and original stress.

3. Basied on the conditions and strain-displacement equations.

$\varepsilon_x = \frac{\partial u}{\partial x} = 2k$ $\qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2ky + 2kx$

$\varepsilon_y = \frac{\partial v}{\partial y} = -6y$ $\qquad \gamma_{yz} = 0$

$\varepsilon_z = \frac{\partial w}{\partial z} = 0$ $\qquad \gamma_{zx} = 0$

The compatibility equation for strain is ~~belo~~ following:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

Subtitute ε_x, ε_y, γ_{xy} into ~~the~~ compatibility equation's ~~left and right~~ each item.

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} = 0 \qquad \frac{\partial^2 \varepsilon_y}{\partial x^2} = 0 \qquad \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0$$

so $\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$, the strain components satisfy the compatibility equation. for strain.

4. Generalised Hooke's law ~~in~~

stress- strain relations:

$$\begin{cases} \sigma_x = 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\ \sigma_y = 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\ \sigma_z = 2G\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) \\ \tau_{xy} = G\gamma_{xy} \\ \tau_{yz} = G\gamma_{yz} \\ \tau_{zx} = G\gamma_{zx} \end{cases}$$

strain-stress relations:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z))$$
$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y))$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\gamma_{zx} = \frac{1}{G}\tau_{zx}$$

shear modulus: $G = \frac{E}{2(1+\nu)}$ unit: Mpa. pa.

Lame constant: $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$ unit: Mpa. pa

Bulk modulus: $k = \frac{E}{3(1-2\nu)}$ unit: Mpa. pa.

Because $E = \frac{\sigma}{\varepsilon}$, so the unit of E is similar with σ

and $\nu = -\frac{\varepsilon_{tran s}}{\varepsilon_{axis}}$, so the unit of ν is ~~similar with~~ none.

5. (1) Using stress formulation steps:

First, use stress-strain relations to express stress by strain

$$\left.\begin{aligned} \sigma_x &= 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) & \tau_{xy} &= G\gamma_{xy} \\ \sigma_y &= 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) & \tau_{yz} &= G\gamma_{yz} \\ \sigma_z &= 2G\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) & \tau_{zx} &= G\gamma_{zx} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \varepsilon_x &= \cdots \\ \varepsilon_y &= \cdots \\ \varepsilon_z &= \cdots \end{aligned}\right.$$

Second, substitude stress-strain relation to compatibility equations:

$$\cancel{\frac{\partial \sigma_x}{}}\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$\vdots$

to express τ_{xy}, τ_{yz}, τ_{zx} by σ_x, σ_y, σ_z

Third, substitude τ_{xy}, τ_{yz}, τ_{zx} expressed by σ_x, σ_y, σ_z into equilibrium equation:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0$$

And lastly, we can obtain the equation expressed by stress formulation.

(2) Using the displacement formulation:

~~First, use strain-stress relation to express strain by stress.~~

~~$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_y))$~~

$\vdots$

~~Second, substitudte strain compoments by stress compoments in compatiblit~~.

First, use strain-displacement to express strain by displacement

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_y = \frac{\partial v}{\partial y},\ \varepsilon_z = \frac{\partial w}{\partial z},\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{cases}$$

Second, use stress-strain relation and substitute strain by displacement.

Third, substitude σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx} by displacement from second step, into equilibrium equation.

Lastly, we can obtain the equation expressed by displacement formulation.

6. Based on the conditions of question and the stress-strain relation, σ_z, ε_x, ε_y can be calculated below:

~~$\sigma_z = 2G\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) = \lambda(\varepsilon_x + \varepsilon_y) = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y)$~~

$$\sigma_z = \nu(\sigma_x + \sigma_y) = 0.3 \times (35 + 25) = 18\ N/mm^2$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = 1.48 \times 10^{-4}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = 4.33 \times 10^{[illegible]}$$

7. Basied on the conditions of question

$$\varepsilon = -3.6 \times 10^{-5}, \quad P = 0.45 \ N/mm^2, \quad \nu = 0.3$$

The Bulk modulus $k = \frac{-P}{\varepsilon} = \frac{E}{3(1-2\nu)}$

so $E = \frac{-P}{\varepsilon} \cdot 3(1-2\nu) = 1.5 \times 10^4 \ N/mm^2$