MAE5009 · Continuum Mechanics B

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Homework 3

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3-2

According to the conditions of the question,$\sigma_y = \sigma_z = 0$, we can get:

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\cdot 0 + \alpha T = \frac{\sigma_x}{E} + \alpha T$$

$$\varepsilon_y = \frac{1}{E}\cdot 0 - \frac{\nu}{E}\sigma_x + \alpha T = -\frac{\nu\sigma_x}{E} + \alpha T$$

$$\varepsilon_z = \frac{1}{E}\cdot 0 - \frac{\nu}{E}\sigma_x + \alpha T = -\frac{\nu\sigma_x}{E} + \alpha T$$

Due to :

$$\varepsilon_x = 0$$

Therefore:

$$0 = \frac{\sigma_x}{E} + \alpha T$$

$$\sigma_x = -E\alpha T$$

Substituting $\sigma_x = -E\alpha T$ into the expressions for ε_y and ε_z:

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \alpha T = -\frac{\nu(-E\alpha T)}{E} + \alpha T = \nu\alpha T + \alpha T = \alpha T(1+\nu)$$

Similarly:

$$\varepsilon_z = \alpha T(1+\nu)$$

3-1

Let

$$\varepsilon_{0-1} = \varepsilon_x = \varepsilon_{x_1}, \quad \varepsilon_{0-2} = \varepsilon_{x_2}, \quad \varepsilon_{0-3} = \varepsilon_{x_3}$$

We know:

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \varepsilon_{xy}\sin 2\alpha$$

Then, we can get:

$$\varepsilon_{0-2} = \varepsilon_{x_2} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 60^\circ + \varepsilon_{xy}\sin 60^\circ$$

$$\varepsilon_{0-3} = \varepsilon_{x_3} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 120^\circ + \varepsilon_{xy}\sin 120^\circ$$

$$\varepsilon_{y_1} = 5\times 10^{-4} \quad \varepsilon_{xy} = \frac{4\sqrt{3}}{3}\times 10^{-4}$$

Due to:

$$\begin{cases} \sigma_x = 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y) \\ \sigma_y = 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y) \\ \tau_{xy} = G\gamma_{xy} = 2G\varepsilon_{xy} \end{cases}$$

we can get:

$$\sigma_x = 96\times 10^{-4}\ \text{GPa}, \quad \sigma_y = 19\times 10^{-4}\ \text{GPa}, \quad \tau_{xy} = 32\sqrt{3}\times 10^{-4}\ \text{GPa}$$

Therefore, the principle stress:

$$\sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.73 \text{ MPa}$$

$$\sigma_{\min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.07 \text{ MPa}$$

The direction of principle stress:

$$\tan 2\alpha = \frac{\gamma_{xy}}{\sigma_x - \sigma_y} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \alpha = -24.55^\circ \text{ or } 65.45^\circ$$

The maximum shear stress:

$$\tau_{\max} = \frac{\sigma_x - \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.33 \text{ MPa}$$

The direction of maximum shear stress:

$$\tan 2\alpha = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \Rightarrow \alpha = -20.45^\circ \text{ or } 69.55^\circ$$

3-3

Since the bar is restrained in the x, y but free to expand in z, we can get:

$$\sigma_z = 0 \quad \varepsilon_x = 0 \quad \varepsilon_y = 0$$

$$\begin{cases} \varepsilon_z = -\frac{v}{E}(\sigma_x + \sigma_y) + \alpha T \\ \varepsilon_x = 0 = \frac{1}{E}(\sigma_x - v\sigma_y) + \alpha T \\ \varepsilon_y = 0 = \frac{1}{E}(\sigma_y - v\sigma_x) + \alpha T \end{cases}$$

Therefore:

$$\sigma_x = \sigma_y = \frac{E\alpha T}{v - 1}$$
$$\varepsilon_z = \frac{1 + v}{1 - v}\alpha T$$

3-9

(a)
Since $\delta_X = 0\ \delta_y = 0\ \tau_{xy} = p = 14140$psi we can get:

$$\delta_{x'} = \frac{\delta_x + \delta_y}{2} + \frac{\delta_x - \delta_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = 28280\text{psi}$$
$$\varepsilon_x = \frac{1}{E}\delta_x = 9.43 \times 10^{-4}$$
$$\varepsilon_y = -v\varepsilon_x = -2.83 \times 10^{-4}$$
$$\gamma_{xy} = \frac{\tau}{G} = 1.23 \times 10^{-3}$$

Therefore

$$\delta_{AB} = (\frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta)L_{AB} = 1.9 \times 10^{-4}$$

(b)
Since,

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

we can get:

$$\varepsilon_1 = 1.2 \times 10^{-4} \quad \varepsilon_2 = -5.4 \times 10^{-4}$$

The direction of principle strain is:

$$\tan 2\alpha = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -22.5^\circ \text{ or } 67.5^\circ$$

3-11

We know that:

$$G = \frac{E}{2(1+v)}, \quad \lambda = \frac{vE}{(1+v)(1-2v)}, \quad \text{and} \quad K = \frac{E}{3(1-2v)}$$

Since $K = \frac{E}{3(1-2v)}$, then

$$1 - 2v = \frac{E}{3K} \quad \Rightarrow \quad -2v = \frac{-3K+E}{3K}$$

we can get

$$v = \frac{3K-E}{6K}$$

Since $G = \frac{E}{2(1+v)}$ and $K = \frac{E}{3(1-2v)}$, then

$$2G = \frac{E}{(1+v)} \quad \text{and} \quad 3K = \frac{E}{(1-2v)}$$

$$3K - 2G = \frac{3vE}{(1+v)(1-2v)} = 3\lambda$$

we can get

$$\lambda = \frac{3K-2G}{3}$$

Since $\lambda = \frac{vE}{(1+v)(1-2v)}$, then

$$E = \frac{\lambda(1+v)(1-2v)}{v}$$