SDM5008 · Advanced Control for Robotics

Homework 1

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1 Spatial Velocity

(a) What is the linear velocity of the point C?

$$v_{Cx} = \frac{dC_x(t)}{d_t} = v$$

Therefore,

$$v_C = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$$

(b) What is the linear velocity of the point A?
Similarly to (a)

$$v_A = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$$

(c) What is velocity of the body-fixed point currently coincides with C?

$$v_c = v_o + \omega \times \overrightarrow{roC} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(d) What is velocity of the body-fixed point currently coincides with A?
Similarly to (c)

$$v_A = v_o + \omega \times \overrightarrow{roA} = \begin{bmatrix} 2v \\ 0 \\ 0 \end{bmatrix}$$

(e) What is the spatial velocity of the cylinder in {0}-frame?

$${}^0V = \begin{bmatrix} {}^0\omega \\ {}^0v \end{bmatrix}$$

where,

$${}^0\omega = \begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix}, {}^0v = 0 + {}^0\omega \times C_x(t)$$

Therefore,

$${}^0V = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ \frac{vC_x(t)}{r} \end{bmatrix}$$

(f) What is the spatial velocity of the cylinder in frame $\{C\}$?
Similarly to (e)

$${}^CV = \begin{bmatrix} {}^C\omega \\ {}^Cv \end{bmatrix}$$

, where

$$^{C}v = v + {}^{C}\omega \times \overrightarrow{roC}$$

Therefore,

$$^{0}V = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2 Modern Robotics: Exercise 3.21

(a) From the figure, we know $p_a = P_{ab} + r_a$, so

$$r_a = p_a - P_{ab}$$

Among them, P_{ab}can be derived from $^{A}T_{B}$ as $\begin{bmatrix} -100 \\ 300 \\ 500 \end{bmatrix}$

So

$$r_a = \begin{bmatrix} 100 \\ 500 \\ -500 \end{bmatrix}$$

And because

$$r_b =^{B} R_A * r_a$$

$$^{B}R_A = ({}^{A}R_B)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So

$$r_b = \begin{bmatrix} 500 \\ -100 \\ -500 \end{bmatrix}$$

(b) From the figure, we know

$$^{A}T_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 800 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$T_{bc} =^{B} T_C =^{B} T_A *^{A} T_C = ({}^{B}T_A)^{-1} *^{A} T_C = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 500 \\ -1 & 0 & 0 & -100 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 Modern Robotics: Exercise 3.28

(a)

$$w_b =^{b} R_s * w_s = R^{-1} * w_s = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

4 Modern Robotics: Exercise 5.5

(a) From the geometric relationship, we can know that:

$$^{s}P = (L + d\sin\theta)\hat{\mathrm{x}}_{\mathrm{s}} + (L - d\cos\theta)\hat{\mathrm{y}}_{\mathrm{s}}$$

(b) the velocity of point P is :

$$\frac{d_{^sP}}{d_t} = \begin{bmatrix} d\dot{\theta}\cos\theta \\ d\dot{\theta}\sin\theta \\ 0 \end{bmatrix}$$

(c)

$$t_{sb} = {}^{s}T_{b} = \begin{bmatrix} ^{s}R_{b} & P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & L + d\sin\theta \\ \sin\theta & cos\theta & 0 & L - d\sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) Because:

$$^{b}V = \begin{bmatrix} ^{b}\omega \\ ^{b}v_{ob} \end{bmatrix}$$

where,

$$^{b}v_{ob} = \omega * \overrightarrow{rob}$$

so,

$$^{b}V = \begin{bmatrix} ^{b}\omega \\ ^{b}v_{ob} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ d\dot{\theta} \\ 0 \\ 0 \end{bmatrix}$$

(e) Similarly to (d)

$$^{s}V = \begin{bmatrix} ^{s}\omega \\ ^{s}v_{os} \end{bmatrix}$$

where,

$$^{s}v_{os} = \omega * \overrightarrow{ros}$$

so,

$$^{s}V = \begin{bmatrix} ^{s}\omega \\ ^{s}v_{os} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ L\dot{\theta} \\ -L\dot{\theta} \\ 0 \end{bmatrix}$$

(f) It can be concluded that

$$^{s}V =^{s} X_{b} *^{b} V$$

where,

$$^{s}X_{b} =^{s} T_{b} = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

(g) Because $^{b}V_{p} =^{s} R_{b}^{-1} *^{s} V_{p}$, and $^{s}R_{b}\mathrm{is}T_{sb}$, this is the relationship between them.

(h) Beacuse $^{s}V_{os} =^{s} V_{p} +^{s} \omega \times \overrightarrow{opos}$

5 Modern Robotics: Exercise 5.6

(a) From the question, we can find that

$$^{s}\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, {}^{s}v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

we can get

$$^{s}V = \begin{bmatrix} ^{s}\omega \\ ^{s}v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, we can get

$$^{b}V_b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 10 \\ 0 \end{bmatrix}$$

And beacuse

$$^{b}V_s = {}^{b}X_s {}^{s}V$$

其中

$$^{b}X_s = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

As can be known from the figure,

$$^{b}R_s = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}, p = \begin{bmatrix} -10 \\ 0 \\ 20 \end{bmatrix}$$

So we can get

$$^{b}V = {}^{b}V_b + {}^{b}V_s = \begin{bmatrix} ^{b}\omega \\ ^{b}v \end{bmatrix} \begin{bmatrix} \sin t \\ \cos t \\ 1 \\ -20\cos t \\ 20\sin t + 10 \\ -10\cos t \end{bmatrix}$$

Therefore,

$$^{b}\omega = \begin{bmatrix} \sin t \\ \cos t \\ 1 \end{bmatrix}, {}^{b}v = \begin{bmatrix} -20\cos t \\ 20\sin t + 10 \\ -10\cos t \end{bmatrix}$$

(b)

$$\dot{p} = {}^{s}R_b v_b = \begin{bmatrix} -20\cos t - 20\cos t \sin t \\ 10\cos t \\ 20\sin t + 10\sin^2 t - 10\cos^2 t \end{bmatrix}$$