Review

- Tensor notations (张量表示)
- Tensor: definition
- Gauss's law of integration
- Stress is a 2nd-order tensor
- Define the symmetric part of velocity gradient as strain, its components is associated with the 6 strain components
- Elastic constants c_{ijkl} is a 4_{th} order tensor
- Hooke's law with tensor

$$\varepsilon_{ij} = \frac{1+\nu}{E}\tau_{ij} - \frac{\nu}{E}\delta_{ij}\Theta \quad \text{where } \Theta = \tau_{kk}$$

$$\tau_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon\delta_{ij} \quad \text{where } \varepsilon = \varepsilon_{kk}$$

- express equilibrium equations with tensor notations
- Strain energy
- Navier's equation

$$(\lambda + G)u_{j,ji} + G\nabla^2 u_i + f_i = 0$$

Continuum Mechanics (B)

Session 07: Basics of Fluid Mechanics

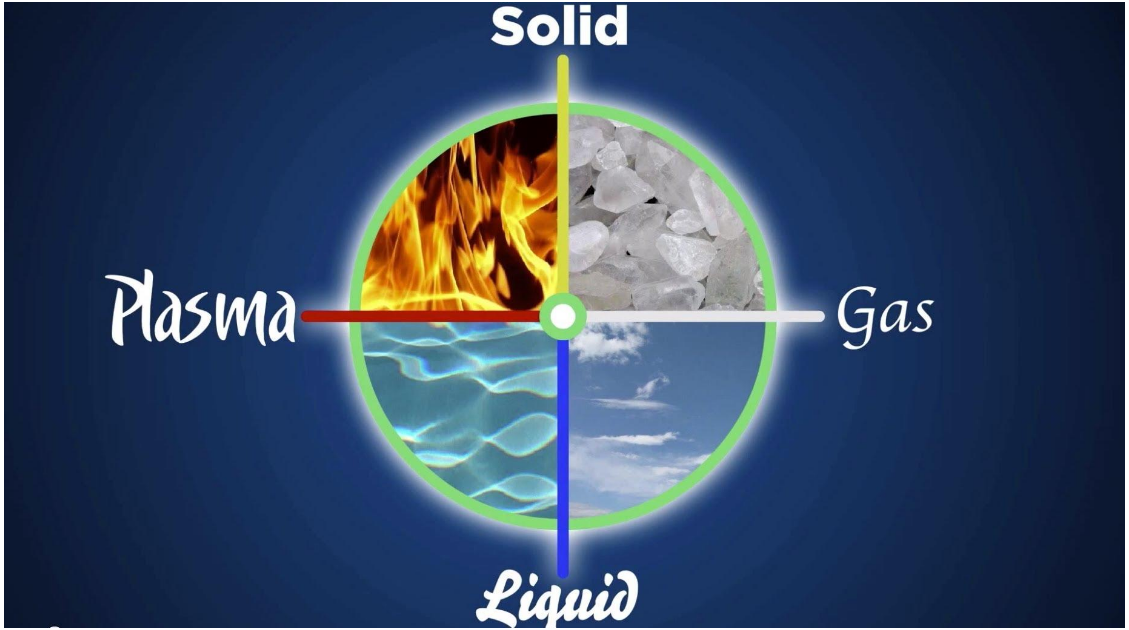
Lecturer: Ting Yang 杨亭
Department: ESS 地空系
E-mail: yangt3@sustech.edu.cn

Content

- Definition of Fluid and Fluid Mechanics
- Lagrangian and Eulerian Viewpoints, Substantial Derivatives (物质导数、随流导数)
- Decomposition of Fluid Motion
- Fluid Viscosity
- Classification of Flow Phenomena
- Streamline, Pathline, Streakline
- The study of Fluids
  - theoretical, experimental, computational

Fluid mechanics (流体力学)

continuum mechanics { Elasticity / **Fluid mechanics**



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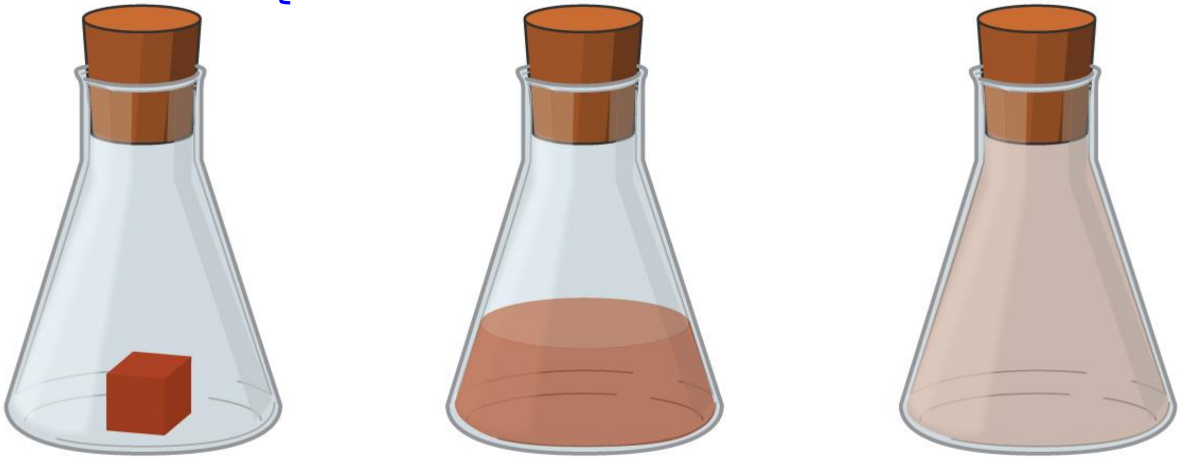
Definition of Fluid and Fluid Mechanics: Fluidity of Fluid

Fluid

- **Liquid**
  - Possess volume but no definite shape
  - Less compressible (K=2000 MPa), density varies little with temperature/pressure (α= 21e-5/K)
- **Gas**
  - No definite volume/shape, fill any container into which it is placed
  - More compressible (K~=1E5 Pa), density varies significantly with T (α= 340e-5/K) /P
- **Plasma**
  - Like gas, with electricity.



Solid

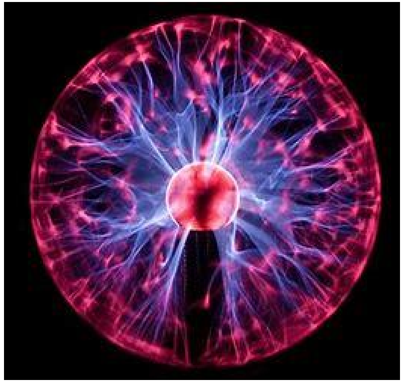
Has fixed shape and volume

Liquid

Takes shape of container
Forms horizontal surface
Has fixed volume

Gas

Expands to fill container



Plasma

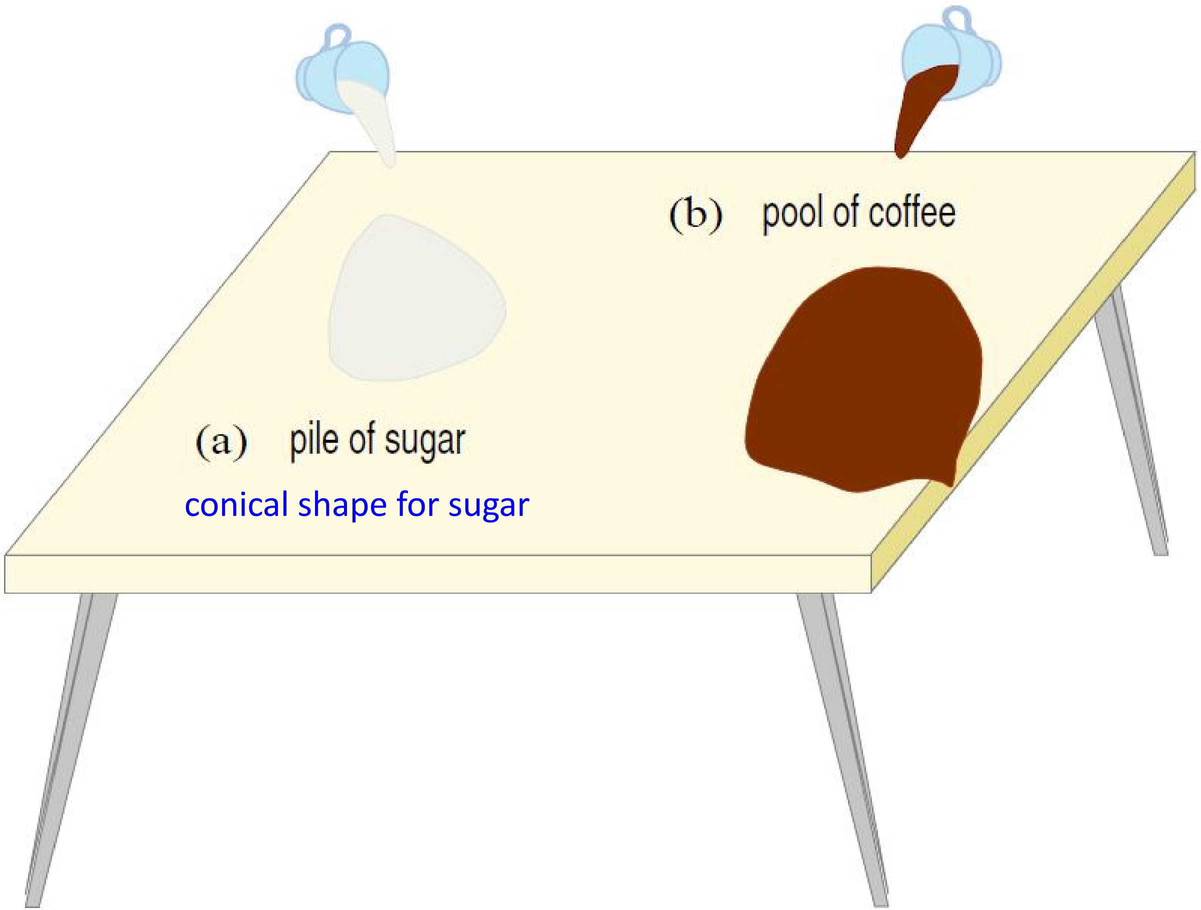
Definition of Fluid and Fluid Mechanics

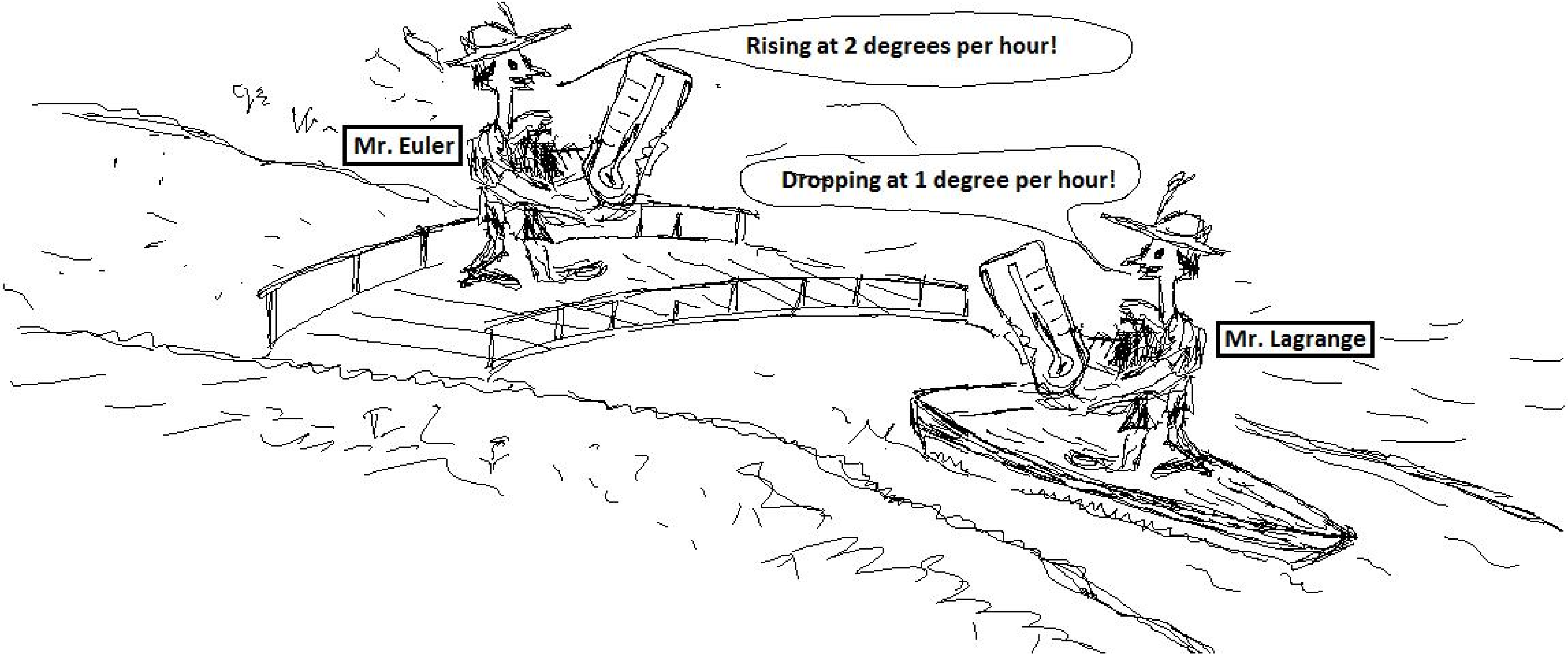
- The most fundamental difference between fluid and solid is that the fluid can flow
  - **Flow**: Material deforms continuously when subjected to shear stress
  - → Fluid has no specific shape
- **Definition of Fluid:**
  - material that deforms (flows) continuously under an applied shear stress, no matter how small the stress is
  - A fluid at rest must be in a state of zero shear stress
- **Fluid Mechanics:**
  - the study of fluid (liquids, gases, and plasmas) behaviors at rest or in motion.
    - fluid statics, fluid kinematics, fluid dynamics (流体静力学、运动学、动力学)

Definition of Fluid and Fluid Mechanics

Not all objects that can flow are fluids

- The shear stress induced by gravity cannot be supported by coffee but is supported by sugar.
- Sugars are composed of small but macroscopic solid grains.
    - Shear stress are supported by friction between solid grains
- **Granular sugar can flow, but is not fluid**
- Some common materials that can flow (sugar, salt, flour, many spices) are not fluid.
- However, the governing equations of fluid mechanics usually provide reasonable approximation to flow in solid grains.




Rising at 2 degrees per hour!
Mr. Euler
Dropping at 1 degree per hour!
Mr. Lagrange

Lagrangian and Eulerian Viewpoints, Substantial Derivatives

The **Lagrangian viewpoint** of fluid mechanics focuses on material particles as they move through the flow.

- Each particle (parcel/element) in the flow is identified by its original position x^0_i.

The temperature in Lagrangian variables is given by

$$T = T_L(x_i^0, \hat{t})$$

The particle position r_i is given by

$$r_i = \tilde{r}_i(x_i^0, \hat{t})$$

The velocity and acceleration of a particle are defined by

$$v_i = \frac{\partial \tilde{r}_i}{\partial \hat{t}} \quad \text{and} \quad a_i = \frac{\partial^2 \tilde{r}_i}{\partial \hat{t}^2}$$

The Lagrangian viewpoint is a natural extension of particle mechanics (质点力学)

Lagrangian and Eulerian Viewpoints, Substantial Derivatives

The **Eulerian viewpoint** focuses on a fixed point in space x_i as time t proceeds.
All flow properties, such as position r_i and velocity v_i are considered as functions of x_i and t.
The temperature of the fluid is given by $T = T_E(x_i, t)$.

- At a fixed time, $T_E(x_i, t)$ tells how the temperature changes in space;
- at a fixed point, $T_E(x_i, t)$ gives the local temperature history.

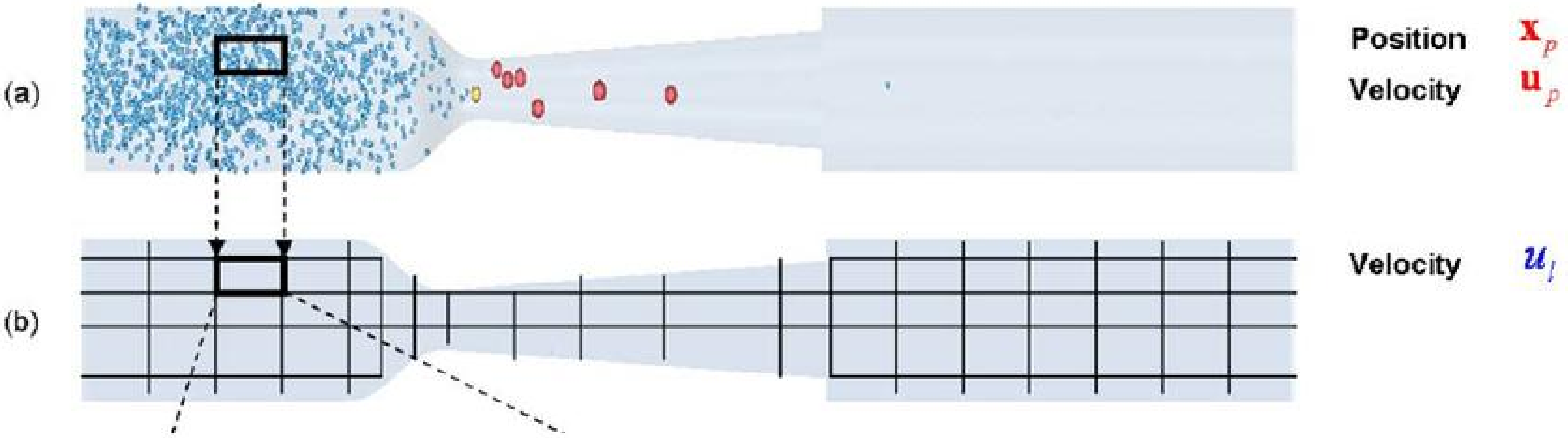
The particle position vector in Eulerian variables is simply

$$r_i = x_i$$

The Eulerian variable x_i, t are connected with the Lagrangian variables x_i^0, $\hat{t}$:

$$t = \hat{t} \quad x_i = r_i = \tilde{r}_i(x_i^0, \hat{t}) = \tilde{r}_i(x_i^0, t)$$

Lagrangian and Eulerian Viewpoints, Substantial Derivatives



https://www.researchgate.net/publication/260312605_A_cavitation_model_based_on_Eulerian_stochastic_fields/figures?lo=1

Lagrangian and Eulerian Viewpoints, Substantial Derivatives (随流导数，物质导数)

Let F be a property of the flow under consideration, we have

$$F = F_L(x_i^0, \hat{t}) = F_E(x_i, t)$$

Lagrangian Eulerian

For a particular particle, $F = F_E(x_i, t) = F_E(\tilde{r}_i(x_i^0, \hat{t}), \hat{t})$

position x_i is changing with time for a particular particle

The rate of change of F for this particle is

$$\frac{\partial F_L}{\partial \hat{t}} = \frac{dF_L}{d\hat{t}} = \frac{\partial F_E}{\partial x_i}\frac{\partial \tilde{r}_i}{\partial \hat{t}} + \frac{\partial F_E}{\partial t}\frac{\partial t}{\partial \hat{t}} = \frac{\partial F_E}{\partial t} + v_i \frac{\partial F_E}{\partial x_i}$$

$v_i = \partial \tilde{r}_i / \partial \hat{t}$ is the particle velocity

The substantial (material) derivative:

$$\frac{d(\ )}{dt} = \frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + v_i\, \partial_i(\ )$$

tensor notation

local rate of change

convective change

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + (\mathbf{v}\cdot\nabla)(\ )$$

symbolic notation

Lagrangian and Eulerian Viewpoints, Substantial Derivatives

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + v_i\ \partial_i(\ )$$

Note that velocity of x_i in the Eulerian coordinate is set as the velocity of the partical velocity passing through it.

Express the velocity and acceleration of a particular point with the Eulerian variables

$$\begin{aligned}\frac{Dr_j}{Dt} &= \frac{\partial r_j}{\partial t} + v_i\ \partial_i r_j \\ &= 0 + v_i\ \partial_i x_j \\ &= v_i\ \delta_{ij} \\ &= v_j\end{aligned}$$

Decomposition of Fluid Motion

Assume a point P moves at a velocity of v_i^P,
Let's consider the motion of point P' that is very close to point P :
PP'=dx_i=dr_i=α_ids
α_i is unit vector in PP' direction and ds the length of PP'.

the velocity of point P' is

$$v_i^{P'} = v_i^P + dv_i = v_i^P + \frac{\partial v_i}{\partial x_j} dx_j$$

$$\frac{\partial v_i}{\partial x_j} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) = \dot{\varepsilon}_{ij} + \dot{\omega}_{ij}$$

Symmetric 2nd tensor   Antisymmetric 2nd tensor

Strain rate tensor $S_{ij} = \dot{\varepsilon}_{ij}$,
Rotation rate tensor $\dot{\omega}_{ij}$

The velocity of point P' is

$$v_i^{P'} = v_i^P + dv_i = v_i^P + dv_i^S + dv_i^R = v_i^P + S_{ij} dx_j + \dot{\omega}_{ij} dx_j$$

Deform Rotation

Motion of point P': (a) translation with P, (b) rigid rotation, (c) deformation (strain)

Decomposition of Fluid Motion

$$dv_i^R = \dot{\omega}_{ij} dx_j$$

$$\dot{\omega}_{ij} = -\frac{1}{2}\varepsilon_{ijk}\dot{\omega}_k \qquad \dot{\omega}_k = \nabla \times \mathbf{v} = \varepsilon_{kmn}\nabla_m v_n$$

$\dot{\omega}_k$: Vorticity (速度的旋度curl在流体力学中称为涡量、涡度)

Velocity associated with rotation tensor is

$$dv_i^R = \dot{\omega}_{ij} dx_j = -\frac{1}{2}\varepsilon_{ijk}\dot{\omega}_k dx_j = \varepsilon_{ikj}(\frac{1}{2}\dot{\omega}_k) dx_j$$

The rigid-rotation velocity of P’ relative to P is

$$\mathbf{V} = \mathbf{\Omega} \times d\mathbf{x} = \varepsilon_{ikj}\Omega_k dx_j$$

Vorticity (涡量) is twice the angular velocity of the rigid rotation of P' relative to P: $\dot{\omega}_k = 2\Omega_k$

dv_i^R represents the rotation of P’ relative to P at an angular velocity of $\dot{\omega}_k / 2$

Decomposition of Fluid Motion

$$dv_i^S = S_{ij}dx_j = S_{ij}\alpha_j ds$$

α_i is unit vector in PP' direction and d*s* the length of PP'.

$$\frac{dv_i^S}{ds} = S_{ij}\alpha_j = d_i$$

d_i is **strain rate vector**
In symbolic notation, **d=α·S**

d_i represent the relative velocity of P' per unit length with respect to P due to deformation.

Decomposition of Fluid Motion

Consider a 1D linear shear flow:

$$v_1 = cx_2$$

where c is a constant

The strain rate tensor is

$$S_{ij} = \frac{1}{2}[\partial_i v_j + \partial_j v_i]$$

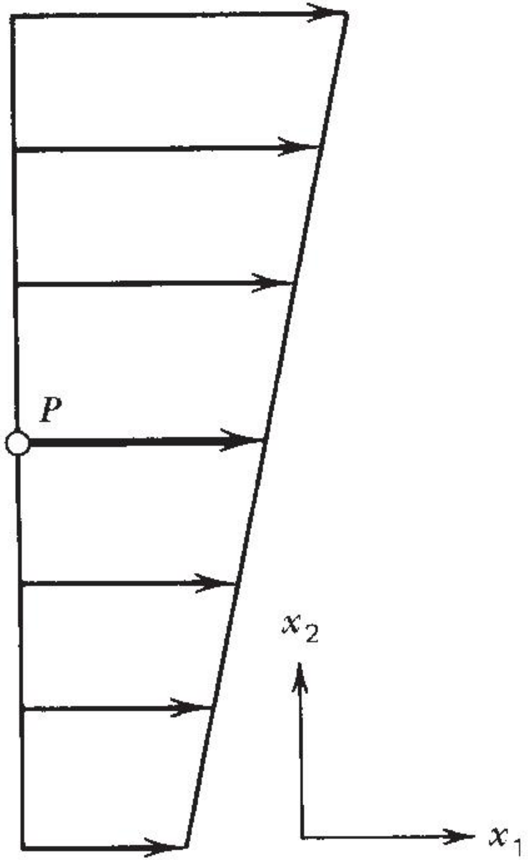
$$S_{11} = S_{22} = 0$$

$$S_{12} = S_{21} = \frac{1}{2}[\partial_1 v_2 + \partial_2 v_1] = \frac{c}{2}$$

The vorticity is

$$\omega_3 = -\partial_2 v_1 = -c$$

Only one vorticity component (涡度分量) is nonzero



Velocity profile of 1D shear flow

Classroom exercise

1. True or False:

Solid can sustain (承受、支持) compression, tension, bending, shear, torsion forces

Fluid can only sustain compression forces

2. For a 2-D flow field, the velocity fields are u=(u,v),
where $u=x^2-y^2$, and $v=2xy$.
For a scalar field $\phi(x,y,t)=x^2+y^2+t$, calculate the material derivative $D\phi/Dt$

$$\frac{D\phi}{Dt} = 1 + 2x^3 + 2xy^2$$