

## MAE5009: Continuum Mechanics B

### Assignment 01: Stress

Due September 29, 2021

1. Given  $\sigma_x = -14$  MPa,  $\sigma_y = 6$  MPa, and  $\tau_{xy} = -17$  MPa, determine both by formulas and by the Mohr's circle,
  - (a) the principal stresses and their directions,
  - (b) the direction having the maximum shear stress and the corresponding shear and normal stress magnitudes,
  - (c) the stress components on the  $x'$  and  $y'$  planes when  $\alpha = 45^\circ$
2. Given a three-dimensional stress state with
$$\sigma_x = 10 \text{ MPa}, \sigma_y = 20 \text{ MPa}, \sigma_z = -10 \text{ MPa}$$
$$\tau_{xy} = 5 \text{ MPa}, \tau_{xz} = -10 \text{ MPa}, \tau_{yz} = -15 \text{ MPa}$$
  - (a) find the magnitude and direction of the stress vector  $p$  on the  $x'$  plane where the  $x'$  direction is defined by
$$\cos(x', x) = 1/2, \cos(x', y) = 1/\sqrt{2} \text{ and } \cos(x', z) \text{ is positive}$$
  - (b) find  $\sigma$  and  $\tau$  on this plane
  - (c) determine the angle between  $p$  and  $\sigma$
  - (d) solve for  $\tau_{x'y'}$  and  $\tau_{x'z'}$ , if  $\cos(x, y') = 1/2$  and  $\cos(z, y')$  is negative. Please note that both the  $x$ - $y$ - $z$  and  $x'$ - $y'$ - $z'$  coordinates system follow the right-hand rule.
  - (e) evaluate all of the stress components acting on the  $x'$ ,  $y'$  and  $z'$  planes
  - (f) determine the principal stresses and the direction cosines of the principal axes with respect to the  $x$ ,  $y$  and  $z$  axis.

3. Given the following stress functions,

$$\begin{aligned}\sigma_x &= 3x^2 + 3y^2 - z & \tau_{xy} &= z - 6xy - \frac{3}{4} \\ \sigma_y &= 3y^2 & \tau_{xz} &= x + y - \frac{3}{2} \\ \sigma_z &= 3x + y - z + \frac{5}{4} & \tau_{yz} &= 0\end{aligned}$$

(a) show that the above stress state is in equilibrium

(b) for the stress state at point  $x = 1/2$ ,  $y = 1$ , and  $z = 3/4$ , determine the principal stresses.