南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

地球与空间科学系 DEPARTMENT OF EARTH AND SPACE SCIENCES

MAE5009
Continuum Mechanics B
Session 03: Stress Strain Relations

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自己推！记住！

So far we have…

Unknowns

- Stress (6)

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ sym. & & \sigma_z \end{bmatrix}$$

- Strain (6)

$$\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_y & \varepsilon_{yz} \\ sym. & & \varepsilon_z \end{bmatrix}$$

- Displacement (3)

$$[u, v, w]$$

Equations

- Equilibrium equations (3)

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} + \dfrac{\partial \tau_{zx}}{\partial z} + f_x = 0 \\ \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{zy}}{\partial z} + f_y = 0 \\ \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \sigma_z}{\partial z} + f_z = 0 \end{cases}$$

- Strain-displacement (6)

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

- Compatibility equations (3/6)

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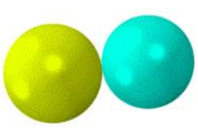
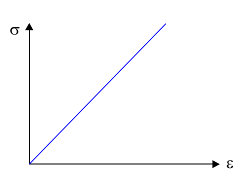
Assumptions

- Elastic
  - Regain its original dimensions after the forces are removed
- Isotropic
  - Properties are the same in any direction
- Homogeneous
  - Properties are independent of position
- Linear stress-strain relations





Linear elastic materials

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Constitutive equations

- Equations relating stress, strain, stress rate, and strain rate
- Depend upon the material properties
- Elastic solids
  - Generalized Hooke's law
  - Only involves stress and strain
  - Independent of stress-rate and strain-rate

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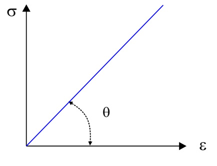
Generalized Hooke's law in 1D

- Most engineering materials exhibit a well-defined elastic range under uniaxial normal stress



$$\sigma_x = E\varepsilon_x$$

E: modulus of elasticity or Young's modulus

Robert Hooke (1635-1703)

Thomas Young (1773-1829)

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Generalized Hooke's law in 3D

- Each stress component is a linear function of six strain components, and vice versa

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

- For homogeneous material, c_{ij} are independent of position, and thus are constants, or elastic constants

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Elastic constants

- Anisotropic material:
  - $c_{ij} = c_{ji}$
  - 21 constants
- Orthotropic material:
  - $c_{15} = c_{16} = c_{25} = c_{26} = 0$
  - $c_{35} = c_{36} = c_{45} = c_{46} = 0$
  - $c_{14} = c_{24} = c_{34} = c_{56} = 0$
  - 9 constants

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}$$

Wood is an example of an orthotropic material. Material properties in three perpendicular directions (axial, radial, and circumferential) are different.

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Elastic constants

- Transversely isotropic material:
  - $c_{11} = c_{33}, c_{21} = c_{23}, c_{44} = c_{55}$
  - 5 constants
- Isotropic material
  - $c_{11} = c_{22} = c_{33},$
  - $c_{12} = c_{13} = c_{23},$
  - $c_{44} = c_{55} = c_{66}$
  - 2 constants: *E* and *v*

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}$$

Sedimentary rocks are transversely isotropic. Each layer has approximately the same properties in-plane but different properties through-the-thickness.

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Poisson's ratio

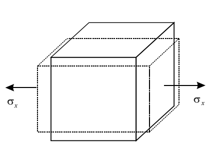
- Poisson's ratio is a measure of the Poisson effect, the phenomenon in which a material tends to expand in directions perpendicular to the direction of compression

Siméon Poisson (1781–1840)

Material	Poisson's ratio
Rubber	0.4999
Gold	0.42–0.44
Rock	0.15–0.40
Cork	0.0

$$\nu = -\frac{\varepsilon_{trans}}{\varepsilon_{axial}}$$

$$\varepsilon_{trans} = -\nu\varepsilon_{axial}$$



- *The Poisson's ratio of a stable, isotropic, linear elastic material must be between −1.0 and +0.5, because of the requirement for E, G and K to have positive values*
- *Most materials have Poisson's ratio values ranging between 0.0 and 0.5*
- *A perfectly incompressible isotropic material would have a Poisson's ratio of 0.5*
- *Some materials, e.g. some polymer foams exhibit negative Poisson's ratio*

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Derivation of generalized Hooke's law (assumptions)

- Normal stress σ_x does not produce shear strain on the *x*, *y* and *z* planes
- Shear stress τ_{xy} does not cause normal strain on the *x*, *y* and *z*
- Shear stress component τ_{xy} only cause one shear strain component γ_{xy}
- **Principal of superposition** may be applied to determine the strain components produced by more than one stress component

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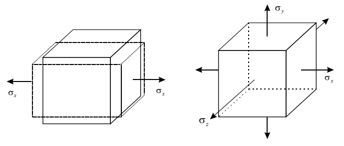
Normal strain VS normal stress

1. Apply σ_x

$$\varepsilon_x = \frac{\sigma_x}{E}, \varepsilon_y = \varepsilon_z = -\nu\frac{\sigma_x}{E}$$

$$L_x = (1+\varepsilon_x)L_{x0} = 1+\frac{\sigma_x}{E}$$

$$L_y = L_z = (1+\varepsilon_y)L_{y0} = 1-\nu\frac{\sigma_x}{E}$$



$$L_{x0} = L_{y0} = L_{z0} = 1$$

2. Apply σ_y

$$\varepsilon_y = \frac{\sigma_y}{E}, \varepsilon_x = \varepsilon_z = -\nu\frac{\sigma_y}{E}$$

$$L_x = (1+\varepsilon_x)L_x = \left(1-\nu\frac{\sigma_y}{E}\right)\left(1+\frac{\sigma_x}{E}\right)$$

$$L_y = (1+\varepsilon_y)L_y = \left(1+\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$

$$L_z = (1+\varepsilon_z)L_z = \left(1-\nu\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$

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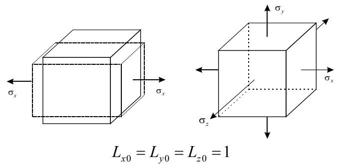
Normal strain VS normal stress

3. Apply σ_z

$$\varepsilon_z = \frac{\sigma_z}{E}, \varepsilon_x = \varepsilon_y = -\nu\frac{\sigma_z}{E}$$



$$L_{x0} = L_{y0} = L_{z0} = 1$$

$$L_x = (1+\varepsilon_x)L_x = \left(1-\nu\frac{\sigma_z}{E}\right)\left(1-\nu\frac{\sigma_y}{E}\right)\left(1+\frac{\sigma_x}{E}\right)$$

$$L_y = (1+\varepsilon_y)L_y = \left(1-\nu\frac{\sigma_z}{E}\right)\left(1+\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$

$$L_z = (1+\varepsilon_z)L_z = \left(1+\frac{\sigma_z}{E}\right)\left(1-\nu\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$

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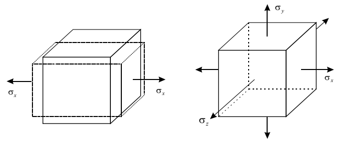
Normal strain VS normal stress

$$L_x = (1+\varepsilon_x)L_x = \left(1-\nu\frac{\sigma_z}{E}\right)\left(1-\nu\frac{\sigma_y}{E}\right)\left(1+\frac{\sigma_x}{E}\right)$$

$$L_y = (1+\varepsilon_y)L_y = \left(1-\nu\frac{\sigma_z}{E}\right)\left(1+\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$

$$L_z = (1+\varepsilon_z)L_z = \left(1+\frac{\sigma_z}{E}\right)\left(1-\nu\frac{\sigma_y}{E}\right)\left(1-\nu\frac{\sigma_x}{E}\right)$$



$$L_{x0} = L_{y0} = L_{z0} = 1$$

Neglect negligible items

$$\varepsilon_x = \frac{L_x - L_{x0}}{L_{x0}} = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right)$$

$$\varepsilon_y = \frac{L_y - L_{y0}}{L_{y0}} = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right)$$

$$\varepsilon_z = \frac{L_z - L_{z0}}{L_{z0}} = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right)$$

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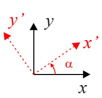
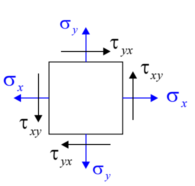
Shear strain VS shear stress

Suppose: $\gamma_{xy} = \frac{1}{G}\tau_{xy}$ $\gamma_{yz} = \frac{1}{G}\tau_{yz}$ $\gamma_{zx} = \frac{1}{G}\tau_{zx}$

G is the modulus of elasticity in shear, or shear modulus, or the modulus of rigidity





Plane stress: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \qquad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

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Shear strain VS shear stress

Stress transformation:

$$\begin{cases} \sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha \\ \sigma_{y'} = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{\sigma_x - \sigma_y}{2}\cos 2\alpha - \tau_{xy}\sin 2\alpha \end{cases}$$

Strain transformation:

$$\begin{cases} \varepsilon_{x'} = \dfrac{\varepsilon_x + \varepsilon_y}{2} + \dfrac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \varepsilon_{xy}\sin 2\alpha \\ \varepsilon_{y'} = \dfrac{\varepsilon_x + \varepsilon_y}{2} - \dfrac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha - \varepsilon_{xy}\sin 2\alpha \end{cases}$$

Stress-strain relations:

$$\begin{cases} \varepsilon_{x'} = \dfrac{1}{E}\left(\sigma_{x'} - \nu\sigma_{y'}\right) \\ \varepsilon_{y'} = \dfrac{1}{E}\left(\sigma_{y'} - \nu\sigma_{x'}\right) \end{cases}$$

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Shear strain VS shear stress

Combine these equations:

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy} \rightarrow G = \frac{E}{2(1+\nu)}$$

There are only two independent elastic constants for an isotropic material, E and ν

Stress-strain relations (generalized Hooke's law):

$$\begin{cases} \varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) \\ \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right) \\ \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ \gamma_{zx} = \frac{1}{G}\tau_{zx} \end{cases}$$

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Shear strain VS shear stress

$$\begin{cases} \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ \gamma_{zx} = \frac{1}{G}\tau_{zx} \end{cases}$$

Principal directions of stress and strain:

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$
$$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$$

For isotropic, elastic materials, the principal axes of stress and principal axes of strain ***coincide***

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Stress in terms of strain

$$\begin{cases} \varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) \\ \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right) \\ \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ \gamma_{zx} = \frac{1}{G}\tau_{zx} \end{cases}$$

$$\downarrow$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\begin{aligned} \sigma_x &= E\varepsilon_x + \nu\left(\sigma_y + \sigma_z\right) = E\varepsilon_x - \nu\sigma_x + \nu\sigma_x + \nu\left(\sigma_y + \sigma_z\right) \\ &= E\varepsilon_x - \nu\sigma_x + \nu\left(\sigma_x + \sigma_y + \sigma_z\right) \end{aligned}$$

$$(1+\nu)\sigma_x = E\varepsilon_x + \nu\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\rightarrow$$

$$\begin{aligned} \sigma_x &= \frac{E}{1+\nu}\varepsilon_x + \frac{\nu}{1+\nu}\left(\sigma_x + \sigma_y + \sigma_z\right) \\ &= 2G\varepsilon_x + \frac{\nu}{1+\nu}\cdot\frac{E}{1-2\nu}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \\ &= 2G\varepsilon_x + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{aligned}$$

$$\sigma_y = 2G\varepsilon_y + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right)$$

$$\sigma_z = 2G\varepsilon_z + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right)$$

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Stress in terms of strain

$$\begin{cases} \varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) \\ \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right) \\ \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ \gamma_{zx} = \frac{1}{G}\tau_{zx} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \sigma_x = 2G\varepsilon_x + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \\ \sigma_y = 2G\varepsilon_y + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \\ \sigma_z = 2G\varepsilon_z + \lambda\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{cases} \qquad \begin{cases} \tau_{xy} = G\gamma_{xy} \\ \tau_{yz} = G\gamma_{yz} \\ \tau_{zx} = G\gamma_{zx} \end{cases}$$

where

$$G = \frac{E}{2(1+\nu)} \qquad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$ Lame constant

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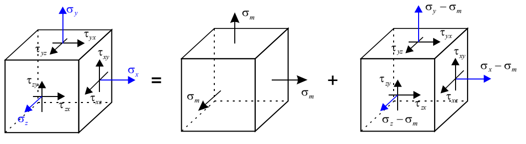
Decomposition of stress



$$\sigma_{ij} = \sigma_m \delta_{ij} + S_{ij}$$

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} \sigma_m & & \\ & \sigma_m & \\ & & \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_m \end{bmatrix}$$

Hydrostatic stress tensor
Volumetric stress tensor
Mean normal stress tensor
Spherical stress tensor

Stress deviator tensor

where $\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_y}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{1}{3} I_1$ *Hydrostatic component of stress, or spherical component of stress*

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Decomposition of strain

$$\varepsilon_{ij} = \varepsilon_m \delta_{ij} + e_{ij}$$

$$\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} \varepsilon_m & & \\ & \varepsilon_m & \\ & & \varepsilon_m \end{bmatrix} + \begin{bmatrix} \varepsilon_x - \varepsilon_m & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y - \varepsilon_m & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z - \varepsilon_m \end{bmatrix}$$

Hydrostatic strain tensor
Volumetric strain tensor
Mean normal strain tensor
Spherical strain tensor

Strain deviator tensor

where

$$\varepsilon_m = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_y}{3} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3} = \frac{1}{3} I_1'$$

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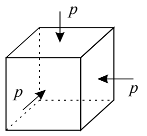
Bulk modulus of elasticity

$$\sigma_x = \sigma_y = \sigma_z = -p$$
$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$



$$\varepsilon_x = \varepsilon_y = \varepsilon_z = -\frac{1-2\nu}{E}p$$
$$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$$

$$(1+\varepsilon_x)dx \cdot (1+\varepsilon_y)dy \cdot (1+\varepsilon_z)dz = (1+\varepsilon)dxdydz$$

Volumetric strain: $\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z$

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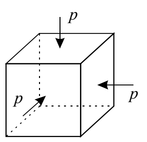
Bulk modulus of elasticity

$$\varepsilon_x = \varepsilon_y = \varepsilon_y = -\frac{1-2\nu}{E}p$$



Volumetric strain:

$$\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = -\frac{3(1-2\nu)}{E}p = -\frac{1}{K}p \quad \Rightarrow \quad K = \frac{E}{3(1-2\nu)}$$

Bulk modulus of elasticity

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$
$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_y}{3}$$

$$\Rightarrow \quad \varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(3\sigma_m) = \frac{3(1-2\nu)}{E}\sigma_m$$
$$\varepsilon = \frac{1}{K}\sigma_m$$

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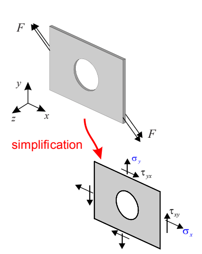
Plane stress

- For thin flat plates acted upon only by load forces that are parallel to them, the stress analysis can be considerably simplified to **plane stress**
- Stress components perpendicular to the plate are negligible compared to those parallel to it



$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Stress $\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$ *Strain* $\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$

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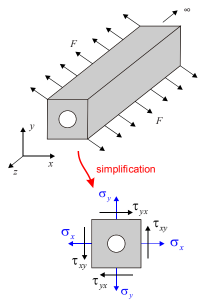
Plane strain

- If one dimension is very large compared to the others, the principal strain in the direction of the longest dimension is constrained and can be assumed as zero, the stress analysis can be considerably simplified to **plane strain**



$$\varepsilon_z = \varepsilon_{xz} = \varepsilon_{yz} = 0$$

$$\downarrow$$

$$\sigma_x = 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y)$$

$$\sigma_y = 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y) \quad \tau_{xy} = G\gamma_{xy} \quad \sigma_z = \lambda(\varepsilon_x + \varepsilon_y)$$

Stress $\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$ *Strain* $\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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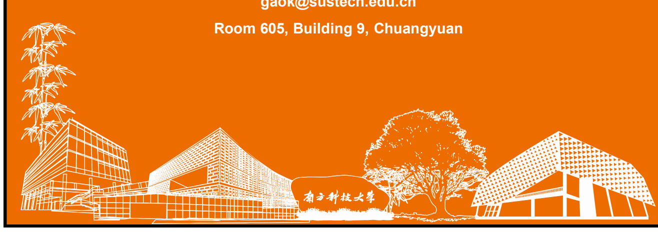

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