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1. $\because T(x_1, x_2)$

given $x_i = b_i(t)$

$\therefore T(x_1, x_2) = T(b_1(t), b_2(t))$

$$\therefore \frac{dT}{dt} = \frac{\partial T}{\partial x_1}\frac{db_1}{dt} + \frac{\partial T}{\partial x_2}\frac{db_2}{dt}$$

2. (a) $\omega = \nabla \times V$

$$= \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{vmatrix}$$

$$= \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z}\right)\hat{e}_r + \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right)\hat{e}_\theta + \frac{1}{r}\left(\frac{\partial (rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right)\hat{e}_z$$

$\omega_r = 0$,

$\omega_\theta = 0 - \frac{\partial v_z}{\partial r} = \frac{2V_0 r}{R^2}$

$\omega_z = 0$

$\therefore \omega_\theta = \frac{2V_0 r}{R^2}$

(b) $\omega_z = \frac{1}{r}\frac{\partial (rv_\theta)}{\partial r} - 0$

$= \frac{1}{r}\frac{\partial (\frac{\Gamma}{2\pi})}{\partial r}$

$= 0$

3. For 2D flow:

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

Given: $v_1 = c x_1$, $v_2 = -c x_2$

$$\therefore \dot{\epsilon}_{11} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right) = \frac{1}{2}(c+c) = c$$

$$\dot{\epsilon}_{12} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right) = \frac{1}{2}(0+0) = 0$$

$$\dot{\epsilon}_{21} = \frac{1}{2}\left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}\right) = \frac{1}{2}(0+0) = 0$$

$$\dot{\epsilon}_{22} = \frac{1}{2}\left(\frac{\partial v_2}{\partial x_2} + \frac{\partial v_2}{\partial x_2}\right) = \frac{1}{2}(-c-c) = -c$$

$$\therefore \cancel{\dot{\epsilon}_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

$$\therefore \dot{\epsilon}_{ij} = \begin{bmatrix} c & 0 \\ 0 & -c \end{bmatrix}$$

4.

streamline：某一瞬时流体微团的运动方向，由速度向量与曲线相切定义。

Pathline：单个流体质点在一段时间内的运动轨迹，通过求解运动方程得到。

Streakline：一段时间内通过流场中某一固定点的所有流体质点依次连接起来的曲线，常用于实验观察。

在稳态流动中，streamline，pathline和streakline重合，而在非稳态流动中，streamline随时间变化不同，Pathline显示的是粒子历史轨迹，而streakline显示的是固定点释放粒子的瞬时轨迹，它们是不同的。