Homework 6

Due Dec 9 2021

1. Show that the following Airy stress function

$$Ax^4 + Bx^2y^2 + Cy^4$$

will not be able to give a solution to the plane linear elasticity problem unless $3A + B + 3C = 0$. Determine the stress based on this stress function.

2. In the absence of body forces, show that the following stresses satisfy the plane strain stress formulation relations.

$$\sigma_x = kxy, \quad \sigma_y = kx, \quad \sigma_z = \nu kx(1+y), \quad \tau_{xy} = -\frac{1}{2}ky^2, \quad \tau_{xz} = \tau_{yz} = 0,$$

in which k is a constant.

3. Consider a plane stress problem for a straight beam subjected to end moments. The dimension of the beam is $-l \leq x \leq l$ and $-c \leq y \leq c$. The boundary conditions are written as

$$\sigma_{yy}(x, \pm c) = 0, \quad \tau_{xy}(x, \pm c) = 0, \quad \tau_{xy}(\pm l, y) = 0,$$
$$\int_{-c}^{c} \sigma_{xx}(\pm l, y)dy = 0, \quad \int_{-c}^{c} \sigma_{xx}(\pm l, y)ydy = -M.$$

Verify that the Airy stress function $\phi = Ay^3$ solves the problem. Determine the stress and strain of this problem. If the fixity displacement boundary condition is specified as $u_x(-l, 0) = 0$ and $u_y(\pm l, 0) = 0$, determine the displacement of the beam.

4. Consider the above problem in the plane strain setting , determine the displacement field. Next, plot for each case of the y-displacement along the x-axis (y=0) with Poisson's ratio $\nu = 0.4$. Use dimensionless variables and plot $u_y(x, 0)/(Ml^2/EI)$ versus x/l. Here I is the moment of inertia and equals $2c^3/3$.

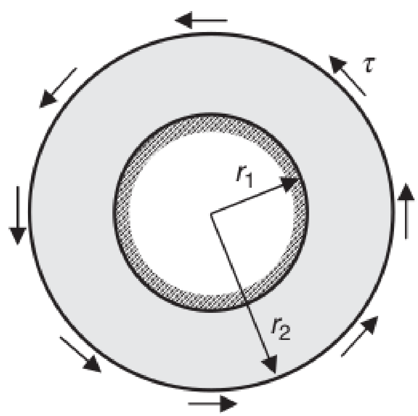
5. Determine the two dimensional strains e_{rr}, $e_{\theta\theta}$, and $e_{r\theta}$ from the following displacement field,

$$u_r = \frac{A}{r}, \quad u_\theta = Bcos(\theta),$$

where A and B are constants.

6. Consider the axisymmetric problem of an annular disk with a fixed inner radius and loaded with uniform shear stress over the outer radius (see the following figure). Using the Airy stress function $a\theta$, show that the stress and displacement solution for this problem is

$$\sigma_{rr} = \sigma_{\theta\theta} = 0, \quad \sigma_{r\theta} = \tau\frac{r_2^2}{r^2},$$
$$u_r = 0, \quad u_\theta = \frac{1+\nu}{E}\tau r_2^2\left(\frac{r}{r_1^2} - \frac{1}{r}\right).$$



τ
r_1
r_2