Homework 8

Due Jan 6 2022

1. (a) Consider a body fully immersed in a static fluid with constant density ρ. Show that the resultant force $\boldsymbol{F}$ acting on the body is $-\rho \boldsymbol{g} V$, where V is the volume of the body.

   (b) Show that the resultant moment $\boldsymbol{L} := \int_{\partial\Omega} \boldsymbol{x} \times (-p\boldsymbol{n}) dA$ equals $\boldsymbol{F} \times \boldsymbol{x}_c$, where $\boldsymbol{x}_c$ is the centroid of the body given by $\int_{\Omega} \boldsymbol{x} dV/V$.

2. Consider the tank shown as in Figure 1. Ignore p_a, determine the hydrostatic force and the center of pressure (CP) point on the panel AB, which is 1.2m long and 0.8 m into the paper. The specific gravity (SG) of the fluid means the ratio of the fluid density to the water density, which is $1000 kg/m^3$.

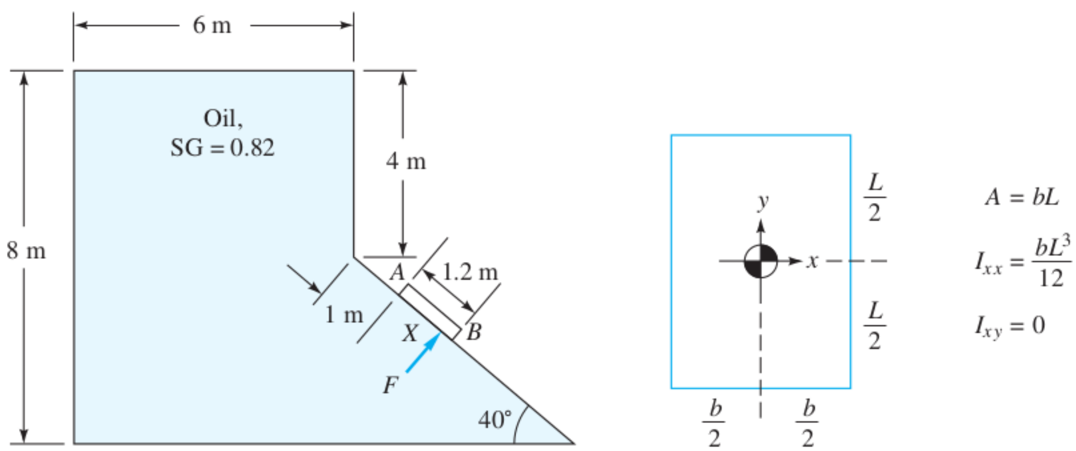

Figure 1: Problem setting (left) and the centroidal mements of a rectangle panel.

3. Consider a dam with a parabolic shape $z/z_0 = (x/x_0)^2$, as shown in 2. The geometry setting is that $x_0 = 10$ m and $z_0 = 25$ m. The width of the dam is 50 m. The fluid is water with $\rho g = 9790$ N/m^3, and the atmosphere pressure can be ignored. The resultant force acting on the dam from the hydrostatic force can be decomposed into a vertical component F_V and a horizontal component F_H. Determine F_V and F_H.

4. Consider a compressible flow flow of an inviscid fluid. The fluid is barotropic if the pressure if a function of density only (that is, the pressure is independnet of the temperature).

   (a) If we define the following

$$\mathcal{P}(\rho) := \int \frac{dp}{\rho},$$

   show that

$$\nabla \mathcal{P} = \frac{1}{\rho} \nabla p.$$

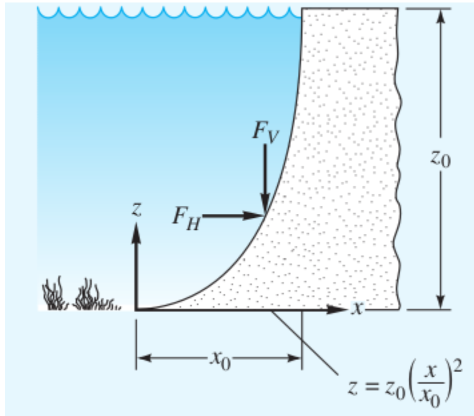

Figure 2: Problem setting for the dam.

(b) If the fluid is barotropic and irrotational, show that for steady flow one has the following Bernoulli equation,

$$\mathcal{P} + \frac{|\boldsymbol{v}|^2}{2} + V = C,$$

where C is a constant.

5. Given a velocity field with Cartesian components $v_x = k(x^2 - y^2)$, $v_y = -2kxy$, and $v_z = 0$.

   (a) Determine the rate of strain tensor, the rate of rotation tensor, the vorticity.

   (b) Determine the stream function for this velocity field. Draw the streamlines.

   (c) Is the flow rotational or irrotational? If it is irrotational, determine the potential for the velocity field.