Answers for Assignment 1:

If you find mistakes in these answers, please contact us and thank you for your correction.
Due to the calculation accuracy, some answers will be slightly different from your answers after the decimal point.

1.

(a) $\sigma_{\max} = \sigma_1 = 15.72\text{MPa}$; $\sigma_{\min} = \sigma_2 = -23.72\text{MPa}$; $\alpha_p = 29.77°$

(b) $\alpha_{\max s} = -15.23° or 74.77°$; $\tau_{\max} = \pm 19.72\text{MPa}$; $\sigma_x = \sigma_y = -4\text{MPa}$

(c) $\sigma_{x45} = -21\text{MPa}$; $\sigma_{y45} = 13\text{MPa}$; $\tau_{xy45} = 10\text{MPa}$

2.

(a) $\cos(x', z) = 1/2$; magnitude: $|\mathbf{p}|$=22.82MPa; direction: $\frac{\mathbf{p}}{|\mathbf{p}|}$=(0.16, 0.40, −0.90)

(b) In x' direction, $\sigma_{x'}$=-2.07MPa ; unit vector in x' direction: $\mathbf{e}_{x'}=(\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2})$; so

vector form: $\boldsymbol{\sigma}_{x'}=\sigma_{x'}\mathbf{e}_{x'}$=(-1.035,-1.464,-1.035);

$\boldsymbol{\tau}_{\perp x'}=\mathbf{p}\text{-}\boldsymbol{\sigma}_{x'} = (3.65, 9.13, -20.54) - (-1.035, -1.464, -1.035) = (4.69, 10.59, -19.50)$

(c) $\alpha_{\sigma p}$=84.64°

(d) 1) $\mathbf{R}=\begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3\sqrt{2}} & -\frac{5}{6} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} & \frac{\sqrt{2}}{6} \end{pmatrix}$, note $\mathbf{RR}^T=\mathbf{E}$; $\tau_{x'y'} = 21.10\text{MPa}$; $\tau_{x'z'} = -8.45\text{MPa}$;

or 2) $\mathbf{R}=\begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3\sqrt{2}} & -\frac{5}{6} \\ -\frac{1}{\sqrt{2}} & \frac{2}{3} & -\frac{\sqrt{2}}{6} \end{pmatrix}$, note $\mathbf{RR}^T=\mathbf{E}$; $\tau_{x'y'} = 21.10\text{MPa}$; $\tau_{x'z'} = 8.45\text{MPa}$;

(e) 1) $\boldsymbol{\sigma}=\begin{pmatrix} -2.07 & 21.10 & -8.45 \\ 21.10 & 12.07 & -2.93 \\ -8.45 & -2.93 & 10 \end{pmatrix}$;

or 2) $\boldsymbol{\sigma}=\begin{pmatrix} -2.07 & 21.10 & 8.45 \\ 21.10 & 12.07 & 2.93 \\ 8.45 & 2.93 & 10 \end{pmatrix}$

(f) σ_1=30MPa ; σ_2=8.23MPa ; σ_3=-18.23MPa ; $\mathbf{R}=\begin{pmatrix} \pm[-0.408 & -0.817 & 0.408] \\ \pm[0.874 & -0.479 & -0.085] \\ \pm[0.265 & 0.322 & 0.909] \end{pmatrix}$

Note $\sigma_1 > \sigma_2 > \sigma_3$, and $\mathbf{R}$ satisfy $\mathbf{R}\boldsymbol{\sigma}\mathbf{R}^{\mathrm{T}}=\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$

3.

(a)hints: show $\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}=0$, $\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}=0$ and $\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=0$

(b) $\boldsymbol{\sigma}=\begin{pmatrix} 3 & -3 & 0 \\ -3 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$; σ_1=6MPa ; σ_2=3MPa ; σ_3=0MPa ; $\mathbf{R}=\begin{pmatrix} \pm[\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0] \\ 0 & 0 & \pm 1 \\ \pm[\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0] \end{pmatrix}$

Note $\sigma_1 > \sigma_2 > \sigma_3$, and $\mathbf{R}$ satisfy $\mathbf{R}\boldsymbol{\sigma}\mathbf{R}^{\mathrm{T}}=\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$