Continuum Mechanics (B)

Session 01: Stress analysis

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- Three-Dimensional State of Stress at a Point (3D stress transformation)

Body and Surface Forces, Stress

When a body is subjected to external loads (加载)

- induced internal deformation
- induced internal forces

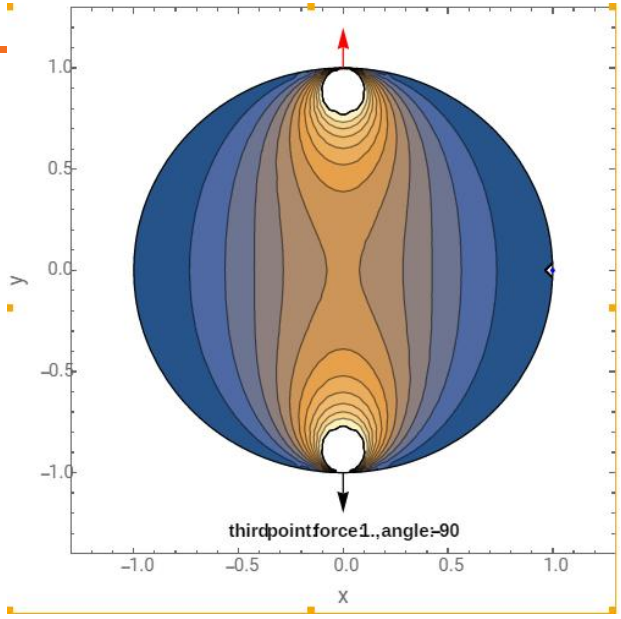
External forces (loads):

- **Body force** F: associated with body mass
  - gravitational, magnetic and inertia forces
  - distributed through the whole volume of the body
  - unit: N/m^3
    - force per unit volume
- **Surface forces**: induced by physical contact
  - red and black arrows in the right figure

Internal force is forces across an imaginary plane inside a body.

Internal force must be surface forces.

Point out the external forces applied to the honey.

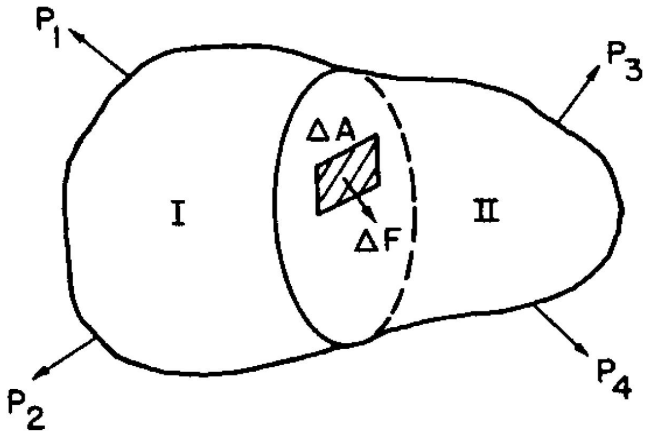


stress (τ_{max}) distribution in a circular plate with concentrated loadings (https://demonstrations.wolfram.com/StressDistributionInACircularPlateWithConcentratedRadialLoad/)

Body and Surface Forces, Stress

Free body (隔离体): A body that is singled out from other bodies for the purposes of dynamic or static analysis, usually associated with free body diagram (受力分析图).

Consider a body subjected to external forces P1-P4.



Consider the force applied by part II to **free body** (隔离体) part I through an imaginary plane ΔA is ΔF

- ΔF is induced by external forces
- ΔF is surface force
- ΔF is external force to part I but is actually internal force for the whole body.

Body and Surface Forces, Stress

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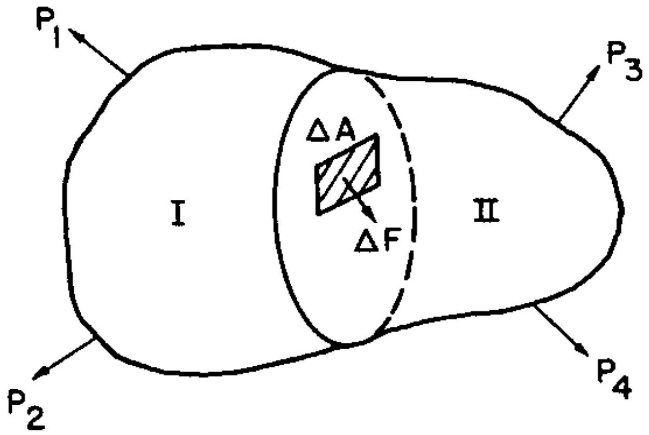


The **stress** at a point is defined as the limitation of force/area

$$\mathbf{p} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A} = \frac{d\mathbf{F}}{dA}$$

The stress ***p*** on the plane ΔA is a vector (**stress vector**) and ***p*** *is parallel to* ΔF

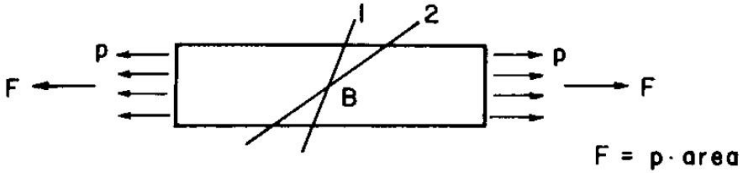
Q: Is the stress vector ***p*** in the direction normal to the surface ΔA?

To fully determine stress vector on a point, we need to know:

- Magnitude, Orientation
- the plane on which stress acts

Body and Surface Forces, Stress

Consider the internal forces at point B due to external force ***F***



The stress vector at point B differs for plane 1 ($\boldsymbol{p}_1$) and plane 2 ($\boldsymbol{p}_2$), why?

The stress vector varies for each plane across point B.

The **stress state** at a point: the set of stress vector acting on the infinitesimal area elements surrounding the point in all possible directions.

Stress state at point B is a 2nd-order tensor:

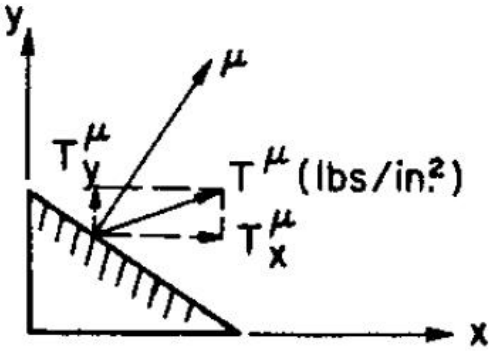
- Stress vector on any plane through a given point can be determined if we know the stress on any two planes passing through the point (in two-dimensional case).

- Check the direction for stress vectors $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$
- Compare the magnitude of the stress vectors $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, and $\boldsymbol{p}$

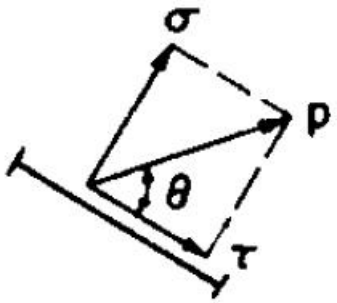
Body and Surface Forces, Stress

We can designate (表示) the stress vector in two ways in a coordinate

- the x, y, and z components
- the normal and tangential (shearing) components



The stress $\mathbf{T}^{\mu}$ is expressed in the x, and y components, outward normal of the plane $\boldsymbol{\mu}$ defines the plane

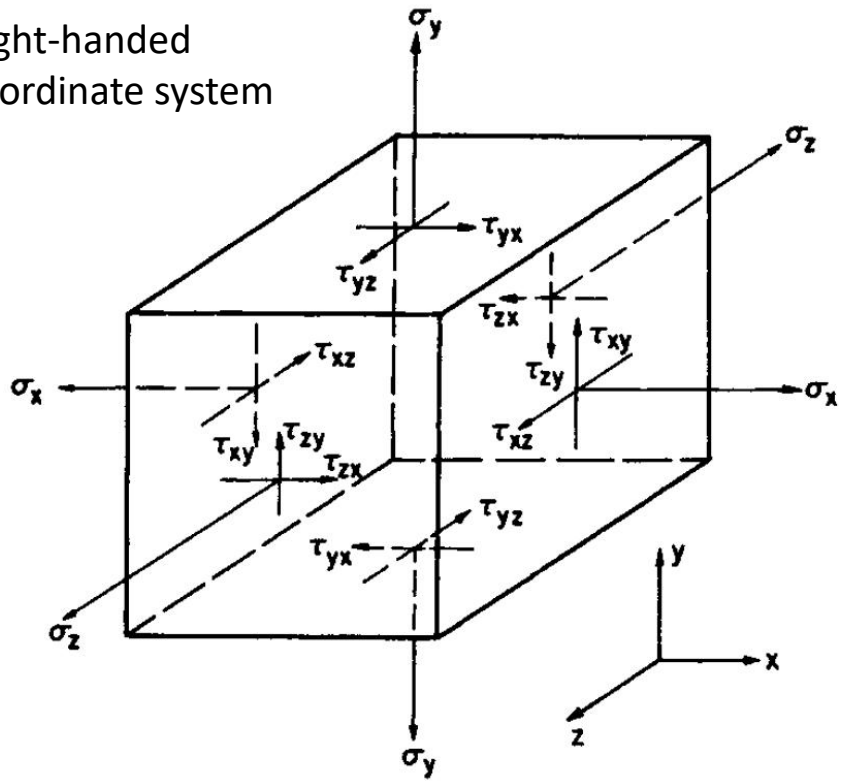


The internal stress ***p*** is expressed in normal (σ) and shear (τ) components.

- **Normal stress**: the component of stress perpendicular to the plane on which it acts.
- **shear stress**: the component of stress which lies in the plane.

Body and Surface Forces, Stress

Right-handed coordinate system



Three-Dimensional State of Stress at an element

Let's give some conventions in this book:

A plane is designated by the direction of its outward normal

The right-hand face of the element in the left figure is called the x plane; the left-hand face the negative x plane.

Sign conventions in this course:

on positive surfaces: stress acting in positive coordinate directions are positive

on negative surfaces: stress acting in negative coordinate directions are positive.

shear stress:

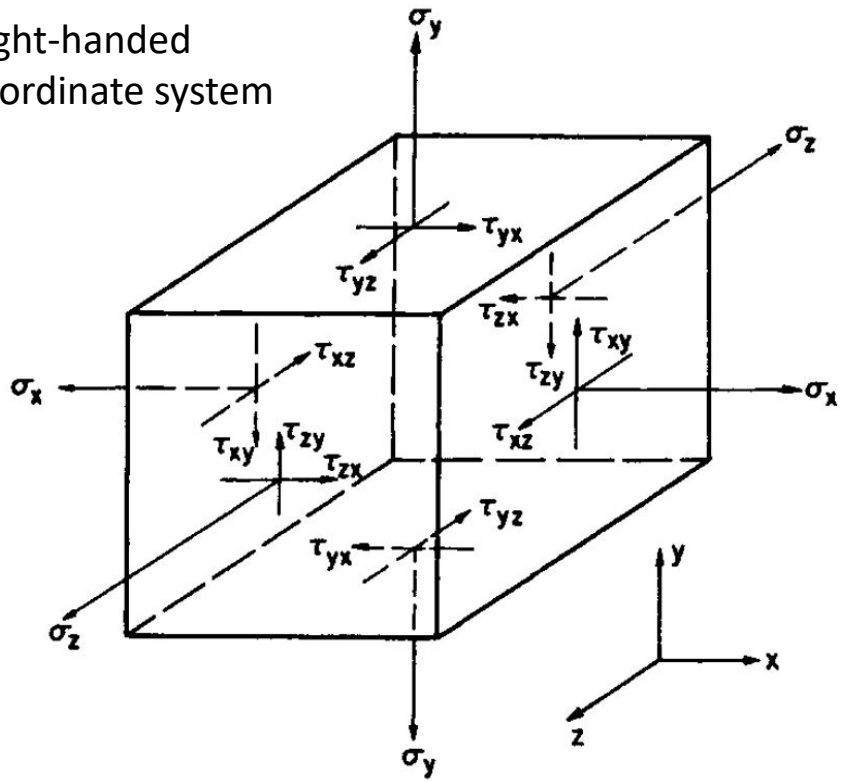
τ_{xy}, τ_{xz}, τ_{yx}, τ_{yz}, τ_{zx}, τ_{zy}

normal stress:

σ_{xx} (σ_x), σ_{yy} (σ_y), σ_{zz} (σ_z)

Body and Surface Forces, Stress

Right-handed coordinate system



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on negative surfaces: stress acting in negative coordinate directions are positive.

- Extension stress is positive and compression negative
- Opposite directions for shear stress on positive and negative planes.

Review

- External vs internal force; Body force vs surface force
- **stress vector** at a point
  - Force per unit area at a single point (Pa=N/m^2)
  - magnitude, direction, on which plane the stress vector acts on
- Stress state at a point
  - stress vectors for all the planes through the point
- Right-handed coordinate systems, sign conventions
- decomposition of the stress vector ***p***
  - normal stress σ and shear stresses τ
  - along x, y, z axes (p_x, p_y)
  - meaning of τ_{xy}, τ_{yz}, σ_x