

Homework 1

Due Sep 23 2021

1. State one scientific or engineering problem that is suitable or unsuitable for the continuum approach. Articulate your reasons.
2. Let δ_{ij} and ε_{ijk} be the Kronecker delta and permutation symbols, respectively. Show the following.

(a) $\delta_{ii} = 3$.

(b) $\delta_{im}T_{mj} = T_{ij}$.

(c) $\varepsilon_{ijk}\varepsilon_{pjk} = 2\delta_{pi}$.

(d) $\varepsilon_{ijk}\varepsilon_{ijk} = 6$.

3. The Cartesian components of a tensor and a vector are given as

$$[\mathbf{A}] = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 1 & 2 \\ 3 & -2 & 3 \end{bmatrix}, \quad [\mathbf{a}] = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

- (a) Calculate A_{ii} , $A_{ij}A_{ji}$, and $A_{ij}a_ia_j$.
 - (b) Decompose the tensor into a symmetric \mathbf{S}^A and an antisymmetric part \mathbf{W}^A .
 - (c) Find the dual vector $\boldsymbol{\omega}^A$ for the antisymmetric part \mathbf{W}^A .
 - (d) Verify that $\mathbf{W}^A\mathbf{a} = \boldsymbol{\omega}^A \times \mathbf{a}$.
4. Show that the contraction of a symmetric tensor \mathbf{S} and an antisymmetric tensor \mathbf{W} is zero, that is, $\mathbf{S} : \mathbf{W} = 0$.
 5. Let a tensor \mathbf{A} be given by

$$\mathbf{A} = 2(\mathbf{I} - \mathbf{e}_1 \otimes \mathbf{e}_1) + 3(\mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1).$$

Determine the eigenvalues and eigenvectors of it.

6. $\mathbf{u}(\mathbf{x}) = x_1x_2x_3\mathbf{e}_1 + x_1x_2\mathbf{e}_2 + x_1\mathbf{e}_3$,

(a) Determine $\text{div}\mathbf{u}$, $\text{curl}\mathbf{u}$, $\text{grad}\mathbf{u}$, and $\nabla^2\mathbf{u}$.

(b) Verify that $\nabla^2\mathbf{u} = \text{grad}(\text{div}\mathbf{u}) - \text{curl}(\text{curl}\mathbf{u})$.