1. (10%) Define the following concepts, explain their physical meanings and give the units: displacement, stress, strain, normal stress, shear stress, normal strain, shear strain, principal stress and principal strain.

2. (30%)

   (1) Write down the equilibrium equations and explain their physical meaning;

   (2) given the following stress distribution, determine, in the absence of body forces, whether equilibrium exists;

   $$\sigma_x = 3x^2 + 4xy - 8y^2$$
   $$\sigma_y = 2x^2 + xy + 3y^2$$
   $$\tau_{xy} = -\frac{1}{2}x^2 - 6xy - 2y^2$$
   $$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

   (3) calculate the stress tensor at point $P(2,2)$ and draw the stress components on an infinitesimal square, and obtain the principal stresses and their directions using Mohr's circle;

   (4) obtain the direction having the maximum shear stress and the corresponding shear and normal stress magnitudes.

3. (10%) For the following displacements:

$u = k\left(2x + y^2\right),\ v = k\left(x^2 - 3y^2\right),\ w = 0$

where k is constant, determine the strain components and check if they satisfy the compatibility equation for strain.

4. (15%) Write down the generalized Hooke's law in terms of both stress-strain and strain-stress relations, and give the expressions for shear modulus, Lame constant and bulk modulus in terms of Young's modulus and Poisson's ratio. What are the units of these three elastic constants?

5. (15%) Explain the procedures of solving an elasticity problem using stress formulation and displacement formulation, give the corresponding equations in each step if possible, and present as much information as possible.

6. (10%) The normal stresses in the x and y directions at a point in an isotropic and homogeneous material are $\sigma_x = 35\ \text{N/mm}^2$ and $\sigma_y = 25\ \text{N/mm}^2$, respectively. All the strains in the z direction are totally constrained. Assuming that the Young's modulus $E = 2.1\times10^5\ \text{N/mm}^2$, Poisson's ratio $\nu = 0.3$, determine σ_z, ε_x and ε_y.

7. (10%) When a small spherical material is subjected to a hydrostatic pressure P = 0.45 N/mm^2, the induced volumetric strain is -3.6×10^{-5}. If the Poisson's ratio of the material is $\nu = 0.3$, determine its Young's modulus.