

# Homework 1

October 11, 2024

## 1 Spatial Velocity

(a) What is the linear velocity of the point C?

$$v_{Cx} = \frac{dC_x(t)}{dt} = v$$

因此,

$$v_C = \begin{bmatrix} -v \\ 0 \\ 0 \end{bmatrix}$$

(b) What is the linear velocity of the point A?

同 (a) 可得,

$$v_A = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$$

(c) What is velocity of the body-fixed point currently coincides with C?

$$v_c = v_o + \omega \times \overrightarrow{roC} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(d) What is velocity of the body-fixed point currently coincides with A?

同 (c) 可得,

$$v_A = v_o + \omega \times \overrightarrow{roA} = \begin{bmatrix} 2v \\ 0 \\ 0 \end{bmatrix}$$

(e) What is the spatial velocity of the cylinder in {0}-frame?

$${}^0V = \begin{bmatrix} {}^0\omega \\ {}^0v \end{bmatrix}$$

其中,

$${}^0\omega = \begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix}, {}^0v = 0 + {}^0\omega \times C_x(t)$$

所以, 可得

$${}^0V = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ -\frac{vC_x(t)}{r} \end{bmatrix}$$

(f) What is the spatial velocity of the cylinder in frame {C}?

同 (e) 可得,

$${}^cV = \begin{bmatrix} {}^c\omega \\ {}^c v \end{bmatrix}$$

, 其中

$${}^C v = v + {}^C \omega \times \overrightarrow{roC}$$

所以, 可得

$${}^0 V = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

## 2 Modern Robotics: Exercise 3.21

(a) 由图可知,  $p_a = P_{ab} + r_a$ , 所以

$$r_a = p_a - P_{ab}$$

其中,  $P_{ab}$  可以由  ${}^A T_B$  得出为  $\begin{bmatrix} -100 \\ 300 \\ 500 \end{bmatrix}$  所以

$$r_a = \begin{bmatrix} 100 \\ 500 \\ -500 \end{bmatrix}$$

又因为

$$r_b = {}^B R_A * r_a$$

$${}^B R_A = ({}^A R_B)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

所以

$$r_b = \begin{bmatrix} 500 \\ -100 \\ -500 \end{bmatrix}$$

(b) 由图可知,

$${}^A T_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 800 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

所以,

$$T_{bc} = {}^B T_C = {}^B T_A * {}^A T_C = ({}^B T_A)^{-1} * {}^A T_C = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 500 \\ -1 & 0 & 0 & -100 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 3 Modern Robotics: Exercise 3.28

(a)

$$w_b = {}^b R_s * w_s = R^{-1} * w_s = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

## 4 Modern Robotics: Exercise 5.5

(a) 由几何关系可知:

$${}^sP = (L + d \sin \theta) \hat{x}_s + (L - d \cos \theta) \hat{y}_s$$

(b) the velocity of point P is :

$$\frac{d {}^sP}{dt} = \begin{bmatrix} d\dot{\theta} \cos \theta \\ d\dot{\theta} \sin \theta \\ 0 \end{bmatrix}$$

(c)

$${}^sT_b = \begin{bmatrix} {}^sR_b & P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & L + d \sin \theta \\ \sin \theta & \cos \theta & 0 & L - d \sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) 因为

$${}^bV = \begin{bmatrix} {}^b\omega \\ {}^bv_{ob} \end{bmatrix}$$

其中,

$${}^bv_{ob} = \omega * \overrightarrow{rob}$$

所以,

$${}^bV = \begin{bmatrix} {}^b\omega \\ {}^bv_{ob} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ d\dot{\theta} \\ 0 \\ 0 \end{bmatrix}$$

(e) 同 (d) 问,

$${}^sV = \begin{bmatrix} {}^s\omega \\ {}^sv_{os} \end{bmatrix}$$

其中,

$${}^sv_{os} = \omega * \overrightarrow{ros}$$

所以,

$${}^sV = \begin{bmatrix} {}^s\omega \\ {}^sv_{os} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ L\dot{\theta} \\ -L\dot{\theta} \\ 0 \end{bmatrix}$$

(f) 可以得出结论

$${}^sV = {}^sX_b * {}^bV$$

其中

$${}^sX_b = {}^sT_b = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

(g) 因为  ${}^bV_p = {}^sR_b^{-1} * {}^sV_p$ , 而  ${}^sR_b$  是  $T_{sb}$  的旋转矩阵, 这是它们之间的关系。

(h) 因为  ${}^sV_{os} = {}^sV_p + {}^s\omega \times \overrightarrow{opos}$

## 5 Modern Robotics: Exercise 5.6

(a) 由题目已知，可以求得

$${}^s\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, {}^sv = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

得到，

$${}^sV = \begin{bmatrix} {}^s\omega \\ {}^sv \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

同理，可以得到

$${}^bV_b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 10 \\ 0 \end{bmatrix}$$

又因为

$${}^bV_s = {}^bX_s {}^sV$$

其中

$${}^bX_s = \begin{bmatrix} R & 0 \\ [p]R & R \end{bmatrix}$$

由图可知，

$${}^bR_s = \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}, p = \begin{bmatrix} -10 \\ 0 \\ 20 \end{bmatrix}$$

所以可以得到

$${}^bV = {}^bV_b + {}^bV_s = \begin{bmatrix} {}^b\omega \\ {}^bv \end{bmatrix} = \begin{bmatrix} \sin t \\ \cos t \\ 1 \\ -20 \cos t \\ 20 \sin t + 10 \\ -10 \cos t \end{bmatrix}$$

Therefore,

$${}^b\omega = \begin{bmatrix} \sin t \\ \cos t \\ 1 \end{bmatrix}, {}^bv = \begin{bmatrix} -20 \cos t \\ 20 \sin t + 10 \\ -10 \cos t \end{bmatrix}$$

(b)

$$\dot{p} = {}^sR_b v_b = \begin{bmatrix} -20 \cos t - 20 \cos t \sin t \\ 10 \cos t \\ 20 \sin t + 10 \sin^2 t - 10 \cos^2 t \end{bmatrix}$$