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Homework 1

1. Calculate people flow rate of a crowded scenic spot is suitable for the continuum approach, because people are squeezed like water flow, which is continuous.

2. (a) $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 1+1+1 = 3.$

(b)
$$\delta_{im} T_{mj} = \delta_{i1} T_{1j} + \delta_{i2} T_{2j} + \delta_{i3} T_{3j}$$
$$= \delta_{11} T_{1j} + \delta_{22} T_{2j} + \delta_{33} T_{3j}$$
$$= T_{1j} + T_{2j} + T_{3j}$$
$$= T_{ij}.$$

(c)
$$\varepsilon_{ijk}\varepsilon_{pjk} = \varepsilon_{ij1}\varepsilon_{pj1} + \varepsilon_{ij2}\varepsilon_{pj2} + \varepsilon_{ij3}\varepsilon_{pj3}$$
$$= (\varepsilon_{i21}\varepsilon_{p21} + \varepsilon_{i31}\varepsilon_{p31}) + (\varepsilon_{i12}\varepsilon_{p12} + \varepsilon_{i32}\varepsilon_{p32}) + (\varepsilon_{i13}\varepsilon_{p13} + \varepsilon_{i23}\varepsilon_{p23}) \quad ①$$

when $i \neq p$, $① = 0 \quad = 2\delta_{pi}$

when $i = p$, $① = 1+1 = 2\delta_{pi}.$

(d)
$$\varepsilon_{ijk}\varepsilon_{ijk} = \varepsilon_{ij1}\varepsilon_{ij1} + \varepsilon_{ij2}\varepsilon_{ij2} + \varepsilon_{ij3}\varepsilon_{ij3}$$
$$= (\varepsilon_{i21}\varepsilon_{i21} + \varepsilon_{i31}\varepsilon_{i31}) + (\varepsilon_{i12}\varepsilon_{i12} + \varepsilon_{i32}\varepsilon_{i32}) + (\varepsilon_{i13}\varepsilon_{i13} + \varepsilon_{i23}\varepsilon_{i23})$$
$$= (1+1) + (1+1) + (1+1) = 6$$

3. $A_{ii} = A_{11} + A_{22} + A_{33} = 1+1+3 = 5.$

$$A_{ij}A_{ji} = A_{i1}A_{1i} + A_{i2}A_{2i} + A_{i3}A_{3i}$$
$$= (A_{11}A_{11} + A_{21}A_{12} + A_{31}A_{13}) + (A_{12}A_{21} + A_{22}A_{22} + A_{32}A_{23}) + (A_{13}A_{31} + A_{23}A_{32} + A_{33}A_{33})$$
$$= (1+0+15) + (0+1-4) + (15-4+9)$$
$$= 33.$$

$$A_{ij}a_i a_j = A_{i1}a_i a_1 + A_{i2}a_i a_2 + A_{i3}a_i a_3$$
$$= (A_{11}a_1 a_1 + A_{21}a_2 a_1 + A_{31}a_3 a_1) + (A_{12}a_1 a_2 + A_{22}a_2 a_2 + A_{32}a_3 a_2)$$
$$+ (A_{13}a_1 a_3 + A_{23}a_2 a_3 + A_{33}a_3 a_3)$$
$$= (9+0+9) + (0+4-4) + (15+4+3)$$
$$= 40.$$

(b) $[\bar{\bar{A}}^T] = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 1 & -2 \\ 5 & 2 & 3 \end{bmatrix}$, $[\bar{\bar{S}}^A] = ([\bar{\bar{A}}] + [\bar{\bar{A}}^T])/2$ $= \begin{bmatrix} 1 & -\frac{1}{2} & 4 \\ -\frac{1}{2} & 1 & 0 \\ 4 & 0 & 3 \end{bmatrix}$ $[\bar{\bar{W}}^A] = ([\bar{\bar{A}}] - [\bar{\bar{A}}^T])/2$ $= \begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}$

(c) $w^A = W^A_{32}\vec{e}_1 + W^A_{13}\vec{e}_2 + W^A_{21}\vec{e}_3 = -2\vec{e}_1 + \vec{e}_2 + \frac{1}{2}\vec{e}_3$.

(d) $\bar{\bar{W}}^A\vec{a} = \begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 & 2 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{7}{2} \\ -7 \end{bmatrix}$

$$w^A \times \vec{a} = (-2\vec{e}_1 + \vec{e}_2 + \tfrac{1}{2}\vec{e}_2) \times (3\vec{e}_1 + 2\vec{e}_2 + \vec{e}_3)$$
$$= (-3\vec{e}_3 + \tfrac{3}{2}\vec{e}_2) + (-4\vec{e}_3 - \vec{e}_1) + (2\vec{e}_2 + \vec{e}_1)$$
$$= \tfrac{7}{2}\vec{e}_2 - 7\vec{e}_3.$$

4. $\bar{\bar{S}} : \bar{\bar{W}} = tr(\bar{\bar{S}}^T\bar{\bar{W}}) = tr(\bar{\bar{S}}\bar{\bar{W}}) = tr(\frac{1}{2}(\bar{\bar{A}} + \bar{\bar{A}}^T) \cdot \frac{1}{2}(\bar{\bar{A}} - \bar{\bar{A}}^T))$
$= tr(\frac{1}{4}(\bar{\bar{A}}^2 - \bar{\bar{A}}^{T2}))$
$= 0$

5. $[\bar{\bar{A}}] = 2\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) + 3\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

$[A - \lambda I] = \begin{bmatrix} -\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix}$, $|A - \lambda I| = (\lambda^2 - 2\lambda - 9)(2-\lambda) = 0$.

$\therefore \lambda_1 = 2$, $\lambda_2 = 1 + \sqrt{10}$, $\lambda_2 = 1 - \sqrt{10}$.

$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $z_2 = \begin{bmatrix} \frac{3}{1+\sqrt{10}} \\ 1 \\ 0 \end{bmatrix}$, $z_3 = \begin{bmatrix} \frac{3}{1-\sqrt{10}} \\ 1 \\ 0 \end{bmatrix}$.