Part II

8-3

Consider a fourth-order tensor \mathbb{A} , with components A_{ijmn} , and a second-order tensor B, with components B_{kl} :

$$C_{ijk} = w_{ij}u_k, \quad C'_{mnl} = A_{ijmn}w_{ij}B_{kl}u_k = A_{ijmn}B_{kl}w_{ij}u_k = A_{ijmn}B_{kl}C_{ijk},$$

so that the third-order tensor $C_{ijk} = w_{ij}u_k$ can be transformed into $C'_{mnl} = w'_{mn}u'_l$ through sixth-order tensor $A_{ijmn}B_{kl}$.

8-4

$$(AB)_{jl} = A_{jk}B_{kl} \ (AB)_{jl,i} = A_{jk,i}B_{kl} + A_{jk}B_{kl,i} \ (AB)_{jl,ii} = (A_{jk,i}B_{kl} + A_{jk}B_{kl,i})_{,i} = A_{jk,ii}B_{kl} + A_{jk,i}B_{kl,i} + A_{jk,i}B_{kl,i} + A_{jk}B_{kl,ii} \ (AB)_{jl,ii} = B_{kl}A_{jk,ii} + 2A_{jk,i}B_{kl,i} + A_{jk}B_{kl,ii} \ (AB)_{,ii} = AB_{,ii} + 2A_{,i}B_{,i} + BA_{,ii}.$$

8-11

Part I

$$\int_V \phi_{,i} dV = \int_S \mu_i \phi dS, \ \int_V U_{i,i} dV = \int_S \mu_i U_i dS, \ \int_V arepsilon_{ijk} U_{j,i} = \int_S arepsilon_{ijk} \mu_j U_i dS, \ \int_V \mu_k arepsilon_{ijk} U_{j,i} = \int_S U_i dL_i, \ \int_V \phi_{,ii} dV = \int_S \mu_i \phi_{,i} dS.$$

Part II

(a)

$$egin{aligned} \int_{S} m{E} \cdot dm{S} &= rac{Q}{arepsilon_{0}} \ \int_{V}
abla \cdot m{E} dV &= \int_{V} rac{
ho}{arepsilon_{0}} dV \
abla \cdot m{E} &= rac{
ho}{arepsilon_{0}} \ E_{i,i} &= rac{
ho}{arepsilon_{0}} \end{aligned}$$

(b)

$$egin{aligned} \int_{S} m{B} \cdot dm{S} &= 0 \ \int_{V}
abla \cdot m{B} dV &= 0 \
abla \cdot m{B} &= 0 \
abla_{i,i} &= 0 \end{aligned}$$

(c)

$$egin{aligned} \int_L oldsymbol{E} \cdot doldsymbol{l} &= -\int_S \dot{oldsymbol{B}} \cdot doldsymbol{S} \ \int_S oldsymbol{\mu} \cdot
abla imes oldsymbol{E} doldsymbol{S} &= -\int_S \dot{oldsymbol{B}} \cdot doldsymbol{S} \ \int_S
abla imes oldsymbol{E} \cdot doldsymbol{S} &= -\int_S \dot{oldsymbol{B}} \cdot doldsymbol{S} \
abla imes oldsymbol{E} \cdot oldsymbol{B} \cdot doldsymbol{S} \
abla imes oldsymbol{E} \cdot oldsymbol{B} \cdot oldsymbol{B} \cdot oldsymbol{B} \cdot oldsymbol{S} \
abla imes oldsymbol{E} \cdot oldsymbol{B} \cdot oldsymbol{B} \cdot oldsymbol{B} \cdot oldsymbol{S} \
abla imes oldsymbol{E} \cdot oldsymbol{B} \cdot oldsymbo$$

(d)

$$egin{aligned} \int_L oldsymbol{B} \cdot doldsymbol{l} &= -\int_S \mu_0 (oldsymbol{j} + arepsilon_0 \dot{oldsymbol{E}}) \cdot doldsymbol{S} \ \int_S oldsymbol{\mu} \cdot
abla imes oldsymbol{B} doldsymbol{S} &= -\int_S \mu_0 (oldsymbol{j} + arepsilon_0 \dot{oldsymbol{E}}) \cdot doldsymbol{S} \ \int_S
abla imes oldsymbol{B} \cdot doldsymbol{S} &= -\int_S \mu_0 (oldsymbol{j} + arepsilon_0 \dot{oldsymbol{E}}) \cdot doldsymbol{S} \
abla imes oldsymbol{B} &= \mu_0 (oldsymbol{j} + arepsilon_0 \dot{oldsymbol{E}}) \
abla imes oldsymbol{B}_{iji} &= \mu_0 (oldsymbol{j}_k + arepsilon_0 \dot{oldsymbol{E}}_k) \end{aligned}$$

8-2

Consider a second-order transformation tensor a, with components a_{ij} :

$$A_{ijk}=u_iv_jw_k, \quad A'_{mnl}=a_{im}u_ia_{jn}v_ja_{kl}w_k=a_{im}a_{jn}a_{kl}u_iv_jw_k.$$

so that the third-order tensor A_{ijk} can be transformed into A'_{mnl} through sixth-order tensor $a_{im}a_{jn}a_{kl}$.

8-12

Consider the transformation between u_i and u'_i is

$$u_i = b_{ij}u_j',$$

and the transformation between u_i' and u_k'' is

$$u_i' = c_{jk}u_k'',$$

so that

$$u_i=b_{ij}u_j'=b_{ij}c_{jk}u_k''=a_{ik}u_k'',\quad a_{ik}=b_{ij}c_{jk}.$$

The value of u_i can be obtained by u_k'' through a_{ik} , the same as the transformation through $b_{ij}c_{jk}$.

Question 5

$$m{A}m{B} = A_{ij}B_{jk} = egin{bmatrix} 30 & 24 & 18 \ 84 & 69 & 54 \ 138 & 114 & 90 \end{bmatrix} \ m{A}m{B}^{\mathrm{T}} = A_{ij}B_{kj} = egin{bmatrix} 46 & 28 & 10 \ 118 & 73 & 28 \ 190 & 118 & 46 \end{bmatrix} \ m{A}: m{B}^{\mathrm{T}} = A_{ij}B_{ji} = \sum_{i,j=1}^{3} A_{ij}B_{ji} = 189 \ m{A}: m{B}^{\mathrm{T}} = A_{ij}B_{ji} = 189 \ m{A}: m{B} = A_{ij}B_{ij} = 189 \ m{A}: m{B} = A_{ij}B_{ij} = 189 \ \m{A}: m{A}: m{A}:$$

$$m{A}:m{B}=A_{ij}B_{ij}=\sum_{i,j=1}^{3}A_{ij}B_{ij}=165$$

Question 6

$$arepsilon_{ijk}arepsilon_{klm}=(\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}), \quad arepsilon_{ijk}=arepsilon_{klj}.$$

Substitute l=i and m=j, and all m and l are supposed to be converted into i and j, then

$$arepsilon_{ijk}arepsilon_{kij}=arepsilon_{ijk}arepsilon_{ijk}=\delta_{ii}\delta_{jj}-\delta_{ij}\delta_{ij}=9-3=6.$$

$$m{a} imes (m{b} imes m{c}) = arepsilon_{ijk} a_j (arepsilon_{klm} b_l c_m) = arepsilon_{ijk} arepsilon_{klm} a_j b_l c_m = (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) a_j b_l c_m = a_m b_i c_m - a_l b_l c_i = (m{a} \cdot m{c}) m{b} - (m{a} \cdot m{b}) m{c}$$