MAE5009 · Continuum Mechanics B

Homework 2

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2-9

Since,

$$\epsilon_n = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha$$

$$\epsilon_t = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha - \frac{\gamma_{xy}}{2}\sin 2\alpha$$

$$\gamma_{nt} = (\epsilon_y - \epsilon_x)\sin 2\alpha + \gamma_{xy}\cos 2\alpha$$

where$\alpha = 22.5°$

Therefore, we can get

$$\epsilon_n = 0.001$$

$$\epsilon_t = -0.001$$

$$\gamma_{nt} = 0.002$$

2-10

$$\epsilon_{ab} = \epsilon_{x'y'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha$$

where,

$$\alpha = arctan\frac{3}{4} = 36.87°$$

Therefore, we can get

$$\epsilon_{ab} = 0.004296$$

Since,

$$\epsilon_{ab} = \frac{\delta ab}{ab}$$

Therefore,

$$\delta ab = 0.02148''$$

2-6

Since,

$$\tan 2\alpha = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

When α is $45°(\gamma_{xy} = \epsilon_x - \epsilon_y)$, the shear stress is maximum, the direction of the maximum shear strain can be determined by the following formula

$$\gamma_{max} = \sqrt{(\frac{\epsilon_x - \epsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2}$$

2-7

(a) Since,

$$\frac{\delta u}{\delta x} = 2kx, \frac{\delta v}{\delta x} = 2kx, \frac{\delta u}{\delta y} = 2ky, \frac{\delta v}{\delta y} = -6ky$$

we can get

$$A'B' = D'C'\sqrt{(dx + \frac{\delta u}{\delta x}dx)^2 + (\frac{\delta v}{\delta x}dx)^2} = dx\sqrt{20k^2 + 4k + 1}$$

$$A'D' = B'C'\sqrt{(dy + \frac{\delta v}{\delta y}dy)^2 + (\frac{\delta u}{\delta y}dy)^2} = dy\sqrt{40k^2 - 12k + 1}$$

The angular positions of $A'B'$ and $A'D'$ is

$$\theta = \tan\theta = \frac{\delta v}{\delta x} = 2kx = 4 \times 10^{-4}$$

$$-\lambda = -\tan\lambda = \frac{\delta u}{\delta y} = 2ky = 2 \times 10^{-4}$$

(b) The coordinates of point A after the displacement is

$$x' = x + u_A = 2.0005$$

$$y' = 1 + v_A = 1.0001$$

So, the coordinates of point $A' = (2.0005, 1.0001, 0)$

(c)

$$w_z = \frac{1}{2}(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}) = \frac{1}{2}(2kx - 2ky) = 1 \times 10^{-4}$$