Answers for Assignment 2:

If you find mistakes in these answers, please contact us and thank you for your correction.
Due to the calculation accuracy, some answers will be slightly different from your answers after the decimal point.

1.
(a) For six second-order compatibility equations:

$$\begin{cases} \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \\ \frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \\ \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \gamma_{zx}}{\partial z \partial x} \end{cases}$$

$$\begin{cases} 2\frac{\partial^2 \varepsilon_x}{\partial y \partial z} = \frac{\partial}{\partial x}(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}) \\ 2\frac{\partial^2 \varepsilon_y}{\partial z \partial x} = \frac{\partial}{\partial y}(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}) \\ 2\frac{\partial^2 \varepsilon_z}{\partial x \partial y} = \frac{\partial}{\partial z}(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}) \end{cases}$$

For three fourth-order compatibility equations:

$$\begin{cases} \frac{\partial^4 \varepsilon_x}{\partial y^2 \partial z^2} + \frac{\partial^4 \varepsilon_y}{\partial x^2 \partial z^2} = \frac{\partial^4 \gamma_{xy}}{\partial x \partial y \partial z^2} \\ \frac{\partial^4 \varepsilon_y}{\partial x^2 \partial z^2} + \frac{\partial^4 \varepsilon_z}{\partial x^2 \partial y^2} = \frac{\partial^4 \gamma_{yz}}{\partial x^2 \partial y \partial z} \\ \frac{\partial^4 \varepsilon_z}{\partial x^2 \partial y^2} + \frac{\partial^4 \varepsilon_x}{\partial y^2 \partial z^2} = \frac{\partial^4 \gamma_{zx}}{\partial x \partial y^2 \partial z} \end{cases}$$

Or/And:

$$\begin{cases} 2\frac{\partial^4 \varepsilon_x}{\partial y^2 \partial z^2} = \frac{\partial^3}{\partial x \partial y \partial z}(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}) \\ 2\frac{\partial^4 \varepsilon_y}{\partial x^2 \partial z^2} = \frac{\partial^3}{\partial x \partial y \partial z}(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}) \\ 2\frac{\partial^4 \varepsilon_z}{\partial x^2 \partial y^2} = \frac{\partial^3}{\partial x \partial y \partial z}(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}) \end{cases}$$

2.

(a)

$$\begin{cases} L_x = dx + \varepsilon_x \times dx = 1.0002dx \\ L_y = dy + \varepsilon_y \times dy = 0.9994dy \end{cases}$$

$$\theta = 4 \times 10^{-4}\, rad$$
$$\lambda = -2 \times 10^{-4}\, rad$$

(b)

$$A' = (2.0005, 1.0001, 0)$$

(c)

$$\omega_z = 10^{-4}$$

(d)

$$\begin{cases} \varepsilon_{max} = 3 \times 10^{-4} \\ \varepsilon_{min} = -7 \times 10^{-4} \\ \gamma_{max} = 10^{-3} \end{cases}$$

3.

The system of strains is possible.

4.

$$\begin{cases} \varepsilon_n = -9.9985 \times 10^{-4} \\ \varepsilon_t = 9.9985 \times 10^{-4} \\ \gamma_{tn} = 1.9997 \times 10^{-3} \end{cases}$$

5.

$$\Delta L = 0.0215\, cm$$