Homework 9

Due date: ∞

1. What is the fundamental hypothesis of continuum mechanics? Name one example that cannot be modeled by continuum mechanics.

2. Recall the definition of the permutation tensor and calculate $\varepsilon_{ijk}\varepsilon_{ijk}$.

3. Given a vector $\boldsymbol{v}$, calculate $(\nabla \times \boldsymbol{v}) \times \boldsymbol{v}$.

4. Determine if the following can serve as the components of an infinitesimal strain tensor,

$$\begin{bmatrix} 8x^2 & -\frac{y}{2} & \frac{3}{2}x^2 z \\ -\frac{y}{2} & xyz & 0 \\ \frac{3}{2}x^2 z & 0 & x^3 \end{bmatrix}.$$

5. The Cauchy stress in one Cartesian system $\{\boldsymbol{e}_i\}$ takes the following form,

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}.$$

   If one rotate the coordinate system counter-clockwisely by an angle of 45 degrees, determine the components of the Cauchy stress in the new system. How does this rotation of the coordinate system reflect in the Mohr's circle for this stress?

6. In the generalized Hooke's law, what are the physical significance of the following material moduli: Lame's parameters, Young's modulus, Poisson's ratio, shear modulus, and bulk modulus.

7. What is the Voigt's notation and write the generalized Hooke's law using this notation. Also, write the stress-strain relation for plane elasticity using the Voigt's notation.

8. Show that the Airy function

$$\phi = \frac{3P}{4c}\left(xy - \frac{xy^3}{3c^2}\right) + \frac{N}{4c}y^2$$

   solves the following cantilever beam problem as shown in the following figure. Boundary conditions at ends ($x = 0$ and L) should be formulated only in terms of the resultant force, while at $y = \pm c$, the exact pointwise specification should be used.

Figure 1: Problem setting.