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8-11

(a) $\because \nabla\phi = \frac{\partial\phi}{\partial x_i}$

$\therefore \int_V \nabla\phi\, dV = \int_V \frac{\partial\phi}{\partial x_i} dV$, $\int_S n\phi\, dS = \int_S n_i\phi\, dS$

$\therefore \int_V \frac{\partial\phi}{\partial x_i} dV = \int_S n_i\phi\, dS$

(b) $\because \nabla\cdot u = \frac{\partial u_i}{\partial x_i}$

$\therefore \int_V \frac{\partial u_i}{\partial x_i} dV = \int_S n_i u_i\, dS$

(c) $\because \nabla\times u = \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j}$

$\therefore \int_V \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j} dV = \int_S \varepsilon_{ijk} n_j u_k\, dS$

(d) similarly

$\int_S n_i \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j} dS = \int_L u_i\, dL_i$

(e) $\int_V \frac{\partial^2\phi}{\partial x_i^2} dV = \int_S n_i \frac{\partial\phi}{\partial x_i} dS$

Maxwell's equations

(a) Due to theorem of Gauss,

$\oint_S E\cdot dS = \int_V (\nabla\cdot E) dV = \frac{1}{\varepsilon_0}\cdot Q$

$\because Q = \iiint_V \rho\, dV$

$\therefore \nabla\cdot E = \frac{\rho}{\varepsilon_0} \Rightarrow \frac{\partial E_i}{\partial x_i} = \frac{\rho}{\varepsilon_0}$

(b) $\oiint_S B\cdot ds = \int_V (\nabla\cdot B)\,dV = 0$

$\therefore \nabla\cdot B = 0 \Rightarrow \frac{\partial B_i}{\partial x_i} = 0$

(c) $\because\ \oint_L E\cdot dL = -\iint_S \frac{\partial B}{\partial t}ds = \iint_S -\frac{dB}{dt}\,ds = \int_S (\nabla\times E)ds$

$\therefore \nabla\times E = -\frac{\partial B}{\partial t} \Rightarrow \varepsilon_{ijk}\frac{\partial E_k}{\partial x_j} = -\frac{\partial B}{\partial t}$

(d) $\oint_L B\cdot dL = \int_S (\nabla\times B)ds$

$\therefore \nabla\times B = \mu_0(j + \varepsilon_0\frac{\partial E}{\partial t}) \Rightarrow \varepsilon_{ijk}\frac{\partial B_k}{\partial x_j} = \mu_0 j_i + \mu_0\varepsilon_0\frac{\partial E_i}{\partial t}$

8-2

Proof:

$\because u_i' = A_{ip}u_p,\ v_j' = A_{jq}v_q,\ w_l' = A_{lr}w_r$

$\therefore u_i v_j w_l = (A_{ip}u_p)(A_{jq}v_q)(A_{lr}w_r)$

$= A_{ip}A_{jq}A_{lr}(u_p v_q w_r)$

Therefore. $u_i v_j w_l$ follows the transformation rule for the third-order tensor.

8-12

Proof: Assume $u_j' = a_{ji}u_i$

$u_k'' = b_{kj}u_j'$

$u_k'' = c_{ki}u_i$

$\therefore u_k'' = b_{kj}(a_{ji}u_i)$

$\because c_{ki} = \sum_j b_{kj}a_{ji}$

$\therefore u_k'' = b_{kj}a_{ji}u_i$

5. (1) $A_{ij}B_{jk} = \sum_{j}(AB)_{ik} = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix} = AB$

$A_{ij}B_{kj} = \sum_{j}(AB)^T_{ik} = \begin{bmatrix} 46 & 28 & 10 \\ 118 & 73 & 28 \\ 190 & 118 & 40 \end{bmatrix} = AB^T$

(2) $A_{ij}B_{ji} = \sum_{i,j} A_{ij}B_{ji} = 189$

$A_{ij}B_{ij} = \sum_{ij} A_{ij}B_{ij} = 165$

6. proof: $\varepsilon_{ijk}\varepsilon_{ijk} = \sum_{i,j,k} \varepsilon_{ijk}\varepsilon_{ijk}$

According to the: $\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Assume $l=i$, $m=j$, we can get:

$\varepsilon_{ijk}\varepsilon_{ijk} = \delta_{ii}\delta_{jj} - \delta_{ij}\delta_{ij}$ (in 3-dimension ~~3-dimension~~, $\delta_{ii}=3$, $\delta_{ij}=3$)

$= 3 \cdot 3 - 3$

$= 6$

(2) $(b\times c)_j = \varepsilon_{jkl}b_k c_l$

$\therefore (a\times(b\times c))_i = \varepsilon_{imn}a_m(b\times c)_n$

$= \varepsilon_{imn}a_m\varepsilon_{nkl}b_k c_l$

$\because \varepsilon_{imn}\varepsilon_{nkl} = \delta_{ik}\delta_{ml} - \delta_{il}\delta_{mk}$

$\therefore (a\times(b\times c))_i = (\delta_{ik}\delta_{ml} - \delta_{il}\delta_{mk})a_m b_k c_l$

$= \delta_{ik}\delta_{ml}a_m b_k c_l - \delta_{il}\delta_{mk}a_m b_k c_l$

$\because \delta_{ik}\delta_{ml}a_m b_k c_l = a_l b_i c_l = (a\cdot c)b_i$

$\delta_{il}\delta_{ml}a_m b_k c_l = a_k b_k c_i = (a\cdot b)c_i$

$\therefore (a\times(b\times c))_i = (a\cdot c)b_i - (a\cdot b)c_i$

$\therefore a\times(b\times c) = (a\cdot c)b - (a\cdot b)c$