SDM5008 · Advanced Control for Robotics

Homework 3

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4.6

$$
{}^0\hat{S}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -H_1 + H_2 \\ 0 \\ L_1 + L_2 \end{bmatrix}, {}^0\hat{S}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L_1 - L_2 - L_3 + W_1 + W_2 \\ -L_1 - L_2 \\ 0 \end{bmatrix}, {}^0\hat{S}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -H_1 + H_2 \\ 0 \\ L_1 + L_2 \end{bmatrix}, {}^0\hat{S}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L_2 - L_3 + W_1 + W_2 \\ -L_1 - L_2 + W_1 \\ 0 \end{bmatrix},
$$

$$
{}^0\hat{S}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -H_1 + H_2 \\ 0 \\ L_1 + L_2 \end{bmatrix}, {}^0\hat{S}_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -H_1 + H_2 - L_3 \\ 0 \\ L_1 + L_2 \end{bmatrix}, {}^0\hat{S}_7 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -H_1 + H_2 \\ 0 \\ L_1 + L_2 \end{bmatrix}
$$

4.9

the screw axes $\mathcal{S}_i$ in S:

$$
M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0S_1 = \begin{bmatrix} {}^0\omega_1 \\ {}^0v_1 \end{bmatrix}
$$

$$
{}^0\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, {}^0v_1 = v_{\mathtt{q}1} - {}^0\omega_1 \times \mathtt{q}1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^0S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Similarly

$$
{}^0S_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 2L \\ 0 \end{bmatrix}, {}^0S_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, {}^0S_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, {}^0S_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ L \\ 0 \\ 0 \end{bmatrix}, {}^0S_6 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -3L \\ 0 \\ 0 \end{bmatrix}
$$

the screw axes $\mathcal{B}_i$ in b:

$$
{}^{0}B_1 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ -3L \\ 0 \end{bmatrix}, {}^{0}B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ -3L \end{bmatrix}, {}^{0}B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, {}^{0}B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, {}^{0}B_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -L \end{bmatrix}, {}^{0}B_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

5.8

$$
{}^{0}\bar{S}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, {}^{0}\bar{S}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, {}^{0}\bar{S}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2L \\ 0 \\ 0 \end{bmatrix}
$$

$$
J_s(\theta) = [{}^{0}\bar{S}_1 \vdots Ad_{\hat{T}_1} {}^{0}\bar{S}_2 \vdots Ad_{\hat{T}_2} {}^{0}\bar{S}_3]
$$

$$
\hat{T}_1 = e^{[{}^{0}\bar{S}_1]\theta_1}, \hat{T}_2 = e^{[{}^{0}\bar{S}_1]\theta_1} e^{[{}^{0}\bar{S}_2]\theta_2}
$$

Therefore,

$$
J_s(\theta) = \begin{bmatrix} 0 & 0 & \sin\theta_1 \\ 1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 \\ 0 & 0 & (2L + \theta_2)\cos\theta_1 \\ 0 & 1 & 0 \\ 0 & 0 & -(2L + \theta_2)\sin\theta_1 \end{bmatrix}
$$