

时间变化情况判断。例如某些流动开始是瞬态变化,之后 均匀流与非均匀流:均匀流在任意时刻流场各点速度矢量 (大小和方向) 相同, 即 $v_{i,j} = \partial v_i/\partial x_j = 0$; 非均匀流 有旋流与无旋流 (涡量与涡流) : 涡量 $\omega = curlv =
abla imes v$

ing the advection from the energy conservation equation:
$$\rho c_p \frac{DT}{Dt} = -\frac{\hat{c}q_i}{\hat{c}q_i} + r_q \hat{c}_q + \alpha T \frac{Dp}{Dt} + \rho H \qquad \qquad \rho c_p \frac{\partial T}{\partial t} = -\frac{\hat{c}q_i}{\hat{c}q_i} + \rho H$$

So that, the energy conservation reads

The Fourier's law reads

 $\frac{Q}{\Omega D^3} = f\left(\frac{P}{\rho \Omega^3 D^5}, \frac{\mu}{\rho \Omega D^2}\right)$

点持续注入染料形成脉线。在层流和湍流中脉线表现

遊线是单个流体质点的轨迹, 由 $x_i(t=t_1)=x_{1i}$,

 $dx_i = v_i dt$ 积分得到质点在不同时刻位置。在每个瞬间。

 $\rho c_{\mathrm{p}} \frac{\partial T}{\partial t} \delta V = (-\nabla \cdot \boldsymbol{q} + \rho H) \delta V.$ 热传导与热扩散系数: 热传递有辐射、传导、对流三种方

式,傅里叶定律描述热传导, $q_i = B_{ij}\partial_j T$ (q_i 为热流,T

为温度, B_{ij} 为热导率,常假设为各向同性,此时 $\rho c_{\mathrm{p}} \frac{\partial T}{\partial t} = k \nabla^2 T + \rho H.$

本构方程 (constitutive equation) 是描述物质在受力或其他 外部作用下,其内部应力、应变或其他物理量之间关系的方程。

 $\left(\frac{q_x(x) - q_x(x + \delta x)}{\varepsilon} + \frac{q_y(y) - q_y(y + \delta y)}{\varepsilon} + \frac{q_z(z) - q_z(z + \delta z)}{\varepsilon}\right) \delta x \delta y \delta z + \rho H \delta x \delta y \delta z = -(q_{,x} + q_{,y}) \delta x \delta y \delta z$

 $q_i = -k\partial_i T$),热扩散系数 $\kappa = rac{k}{
ho c_p}$ (c_p 为等压热

调方程

$$\frac{\partial^2 \boldsymbol{\varepsilon}_x}{\partial y^2} + \frac{\partial^2 \boldsymbol{\varepsilon}_y}{\partial x^2} = \frac{\partial^2 \boldsymbol{\gamma}_{xy}}{\partial x \partial y} \begin{cases} T_x^{\mu} = \boldsymbol{\sigma}_x \boldsymbol{\mu}_x + \boldsymbol{\tau}_{yx} \boldsymbol{\mu}_y + \boldsymbol{\tau}_{zx} \boldsymbol{\mu}_z & \boldsymbol{\pi} + \boldsymbol$$

化率等,是推导控制方程的

$$\frac{y}{\partial z^2} + \frac{z}{\partial y^2} = \frac{y}{\partial y}$$
 Boundary condition

quilibrium equations (3)
$$au_{y,j}$$
 +

$$\frac{2}{2\tau_{xy}}\left(\alpha\in\left[0,\pi\right]\right)\ \sigma_{\max}=\sigma_{1}=\frac{\sigma_{x}+\sigma_{y}}{2}+\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{xy}^{2}}\ \frac{\partial^{2}\epsilon_{z}}{\partial x^{2}}+\frac{\partial^{2}\epsilon_{x}}{\partial z^{2}}=\frac{\partial^{2}\gamma_{zx}}{\partial z\partial x}\ \text{ Hooke's law (6)}\ \ \epsilon_{y}=\frac{1+v}{E}\tau_{y}-\frac{v}{E}\delta_{y}\Theta\ \tau_{y}=2G\epsilon_{y}+\lambda\epsilon\delta_{y}\Theta\ \tau_{y}=2G\epsilon_{y}+\lambda\epsilon$$

$$\frac{\partial^2 \gamma_{zx}}{\partial z \partial x}$$
 Hook

Hooke's law (6)
$$\epsilon_{ij}$$

$$\frac{1+v}{2}\tau_{ii} - \frac{v}{2}\delta_{ii}\Theta \qquad \tau_{ij} =$$

$$rac{\partial^2 \gamma_{zx}}{\partial z \partial x}$$
 Hoo

$$\frac{\tau_{ij} - \tau_{ij} - \sigma_{ij}\Theta}{\epsilon_{ij}} = 20$$

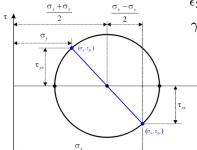
$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$$

$$\frac{\mathbf{v}}{E}\delta_{ij}\Theta$$
 $\mathbf{\tau}_{ij} = 2G\varepsilon_{ij}$

$$\sigma_{v} - \sigma_{w} = \pm \sqrt{\left(\frac{\sigma_{x} - \sigma_{y}}{\sigma_{x} + \sigma_{w}}\right)^{2} + \tau_{w}^{2}}$$
maximum shear stress is
$$\tau_{v \vee mw} = \pm \sqrt{\left(\frac{\sigma_{x} - \sigma_{y}}{\sigma_{x} + \sigma_{w}}\right)^{2} + \tau_{w}^{2}}$$

$$an 2lpha = -rac{\sigma_x - \sigma_y}{2 au_{xy}} \quad lpha \in [0, \pi] \qquad an 2lpha = rac{\pi xy}{\epsilon_x - \epsilon_y}$$
(应力莫尔圆) $rac{\epsilon_1}{\epsilon_2} = rac{\epsilon_x + \epsilon_y}{2} \pm rac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2}$

$$\frac{\tau_{xy'\max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2}}{\text{Mohr's circle of stress (应力莫尔圆)}} + \frac{\epsilon_1}{\epsilon_2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}.$$



$$\gamma_{\mathrm{max}} = \pm \sqrt{\left(\epsilon_x - \epsilon_y\right)^2 + \gamma_{xy}^2}$$
 where $G = \frac{E}{2(1+v)}$ $\lambda = \frac{vE}{(1+v)(1-2v)}$

Unknowns (15)

$$\begin{cases} \varepsilon_x = \frac{1}{E} \left(\sigma_x - v \left(\sigma_y + \sigma_z \right) \right) \\ \varepsilon_y = \frac{1}{E} \left(\sigma_y - v \left(\sigma_z + \sigma_x \right) \right) \\ \varepsilon_z = \frac{1}{E} \left(\sigma_z - v \left(\sigma_x + \sigma_y \right) \right) \end{cases}$$

$$\begin{cases} \gamma_{xy} = \frac{1}{G} \tau_{xy} \\ \gamma_{yz} = \frac{1}{G} \tau_{yz} \end{cases} \longleftrightarrow \begin{cases} \sigma_x = 2G\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y) \\ \sigma_y = 2G\varepsilon_y + \lambda(\varepsilon_x + \varepsilon_y) \\ \sigma_z = 2G\varepsilon_z + \lambda(\varepsilon_x + \varepsilon_y) \end{cases}$$

$$\gamma_{zx} = \frac{1}{G} \tau_{zx}$$

$$\begin{cases} \varepsilon_x = \frac{1}{E} \left(\sigma_x - v \left(\sigma_y + \sigma_z\right)\right) & \begin{cases} \gamma_{xy} = \frac{1}{G} \tau_{xy} & \text{stress expressed by strain} \\ \varepsilon_y = \frac{1}{E} \left(\sigma_y - v \left(\sigma_z + \sigma_x\right)\right) & \begin{cases} \gamma_{xy} = \frac{1}{G} \tau_{xy} & \begin{cases} \sigma_x = 2G \varepsilon_x + \lambda \left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \\ \sigma_y = 2G \varepsilon_y + \lambda \left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{cases} & \begin{cases} \tau_{yz} = G \gamma_{yz} \\ \sigma_z = 2G \varepsilon_z + \lambda \left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{cases} & \begin{cases} \tau_{yz} = G \gamma_{yz} \\ \tau_{zz} = G \gamma_{zz} \end{cases} \\ \begin{cases} \sigma_z = 2G \varepsilon_z + \lambda \left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \end{cases} & \begin{cases} \tau_{yz} = G \gamma_{xz} \\ \tau_{zz} = G \gamma_{zz} \end{cases} \end{cases}$$

 $= \sigma_x a_{11} a_{12} + \sigma_y a_{21} a_{22} + \sigma_z a_{31} a_{32}$ $+ au_{xy}(a_{11} a_{22} + a_{21} a_{12})$ $+ au_{yz}(a_{21} a_{32} + a_{31} a_{22})$ $+ \tau_{zx}(a_{31} a_{12} + a_{11} a_{32})$

 $a_{11}a_{21}a_{31}$

有如下形式 (提示:已知 xyz 坐标系下的应力分量 σ ,则任意平面 μ 上的应力矢量可由公式

Principal stresses

maximum shear stress
$$(\lambda + G) \frac{\partial z}{\partial z}$$
its locator: $\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_1 + \sigma_2 = \sigma_2 = \sigma_3 = \sigma_4 + \sigma_2 = \sigma_4 =$

$$\int_V U_{i,i} dv = \int_S \mu_i U_i dS,$$
 $\int_V arepsilon_{ijk} U_{j,i} = \int_S arepsilon_{ijk} \mu_j U_i dS,$
 $\int_V \mu_k arepsilon_{ijk} U_{j,i} = \int_S U_i dL_i,$

$$m{A}: m{B}^{m{A}} = A_{ij}B_{ji} = \sum_{i,j=1}^{i,j=1} A_{ij}B_{ji}$$
 $m{A}: m{B} = A_{ij}B_{ij} = \sum_{i,j=1}^{3} A_{ij}B_{ij}$
假设原 xyz 坐标系通过 $m{a}$ 变换为任意新 xyz'坐标系,变换乘量定义为:

 $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ $\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + F_{z} = 0$ 2D strain transformation $A_{ijk}=u_iv_jw_k, \quad A'_{mnl}=a_{im}u_ia_{jn}v_ja_{kl}w_k=a_{im}a_{jn}a_{kl}u_iv_jw_k.$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\alpha + \frac{\gamma_{xy}}{2} \sin 2\alpha \quad (\lambda + G)u_{i,i,k} + G\nabla^2 u_k + f_k = 0$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\alpha - \frac{\gamma_{xy}}{2} \sin 2\alpha$$

e momentum equation can be reduced to:
$$\rho \frac{Dv_j}{e} = \rho F_j - \frac{\partial p}{\partial t_j} + \frac{\partial \tau_{jj}}{\partial t_j}$$
 The momentum change of a particle equation

 $\pm \lambda = 2\sqrt{2}b$.

 $p_y = \tau_{xy}a_{11} + \sigma_y a_{21} + \tau_{zy}a_{31}$ $p_z = \tau_{xz}a_{11} + \tau_{yz}a_{21} + \sigma_z a_{31}$ $\sigma_x a_{11} a_{12} + \sigma_y a_{21} a_{22} + \sigma_z a_{31} a_{32}$

 $\gamma_{x'y'} = (\epsilon_y - \epsilon_x) \sin 2\alpha + \gamma_{xy} \cos 2\alpha$

$$rac{\partial
ho}{\partial t} + rac{\partial (
ho v_i)}{\partial x_i} = 0$$
、动量守恒

$$= \tan \theta = \frac{\frac{\partial v}{\partial x} dx}{dx + \frac{\partial u}{\partial x} dx} = \frac{\frac{\partial v}{\partial x}}{1 + \frac{\partial u}{\partial x}} = \frac{\frac{\partial v}{\partial x}}{1 + \frac{\partial$$

共5个变量(3个速度分量,1个压力,1个温度)和5 个方程(1个质量守恒、3个动量守恒、1个能量守恒)。 对于不可压缩流和不可压缩恒温流,方程有相应简化形

热对流条件分析:对于底部加热的流体层(顶部和底部温 缩放 (Scaling)

于扩散热传输速率。扩散时间为 b^2/κ ,对流时间为b/u(对于粘性层流, $u=\Delta
ho g b^2/\eta$, $\Delta
ho$ 为密度差, η 为动力

粘度) ,定义瑞利数 $Ra=rac{
holpha_v(T_1-T_0)gb^3}{}$,当

 $Ra > Ra_c$ ($Ra_c = 700 - 1.1$) 时热对流发生, 且临界 瑞利数与初始温度扰动波长有关,最小临界瑞利数对应波

目的与模型应用:由于构建全尺寸模型进行流体动力学 度分别为 T_1 和 T_0) ,热对流发生条件是对流热传输速率大 $^\circ$ 实验成本高昂,常使用较小模型。需保证缩放模型与原 型之间的几何相似和动力相似,几何相似要求模型与原 $s(\theta)$ 型所有线性长度尺度比例固定(如 $\frac{s(v)}{S(\theta)}$

> 例因子,模型与实际物体表面积比为 α^2 ,体积比为 α^3); 动力相似要求作用在几何相似物体对应位置的力比 例处处相同,即控制流动的无量纲参数相等。

1. 应力基本概念

- **外力与内力**: 外力分为体力(如重力、磁力和惯性力,单位为 N/m^3)和表面力(由物理接触产生)。 内力是物体内部假想平面上的力,且内力一定是表面力。
- **应力的定义**: 应力是单位面积上的力, $p=\lim_{\Delta A o 0} rac{\Delta F}{\Delta A} = rac{dF}{dA}$,应力矢量p与 ΔF 平行,其方向不一定垂

直于表面 ΔA 。要确定一点的应力矢量,需知道其大小、方向和作用平面。

应力状态:一点的应力状态是该点周围所有可能方向上微元面积上的应力矢量的集合。应力矢量可分解为 法向应力 $(\sigma, 垂直于作用平面)$ 和剪应力 $(\tau, 位于平面内)$ 。

应变位移关系 (几何方程)

基本假设: 连续介质假设(材料在介质中每一点都存在且连续)、连续位移假设(位移后材料无间隙)、 单值位移函数假设(变形后质点位置唯一)、小变形假设(位移梯度远小于 1)。

应变叠加原理:在小变形假设下,多个单独应变产生的位移场可叠加得到组合应变的位移,且应变施加顺序 不影响物体最终构形,适用于线性系统。

应力张量可全面描述一点应力状态,其分量确定任意截面应力,对称性降低复杂度

白金汉 π 定理 (Buckingham Π Theorem)

定理内容与应用步骤:对于涉及 N_n 个量纲变量(因变

量 + 参数) 的物理问题,可简化为仅 N_p 个无量纲变量

之间的关系, $N_d = N_{tt} - N_{tt}$ 为描述问题所需独立量纲

化了对固体、液体和气体等物质的力学分析, 使我们能够使用微分方程来描述其行为, 力学中的纳维-斯托克斯方程, 然而, 这一假设在微观尺度或在物质不均匀性显著的 情况下可能失效。

- 要使用分子动力学或统计力学方法。如伯尔兹曼方程、来描述气体行为。 颗粒物质: 如沙子、谷物或粉末等由大颗粒组成的物质 其高散性明显、在研究这些
- 聚粒物质的流动。堆积或传输时,连续介质假设可能无法准确描述其行为。需要采用离 散元方法或其他颗粒动力学模型来分析。 正应变描述材料沿某一方向的相对伸长或压缩, 它表示材料单位长度的长度变化, 定义

其中 ΔL 是长度的变化量 L_0 是初始长度

剪应变描述材料形变时的角度改变,通常由于剪切力引起。它表示材料形状的改变,而不涉及体积的变化、定义为:

其中 θ 是由剪切作用引起的角度改变、角度较小时、可以直接用 θ 近似于 $\tan \theta$