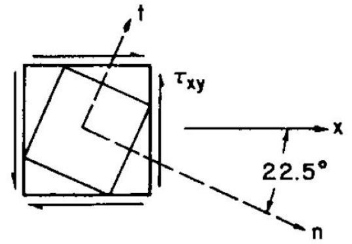
Homework 2 Solution

2-9 Determine ϵ_n, ϵ_t, and γ_{tn} if $\gamma_{xy} = 0.002828$ and $\epsilon_x = \epsilon_y = 0$, for the element shown.



$$\gamma_{xy} = 0.002828, \epsilon_x = \epsilon_y = 0, \alpha = -22.5°$$

According to 2D strain transformation equations:

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha - \frac{\gamma_{xy}}{2}\sin 2\alpha$$

$$\gamma_{x'y'} = (\epsilon_y - \epsilon_x)\sin 2\alpha + \gamma_{xy}\cos 2\alpha$$

Therefore,

$$\begin{aligned} \epsilon_t &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha - \frac{\gamma_{xy}}{2}\sin 2\alpha \\ &= -\frac{0.002828}{2}\sin[2 \times (-22.5°)] \\ &= 9.9985 \times 10^{-4} \end{aligned}$$

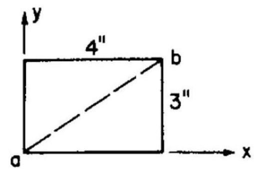
$$\begin{aligned} \epsilon_n &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha \\ &= -9.9985 \times 10^{-4} \end{aligned}$$

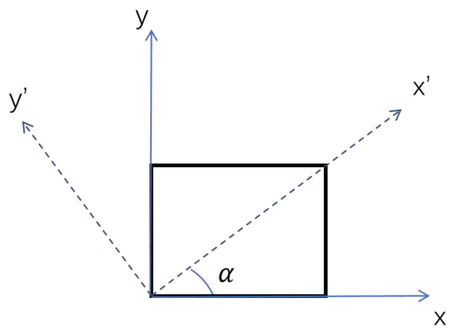
$$\begin{aligned} \gamma_{tn} &= (\epsilon_y - \epsilon_x)\sin 2\alpha + \gamma_{xy}\cos 2\alpha \\ &= 1.9997 \times 10^{-3} \end{aligned}$$

2-10 A thin rectangular plate 3″ by 4″ is acted upon by a stress distribution which results in the uniform strains

$$\epsilon_x = 0.0025, \quad \epsilon_y = 0.0050, \quad \epsilon_z = 0, \quad \gamma_{xy} = 0.001875, \quad \gamma_{xz} = \gamma_{yz} = 0$$

as shown in the figure. Determine the change in length of diagonal ab.





The length change on the diagonal direction should be solved.We can build a new coordinate system $x'Oy'$,with the rotation angle α that

$$\tan\alpha=\frac{3}{4},\cos\alpha=\frac{4}{5},\sin\alpha=\frac{3}{5}$$

$$\begin{aligned}\epsilon_{x'}&=\frac{\epsilon_x+\epsilon_y}{2}+\frac{\epsilon_x-\epsilon_y}{2}\cos2\alpha+\frac{\gamma_{xy}}{2}\sin2\alpha\\&=\frac{0.0025+0.005}{2}+\frac{0.0025-0.005}{2}\times(\cos^2\alpha-\sin^2\alpha)+\frac{0.001875}{2}\cdot2\sin\alpha\cos\alpha\\&=0.0043\end{aligned}$$

$$\Delta L_{ab}=L_{ab}\cdot\epsilon_{x'}=\sqrt{3^2+4^2}\cdot0.0043=0.0215$$

2-6 Derive the equations which define the directions and magnitude of maximum shear strain at a point (two-dimensional). Check the relations by replacing σ by ϵ and τ by $\gamma/2$ in the corresponding stress equations.

The maximum shear strain $\gamma_{x'y'}$ should be derived:

$$\gamma_{x'y'}=(\epsilon_y-\epsilon_x)\sin2\alpha+\gamma_{xy}\cos2\alpha$$

Naturally,

$$\frac{d\gamma_{x'y'}}{d\alpha}=2(\epsilon_y-\epsilon_x)\cos2\alpha-2\gamma_{xy}\sin2\alpha$$

$$\frac{d\gamma_{x'y'}}{d\alpha}=0\Longrightarrow\tan2\alpha=\frac{\epsilon_y-\epsilon_x}{\gamma_{xy}},\alpha\in[0,\pi]$$

$$\sin2\alpha=\pm\frac{\epsilon_y-\epsilon_x}{\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}},\cos2\alpha=\pm\frac{\gamma_{xy}}{\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}}$$

$$\begin{aligned}\gamma_{\max}&=\pm(\epsilon_y-\epsilon_x)\cdot\frac{(\epsilon_y-\epsilon_x)}{\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}}\pm\gamma_{xy}\frac{\gamma_{xy}}{\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}}\\&=\pm\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}\end{aligned}$$

Therefore, the maximum shear strain directions α and magnitude $\gamma_{\max}$ are solved.

Check:by simply replacing σ by ϵ and τ by $\gamma/2$:

$$\tau_{\max}=\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\Longrightarrow\frac{\gamma_{\max}}{2}=\pm\sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

Namely,

$$\gamma_{\max}=\pm\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2}$$

2-7 The following displacement field is applied to a certain body

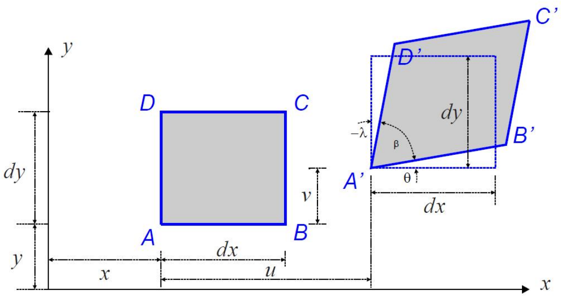
$$u = k(2x + y^2) \qquad v = k(x^2 - 3y^2) \qquad w = 0$$

where $k = 10^{-4}$.

(a) Show the distorted configuration of a two-dimensional element with sides dx and dy and its lower left corner (point A) initially at the point (2, 1, 0), i.e., determine the new length and angular position of each side.

(b) Determine the coordinates of point A after the displacement field is applied.

(c) Find ω_z at this point. $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ represents the average (rigid) rotation of the continuum at point A



Translation and deformation of a 2D element

(a)

$$\frac{\partial u}{\partial x} = 2k,\ \frac{\partial u}{\partial y} = 2ky,\ \frac{\partial v}{\partial x} = 2kx,\ \frac{\partial v}{\partial y} = -6ky$$

$$\begin{aligned} A'B' &= dx + \epsilon_x dx & \qquad A'D' &= dy + \epsilon_y dy \\ &= dx + \frac{\partial u}{\partial x} dx & &= dy + \frac{\partial v}{\partial y} dy \\ &= (1 + 2k)dx & &= dy + (-6ky)dy \\ &= 1.0002dx & &= 0.9994dy \end{aligned}$$

$$\theta = \tan\theta = \frac{\partial v}{\partial x} = 4 \times 10^{-4}$$

$$\lambda = \tan\lambda = -\frac{\partial u}{\partial y} = -2 \times 10^{-4}$$

(b)

$$\begin{aligned} x_{A'} &= x_A + u = 2 + k(2x + y^2) = 2 + 10^{-4}(4 + 1) = 2.0005 \\ y_{A'} &= y_A + v = 1 + k(x^2 - 3y^2) = 1.0001 \\ z_{A'} &= 0 \end{aligned}$$

$$A'(2.0005, 1.0001, 0)$$

(c)

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(2k \times 2 - 2k \times 1) = k = 10^{-4}$$