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1. (a) $\frac{\partial(\rho Q_{lm})}{\partial t} = \rho\frac{\partial Q_{lm}}{\partial t} + Q_{lm}\frac{\partial \rho}{\partial t}$

$\frac{\partial(\rho v_j Q_{lm})}{\partial x_j} = v_j\rho\frac{\partial Q_{lm}}{\partial x_j} + Q_{lm}\frac{\partial(\rho v_j)}{\partial x_j}$

According to: $\frac{DQ_{lm}}{Dt} = \frac{\partial Q_{lm}}{\partial t} + v_j\frac{\partial Q_{lm}}{\partial x_j}$

$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_j)}{\partial x_j} = 0$

$\therefore \frac{\partial(\rho Q_{lm})}{\partial t} + \frac{\partial(\rho v_j Q_{lm})}{\partial x_j} = \rho\left(\frac{\partial Q_{lm}}{\partial t} + v_j\frac{\partial Q_{lm}}{\partial x_j}\right) + Q_{lm}\left(\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_j)}{\partial x_j}\right)$

$= \rho\frac{DQ_{lm}}{Dt}$

(b) $\frac{\partial(\rho v_i)}{\partial t} = v_i\frac{\partial \rho}{\partial t} + \rho\frac{\partial v_i}{\partial t}$

$\frac{\partial(\rho v_j v_i)}{\partial x_j} = v_i v_j\frac{\partial \rho}{\partial x_j} + \rho v_i\frac{\partial v_j}{\partial x_j} + \rho v_j\frac{\partial v_i}{\partial x_j}$

$\because v^2 = v_i v_i$

$\therefore \frac{\partial(\frac{1}{2}\rho v^2)}{\partial t} = \frac{1}{2}\frac{\partial(\rho v_i v_i)}{\partial t}$, $\frac{\partial(\frac{1}{2}\rho v_j v^2)}{\partial x_j} = \frac{1}{2}\frac{\partial(\rho v_j v_i v_i)}{\partial x_j}$

$\therefore$ We can get: $v_i\frac{\partial(\rho v_i}{\partial t} + v_i\frac{\partial(\rho v_i v_j)}{\partial x_j} = \frac{\partial(\frac{1}{2}\rho v^2)}{\partial t} + \frac{\partial(\frac{1}{2}\rho v_j v^2)}{\partial x_j}$

2. According to: $[Q] = \frac{L^3}{T}$

$[p] = \frac{ML^2}{T^3}$

$[D] = L$

$[\Omega] = \frac{1}{T}$

$[\rho] = \frac{M}{L^3}$ $[\mu] = \frac{M}{LT}$

$\therefore \pi_P = \dfrac{P}{\rho\Omega^3 D^5}$, $\pi_\mu = \dfrac{\mu}{\rho\Omega D^2}$

$\therefore \dfrac{Q}{\Omega D^3} = f\left(\dfrac{P}{\rho\Omega^3 D^5}, \dfrac{\mu}{\rho\Omega D^2}\right)$

3. $\rho c_p \dfrac{DT}{Dt} = -\dfrac{\partial q_i}{\partial x_i} + \tau_{ij}\dot{\varepsilon}_{ij} + \alpha T \dfrac{DP}{Dt} + \rho H$

it can be simplify in : $\rho c_p \dfrac{\partial T}{\partial t} = -\dfrac{\partial q_i}{\partial x_i} + \rho H$ $\left(q_i = -k\dfrac{\partial T}{\partial x_i}\right)$

$\because \delta q_x = [q_x(x) - q_x(x+\delta x)]\,\delta y\,\delta z = -\dfrac{\partial q_x}{\partial x}\delta x\,\delta y\,\delta z$.

where $\delta q_y = -\dfrac{\partial q_y}{\partial y}\delta x\,\delta y\,\delta z$

$\delta q_z = -\dfrac{\partial q_z}{\partial z}\delta x\,\delta y\,\delta z$

$\delta q = -\left(\dfrac{\partial q_x}{\partial x} + \dfrac{\partial q_y}{\partial y} + \dfrac{\partial q_z}{\partial z}\right)\delta x\,\delta y\,\delta z$.

$\therefore \delta q = -\nabla \vec{q}\,\delta x\,\delta y\,\delta z$.

$Q_{source} = \rho H\,\delta x\,\delta y\,\delta z$

$Q_{change} = \rho c_p \dfrac{\partial T}{\partial t}\,\delta x\,\delta y\,\delta z = \delta q + Q_{source}$.

$\therefore \rho c_p \dfrac{\partial T}{\partial t}\cdot\delta x\,\delta y\,\delta z = -\nabla\vec{q}\,\delta x\,\delta y\,\delta z + \rho H\,\delta x\,\delta y\,\delta z$

$\Downarrow$

$$\rho c_p \frac{\partial T}{\partial t} = -\nabla\vec{q} + \rho H.$$

$\because \vec{q} = -k\nabla T$

$\therefore \nabla\vec{q} = \nabla(-k\nabla T) = -k\nabla^2 T$

$\therefore$ we have : $\rho c_p \dfrac{\partial T}{\partial t} = k\nabla^2 T + \rho H$.