MAE5009: Continuum Mechanics B

Assignment 05: Notation

Due December 1, 2021

1. Please expand the following Cartesian tensor notations and give final values if possible:

(a) B_{ijj}

$i=1,\ j=1,2,3,\ B_{1jj} = B_{111} + B_{122} + B_{133}$

$i=2,\ j=1,2,3,\ B_{2jj} = B_{211} + B_{222} + B_{233}$

$i=3,\ j=1,2,3,\ B_{3jj} = B_{311} + B_{322} + B_{333}$

$\therefore B_{ijj} = B_{1jj} + B_{2jj} + B_{3jj} =$

$B_{111} + B_{122} + B_{133} + B_{211} + B_{222} + B_{233} + B_{311} + B_{322} + B_{333}$

(b) a_iT_{ij}

$j=1,\ i=1,2,3,\ a_iT_{i1} = a_1T_{11} + a_2T_{21} + a_3T_{31}$

$j=2,\ i=1,2,3,\ a_iT_{i2} = a_1T_{12} + a_2T_{22} + a_3T_{32}$

$j=3,\ i=1,2,3,\ a_iT_{i3} = a_1T_{13} + a_2T_{23} + a_3T_{33}$

$$\begin{bmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$\therefore a_iT_{ij} = a_1T_{11} + a_2T_{21} + a_3T_{31} + a_1T_{12} + a_2T_{22} + a_3T_{32} + a_1T_{13} + a_2T_{23} + a_3T_{33}$

(c) $a_ib_jS_{ij}$

$i=1,\ j=1,2,3,\ a_1b_jS_{1j} = a_1b_1S_{11} + a_1b_2S_{12} + a_1b_3S_{13}$

$i=2,\ j=1,2,3,\ a_2b_jS_{2j} = a_2b_1S_{21} + a_2b_2S_{22} + a_2b_3S_{23}$

$i=3,\ j=1,2,3,\ a_3b_jS_{3j} = a_3b_1S_{31} + a_3b_2S_{32} + a_3b_3S_{33}$

$\therefore a_ib_jS_{ij} =$

$a_1b_1S_{11} + a_1b_2S_{12} + a_1b_3S_{13} + a_2b_1S_{21} + a_2b_2S_{22} + a_2b_3S_{23} + a_3b_1S_{31} + a_3b_2S_{32} + a_3b_3S_{33}$

(d) δ_{ii}

$i=1,\ \delta_{11}=1.\ i=2,\ \delta_{22}=1.\ i=3,\ \delta_{33}=1$

$\therefore\ \delta_{ii}=\delta_{11}+\delta_{22}+\delta_{33}=3$

(e) $\delta_{ij}\delta_{ij}$

$i=j,\ \delta_{ij}\delta_{ij}=1.\ i\neq j,\ \delta_{ij}\delta_{ij}=0$

$i=j=1,\ \delta_{11}\delta_{11}=1.\ i=j=2,\ \delta_{22}\delta_{22}=1.\ i=j=3,\ \delta_{33}\delta_{33}=1$

$\therefore\ \delta_{ij}\delta_{ij}=\delta_{11}\delta_{11}+\delta_{22}\delta_{22}+\delta_{33}\delta_{33}=3$

(f) $\delta_{ij}\delta_{ik}\delta_{jk}$

$i=j,\ \delta_{ij}=1.\ i\neq j,\ \delta_{ij}=0$

$i=j=k=1,\ \delta_{11}\delta_{11}\delta_{11}=1.\ i=j=k=2,\ \delta_{22}\delta_{22}\delta_{22}=1.\ i=j=k=3,\ \delta_{33}\delta_{33}\delta_{33}=1$

$\therefore\ \delta_{ij}\delta_{ik}\delta_{jk}=\delta_{11}\delta_{11}\delta_{11}+\delta_{22}\delta_{22}\delta_{22}+\delta_{33}\delta_{33}\delta_{33}=3$

(g) $\varepsilon_{ijk}\varepsilon_{kij}$

if any two of i, j, k are equal, $\varepsilon_{ijk}=0$

$i=1, j=2, k=3,\ \varepsilon_{123}\varepsilon_{312}=1\times1=1$

$i=1, j=3, k=2,\ \varepsilon_{132}\varepsilon_{213}=(-1)\times(-1)=1$

$i=2, j=1, k=3,\ \varepsilon_{213}\varepsilon_{321}=(-1)\times(-1)=1$

$i=2, j=3, k=1,\ \varepsilon_{231}\varepsilon_{123}=1\times1=1$

$i=3, j=1, k=2,\ \varepsilon_{312}\varepsilon_{231}=1\times1=1$

$i=3, j=2, k=1,\ \varepsilon_{321}\varepsilon_{132}=(-1)\times(-1)=1$

$\therefore\ \varepsilon_{ijk}\varepsilon_{kij}=$

$\varepsilon_{123}\varepsilon_{312}+\varepsilon_{132}\varepsilon_{213}+\varepsilon_{213}\varepsilon_{321}+\varepsilon_{231}\varepsilon_{123}+\varepsilon_{312}\varepsilon_{231}+\varepsilon_{321}\varepsilon_{132}=6$

2. Prove the following:

(a) $\delta_{ik}\varepsilon_{ikm} = 0$

$i=k,\ \delta_{ik}=1.\ \varepsilon_{ikm}=\varepsilon_{iim}=0.\ \therefore\ \delta_{ik}\varepsilon_{ikm}=0$

$i\neq k.\ \delta_{ik}=0.\quad \therefore\ \delta_{ik}\varepsilon_{ikm}=0$

Thus, $\delta_{ik}\varepsilon_{ikm}=0$

(b) $\varepsilon_{ijk}\varepsilon_{ijk} = 6$

if any two of i,j,k are equal. $\varepsilon_{ijk}=0$

$i=1,\ j=2,\ k=3,\ \varepsilon_{123}\varepsilon_{123}=1\times1=1$

$i=1,\ j=3,\ k=2,\ \varepsilon_{132}\varepsilon_{132}=(-1)\times(-1)=1$

$i=2,\ j=1,\ k=3,\ \varepsilon_{213}\varepsilon_{213}=(-1)\times(-1)=1$

$i=2,\ j=3,\ k=1,\ \varepsilon_{231}\varepsilon_{231}=1\times1=1$

$i=3,\ j=1,\ k=2,\ \varepsilon_{312}\varepsilon_{312}=1\times1=1$

$i=3,\ j=2,\ k=1,\ \varepsilon_{321}\varepsilon_{321}=(-1)\times(-1)=1$

$\therefore\ \varepsilon_{ijk}\varepsilon_{ijk}=$

$$\varepsilon_{123}\varepsilon_{123}+\varepsilon_{132}\varepsilon_{132}+\varepsilon_{213}\varepsilon_{213}+\varepsilon_{231}\varepsilon_{231}+\varepsilon_{312}\varepsilon_{312}+\varepsilon_{321}\varepsilon_{321}=6$$

(c) $\varepsilon_{ijk}\varepsilon_{ijp} = 2\delta_{kp}$

if $k=p$. $2\delta_{kp} = 2\delta_{kk} = 2(\delta_{11}+\delta_{22}+\delta_{33}) = 2\times 3 = 6$

$\varepsilon_{ijk}\varepsilon_{ijp} = \varepsilon_{ijk}\varepsilon_{ijk} = 6$

$\therefore \varepsilon_{ijk}\varepsilon_{ijp} = 2\delta_{kp}$

if $k \neq p$, $2\delta_{kp} = 0$

also if any two of ijk, ijp are equal, $\varepsilon_{ijk}=0$ or $\varepsilon_{ijp}=0$

$i=1,\ j=2,\ k=3$

So $p=1$ or 2

Thus $\varepsilon_{ijp}=0$, $\varepsilon_{ijk}\varepsilon_{ijp} = 2\delta_{kp}$

(d) $\delta_{ij}\delta_{jk}\delta_{km} = \delta_{im}$

$i=1$, $\delta_{ij}\delta_{jk} = \delta_{1j}\delta_{jk} = \delta_{11}\delta_{1k} + \delta_{12}\delta_{2k} + \delta_{13}\delta_{3k} = \delta_{1k}$

$i=2$, $\delta_{ij}\delta_{jk} = \delta_{2k}$

$i=3$, $\delta_{ij}\delta_{jk} = \delta_{3k}$

Thus, $\delta_{ij}\delta_{jk} = \delta_{ik}$

$\delta_{ij}\delta_{jk}\delta_{km} = \delta_{ik}\delta_{km}$

With the same way, $\delta_{ik}\delta_{km} = \delta_{im}$

Thus. $\delta_{ij}\delta_{jk}\delta_{km} = \delta_{im}$