A Market Vega Matrix under Local Volatility *

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Abstract

We introduce an algorithm to find a vega matrix on implied volatility surfaces. Instead of directly bumping implied volatility surfaces, we first derive local volatility vega and this can be converted to the market vega matrix. The re-calibarion issue regarding the arbitrage-free condition does not appear in this algorithm. Thus, this may produce better vega matrix by improving locality. The conversion ratio between the market/model vega is the sensitivity of implied volatility with respect to local volatility. Once this ratio matrix is calculated, then this can be cached, i.e., this matrix does not need to be re-calculated for each ELS-vega.

1 The Main Algorithm

To obtain a vega matirx, one would bump a part of the implied volatility surface. However, under the local volatility model, the plain vanilla bumping approach breaks the arbitrage-free conditions. The failure of arbitrage makes the bumping effect propagate to volatility surface areas, the worse it may be even impossible to recalibrate after the bumping. Instead of struggling for the implied volatility, one may be tempted to use the vega matrix of local volatility since bumping of local volatility does not produce any problem. However, the local vega matrix does not tell traders much of anything for hedging. Thus, we will stick to the implied vega matrix and the task will be avoiding directly bumping the implied volatility surface.

To overcome the difficulty, we will introduce a conversion from local to implied (or market) vega matrix. The conversion method was discussed in Chapter 26 in Andersen & Piterbarg (2010) for a different context, interest rate models. The method is still borrowed in a different purpose that is avoiding the recalibration with arbitrage-free conditions. To explain the idea, let V denote the value of a derivative portfolio and σ^{imp}, σ^{loc} denote implied/local volatility respectively. We then formally have:

$$\frac{\partial V}{\partial \sigma^{loc}} = \frac{\partial V}{\partial \sigma^{imp}} \frac{\partial \sigma^{imp}}{\partial \sigma^{loc}}. \tag{1.1}$$

Hence, the implied vega matrix $\partial V / \partial \sigma^{imp}$ can be obtained through the other two terms. The calculation of the two terms do not require any arbitrage-free condition, so the re-calibration issue will not arise.

Note that (1.1) is only a formal representation, since a rank mismatch issue in (1.1) still remains. The local volatility grid is much finer than the bumping grid of implied volatility surface, a change of one grid in σ^{loc} impacts many implied volatility points. For the rank mismatch issue, we will define sensitivities mainly by least-square minimization fashion.

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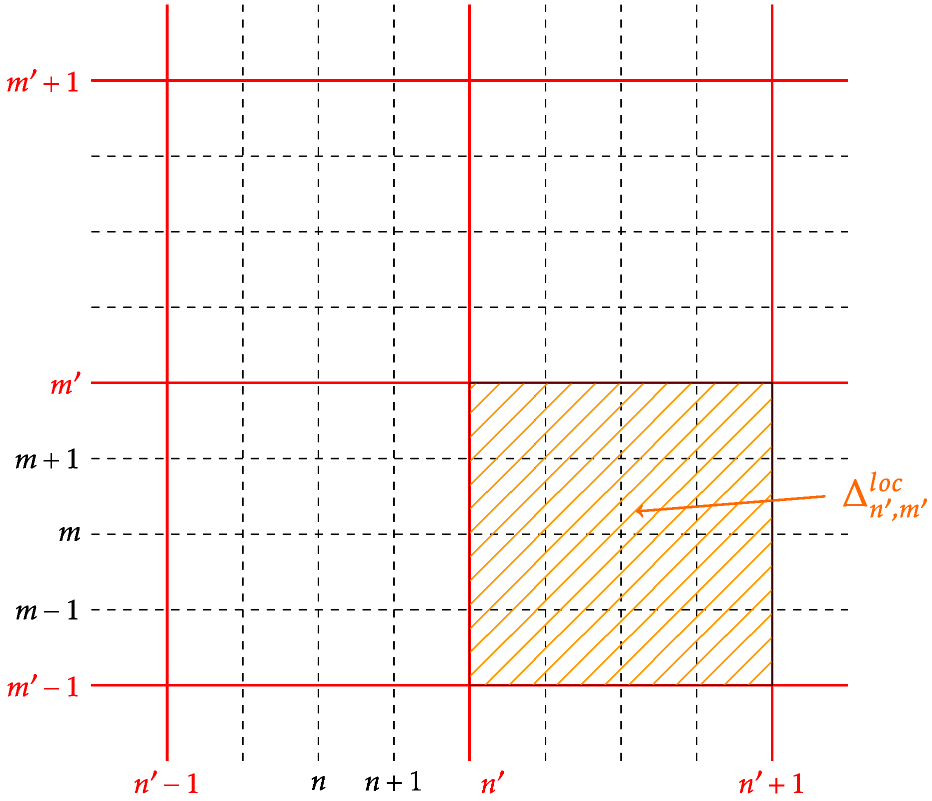

Figure 1: A grid of local volatility and its blocks

Before exmplaining the details, we first introduce some notations. We consider implied/local volatility surfaces $\{\sigma^{imp}_{i,j}\} \in \mathbb{R}^{I\times J}$, $\{\sigma^{loc}_{n,m}\} \in \mathbb{R}^{N\times M}$. Mostly in practice, $I \times J << N \times M$. However, for the purpose of reducing the computational cost in bumping, we can consider a reduced set of block of local volatility, e.g., consider a bumping set of local volatility $\{\Delta^{loc}_{n',m'}\}$, where $n' \le N' < N$, $m' \le M' < M$ such that for some $(k,l) \in \mathbb{N}^2$:

$$\Delta^{loc}_{n',m'} = \begin{cases} 1 \text{ bp}, & k(n'-1) \le n \le kn' \ \ \& \ \ l(m'-1) \le m \le lm' \\ 0, & \text{otherwise.} \end{cases} \tag{1.2}$$

Figure 1 describes (1.2), the grid of local volatility and its contained blocks. For computational efficiency, we can also consider bumping a block of the implied volatility. However, the number of implied volatility points are not overwhelming, so for simplicity of exposition, we consider all points of the implied volatility

We let $\Delta^{i,j}_{n',m'}$ denote the change of the implied volatility at the grid (i,j) with respect to the bumping of (n',m'). Moreover, $\Delta V^{ELS}_{n',m'}$ represents the change of ELS value with respect to the local volatility. Now, by least-square minimization fashion, we define our market vega matrix $\{\mathbb{V}_{i,j}\}$ based on (1.1) as follows:

$$\mathbb{V} := \arg\min_{\{U_{i,j}\}} \sum_{n',m'} \left\{ \frac{\Delta V^{ELS}_{n',m'}}{\Delta^{loc}_{n',m'}} - \sum_{i,j} U_{i,j} \left(\frac{\Delta^{i,j}_{n',m'}}{\Delta^{loc}_{n',m'}} \right) \right\}^2 . \tag{1.3}$$

In (1.3), calculating $\Delta^{i,j}_{n',m'}$ can be demanding. For an efficient algorithm to approximate the implied volatility, readers may refer to Jäckel (2015).

References

Andersen, L. B., & Piterbarg, V. V. (2010). *Interest rate modeling. volume III: Products and risk management.* Atlantic Financial Press.

Jäckel, P. (2015). Let's be rational. *Wilmott*, *2015*(75), 40–53.