Notes on Trade, Macro, Spatial, and IO

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Contents

Chapter A

Spatial Economics

A.1 Spatial Equilibrium

A.1.1 "On the Equilibrium Properties of Spatial Models" (Allen, Arkolakis, and Li, 2024)

A.1.1.1 Setting

Exponential growth in computational power and new micro-data allow the empirical implementation of models where large number of agents (e.g., locations) interact with each other in multiple ways (e.g., spatial linkages).

Consider a system of N locations across which there are H types of interactions whose equilibrium can be reduced to a set of $N \times H$ equations of the following forms:

$$x_{ih} = \sum_{j=1}^{N} f_{ijh}(x_{j1}, x_{j2}, \cdots, x_{jH}) \tag{A.1.1}$$

where $\{x_{ih}\} \in \mathbb{R}_{++}^{N \times H}$ reflect the (strictly positive) equilibrium outcome of each interaction in each locations, and $f_{ijh} : \mathbb{R}_{++}^{H} \to \mathbb{R}_{++}$ are known functions that govern the interactions across locations. In other words, f_{ijh} governs the impact of location j on i's h via H linkages.

A.1.1.2 Overview

This paper provides a new theorem that offers

1. an iterative algorithm for calculating an equilibrium and

2. sufficient and "globally necessary" conditions under which the equilibrium is unique.

In short, this paper approaches the analysis of $N^2 \times H$ functions $\{f_{ijh}\}$ in equation (A.1.1) to a $H \times H$ matrix $(\mathbf{A})_{hh'} \equiv \sup_{ij} \left(| \frac{\partial \ln f_{ijh}}{\partial \ln x_{jh'}} | \right)$ that characterize the strength of the economic interactions.

A.1.1.3 Perov Fixed Point Theorem

Let $\{(X_h, d_h)\}_{h=1,2,\ldots,H}$ be H metric spaces where X_h is a set and d_h is its corresponding metric. Define $X \equiv X_1 \times X_2 \times \ldots \times X_H$, and $d: X \times X \to \mathbb{R}_+^H$ such that for $x = (x_{1,\ldots,}x_H)$, $x' = \left(x'_{1,\ldots,}, x'_H\right) \in X$, $d(x,x') = \begin{pmatrix} d_1(x_1, x'_1) \\ \ldots \\ d_H(x_H, x'_H) \end{pmatrix}$. Given operator $T: X \to X$, suppose for any $x, x' \in X$

$$d\left(T(x), T\left(x'\right)\right) \leq \mathbf{A} d\left(x, x'\right)$$

where $\mathbf{A}$ is a non-negative matrix and the inequality is entry-wise. Denote $\rho(\mathbf{A})$ as the spectral radius of $\mathbf{A}$. If $\rho(\mathbf{A}) < 1$ and for all $h = 1, 2, \ldots, H, (X_h, d_h)$ is complete, there exists a unique fixed point of T, and for any $x \in X$, the sequence of $x, T(x), T(T(x)), \ldots$ converges to the fixed point of T.

A.1.1.4 The Theorem

Table A.1.1: Notations in Allen, Arkolakis, and Li (2024)

Notation	Meaning
f_{ijh}	type h-spatial linkage
x_{ih}	location i's equilibrium outcome h
$\epsilon_{ijh,jh'}(x_j)$	the impact of location j's outcome h' on i's outcome h
$\mathbf{A}$	bounds of the elasticities $\epsilon_{ijh,jh'}(x_j)$
$\rho(\mathbf{A})$	spectral radius of matrix $\mathbf{A}$

NOTES:

Theorem 1 Suppose there exists an H-by-H matrix $\mathbf{A}$ such that for all $i, j \in \mathcal{N}, h, h' \in \mathcal{H}$, and $x_j \in \mathbb{R}_{++}^H, \left|\epsilon_{ijh,jh'}(x_j)\right| \leq (\mathbf{A})_{hh'}$. Then:

(i) If $\rho(\mathbf{A}) < 1$, then there exists a unique solution to equation (1) which can be computed by iteratively applying equation (1) with a rate of convergence $\rho(\mathbf{A})$;

(ii) If $\rho(\mathbf{A}) = 1$ and: a. For all $i \in \mathcal{N}$ and $h, h' \in \mathcal{H}$ when $(\mathbf{A})_{hh'} \neq 0$ there exists some j such that for all $x_j \in \mathbb{R}_{++}^H, \left|\epsilon_{ijh,jh'}(x_j)\right| < (\mathbf{A})_{hh'}$, then equation (1) has at most one solution; b. For all $x_j, \epsilon_{ijh,jh'}(x_j) = \alpha_{hh'} \in \mathbb{R}$ where $|\alpha_{hh'}| = (\mathbf{A})_{hh'}$ for all $i, j \in \mathcal{N}$ and $h, h' \in \mathcal{H}$-i.e. $f_{ijh}(x_j) = K_{ijh} \prod_{h' \in \mathcal{H}} x_{jh'}^{\alpha_{hh'}}$ for some $K_{ijh} > 0$-then there is at most one columnwise up-to-scale solution, i.e. for every two solutions x and x' and $h \in \mathcal{H}$, it must be $x'_{.h} = c_h x_{.h}$ for some scalar $c_h > 0$;

(iii) If $\rho(\mathbf{A}) > 1$ and $N \geq 2H + 1$, then there exists some $\{K_{ijh} > 0\}_{i,j \in \mathcal{N}, h \in \mathcal{H}}$ such that for $f_{ijh}(x_j) = K_{ijh} \prod_{h' \in \mathcal{H}} x_{jh'}^{\alpha_{hh'}}$ where $\alpha_{hh'} \in \mathbb{R}$ and $|\alpha_{hh'}| = (\mathbf{A})_{hh'}$, equation (1) has multiple solutions that are column-wise up-to-scale different, i.e. it has two solutions x and x' such that for some $h \in \mathcal{H}, x'_{.h} \neq c_h x_{.h}$ with every $c_h > 0$.

Proof. Part (i): Notice that equation (A.1.1) can be written as $y_{ih} \equiv \ln x_{ih} = \ln \sum_{j \in \mathcal{N}} f_{ijh} (\exp \ln x_j)$ and furthermore denote its right side as function $g_{ih}(y)$ for matrix y, we thus have:

$$\frac{\partial g_{ih}}{\partial y_{jh'}} = \frac{\epsilon_{ijh,jh'} (\exp y_j) f_{ijh} (\exp y_j)}{\sum_{k \in \mathcal{N}} f_{ikh} (\exp y_j)} \tag{A.1.2}$$

Given any y and y', according to the mean value theorem, for each i and h, there exists $\hat{y} = (1 - t_{ih}) y + t_{ih} y'$ where $t_{ih} \in [0, 1]$ such that:

$$g_{ih}(y) - g_{ih}(y') = \sum_{j \in \mathcal{N}, h' \in \mathcal{H}} \frac{\partial g_{ih}(\hat{y})}{\partial y_{jh'}} \left(y_{jh'} - y'_{jh'}\right) \tag{A.1.3}$$

Equations (A.1.2) and (A.1.3) together with condition $\left|\epsilon_{ijh,jh'} (x_j)\right| \leq (\mathbf{A})_{hh'}$, imply

$$\left|g_{ih}(y) - g_{ih}(y')\right| \leq \sum_{h' \in \mathcal{H}} (\mathbf{A})_{hh'} \max_{j \in \mathcal{N}} \left|y_{jh'} - y'_{jh'}\right| \tag{A.1.4}$$

For any $h \in H$, define metric $d_h (y_h, y'_h) = \max_{j \in \mathcal{N}} \left|y_{jh} - y'_{jh}\right|$ on space $Y_h \equiv \mathbb{R}^N$. Furthermore, define $Y = Y_1 \times \ldots \times Y_H$ and $d(y, y') = [d_1 (y_1, y'_1), \ldots, d_H (y_H, y'_H)]'$ for $y, y' \in Y$. Notice that inequality (A.1.4) then becomes $d(g(y), g(y')) \leqq \mathbf{A} d(y, y')$. Thus we can apply the Perov Fixed Point Theorem to obtain the desired results (existence, uniqueness and computation).

A.1.2 Application to Spatial Models with Input-output Linkages

A.2 Static Models

A.2.1 "Estimates of the Trade and Welfare Effects of NAFTA" (Caliendo and Parro, 2015)

A.3 Dynamic Models

Chapter B

International Trade

B.1 Multinational Production

In this section, I summarize the multinational production literature that explicitly model the role of platform by multinational firms.These flows are important because

1. A large share of foreign affiliates' production is exported to other countries, beyond exports back to their parent countries (Tintelnot, 2017).

2. They also lead to interesting implications for commercial policy and for the welfare gains from trade and multinational production.

The main challenge for modeling export platforms is the ugly corner solutions for production, consumption, and trade. However, it is not surprising that, the same solution that EK (2002) provided for extending the Ricardian trade model to multiple countries is also helpful for the extension of the multinational production models to multiple countries: **A probabilistic structure based on the Frechet distribution.**

$$\Pr(Z_i(j) \leq z) = F_i(z) = \exp\Big(-T_i z^{-\theta}\Big) \tag{B.1.1}$$

where

- $T_i > 0$ governs the **location** of $F_i(z)$. Higher T_i implies higher productivity draw is more likely for any good j. "Absolute advantage."

- $\theta > 1$ determines the dispersion of productivity, where a higher θ means there is less dispersion (common across countries). "Comparative advantage."

B.1.1 "Trade, Multinational Production, and the Gains from Openness" (Ramondo and Rodríguez-Clare, 2013)

Motivation The existing literature on the gains from inter-country interactions studies **trade in goods**, and **MP/FDI** in isolation. The omission of combining these two interactions in analysis is important because trade agreements often combine tariff reductions and removal of barriers to MP.

Table B.1.1: Notations in (Ramondo and Rodríguez-Clare, 2013)

Notation	Meaning
$i = \{1, 2, 3, \cdots, I\}$	country of headquarter (parent)
$l = \{1, 2, 3, \cdots, I\}$	country of production
$n = \{1, 2, 3, \cdots, I\}$	country of destination
$d_{nl} \geq 1$	trade costs
$h_{li}^{g} \geq 1$	MP costs
$v \in [0, 1]$	tradable intermediate goods
$u \in [0, 1]$	non-tradable final goods

NOTES: $h_{li}^{g} \geq 1$ implies that home production is more efficient that those of foreign affiliates.

Some Notations

B.1.1.1 Model

MP Cost Unit cost of the MP input bundle for MP by i in l:

$$c_{li} = \left[(1-a)(c_l h_{li})^{1-\xi} + (a)(c_i d_{li})^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{B.1.2}$$

Productivity Distributions For a home country, she faces a vector $\mathbf{z}_i^s = (z_{1i}, z_{2i}, \cdots z_{Ii}), s = g, f$ that is drawn independently across goods and countries from a Multivariate Frechet distribution:

$$F_i\left(\mathbf{z}_i^s; T_i\right) = \exp\left[-T_i\left(\sum_{l=1}^{I}\left(z_{li}^s\right)^{\frac{-\theta}{1-\rho}}\right)^{1-\rho}\right] \tag{B.1.3}$$

where ρ governs the correlation of productivity across production locations.

Equilibrium Analysis Final goods are non-tradable, so i must produce them in destination country n to obtain positive market share. Therefore, the price of final good u in n is

$$p_n^f(u) = \min_i \frac{c_{ni}^f}{z_{ni}^f}$$

As $z_{ni}^f \sim F(z)$, the share of expenditure by country n on final goods produced in country n with country i technologies is

$$\pi_{ni}^f = \frac{T_i(c_{ni}^f)^{-\theta}}{\sum_j T_j(c_{nj}^f)^{-\theta}} \tag{B.1.4}$$

Compared to non-tradable final goods, intermediate goods are tradable and can be imported from production countries that might differ from the home countries and destination countries ($i \neq l \neq n$).

The price of intermediate good v in n is

$$p_n^g(v) = \min_{i,l} \frac{c_{ni}^g d_{nl}}{z_{li}^g}, \quad d_{nl} \geq 1$$

where z_{li}^g is home-production technology. As $z_{li}^g \sim F(z)$, the share of expenditures by country n on intermediate goods produced in country l with country i technology is:

$$\pi_{nli}^g = \frac{T_i \left(\tilde{c}_{ni}^g\right)^{-\theta}}{\sum_j T_j \left(\tilde{c}_{nj}^g\right)^{-\theta}} \frac{\left(c_{li}^g d_{nl}\right)^{-\theta/(1-\rho)}}{\sum_k \left(c_{ki}^g d_{nk}\right)^{-\theta/(1-\rho)}} \tag{B.1.5}$$

where $\tilde{c}_{ni}^g = \left(\sum_k \left(c_{ki}^g d_{nk}\right)^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta}$ This expression has a natural interpretation: The first term on the right-hand side is the share of expenditures that country n allocates to intermediate goods produced with country i's technologies independently of the location where they are produced, while the second term on the right-hand side is the share of these goods that are produced in country l.

B.1.2 Calibration

Chapter C

Econometrics

C.1 Shfit-Share Designs

C.1.1 "A Practical Guide to Shift-Share Instruments" (Borusyak, Hull, and Jaravel, 2024)

A recent econometric literature shows two distinct paths for identification with shift-share instruments, leveraging either many exogenous shifts (Borusyak, Hull, and Jaravel, 2022; Adão, Arkolakis, and Esposito, 2019) or exogenous shares (Goldsmith-Pinkham, Sorkin, and Swift, 2020).

This paper presents the core logic of both paths and practical takeaways.

C.1.2 ADH (2003)'s SSIV

The influential China shock paper by ADH constructs an instrumental variable with a shift-share structure:

$$\text{SSIV}_i = \sum_k \text{emp share}_{i,k} \times \text{avg. of growth in Chinese import among non-US countries}_k \tag{C.1.1}$$

where k denotes industry and j denotes commuting zone.

C.1.3 Definition of SSIV

A shift-share structure follows

$$z_i = \sum_{k=1}^{K} \underbrace{s_{ik}}_{\text{Share}} \underbrace{g_k}_{\text{Shift}} \tag{C.1.2}$$

Remarks

- Shifts vary at a different level (e.g. industries) than the unit of analysis (e.g. local labor markets).

- Shares vary across units but are usually predetermined (e.g., employment shares are measured in a pre-period).
- To argue convincingly that SSIV are exogenous, one must explain what properties of the shifts and shares make z_i uncorrelated with ϵ_i (rather than simply stating $\text{Cov}[z_i, \epsilon_i] = 0$).
- $\sum_{k=1}^{K} s_{ik}$ is generally one. For incomplete share see Section C.1.4.

One strategy to ensure that the shift-share instrument z_i is exogenous is to have exogenous shift g_k. The key threat to identification in the exogenous shifts approach is the violation of the following condition: g_k **should be uncorrelated with an average of** ϵ_i **taken across units with weights** s_{ik}**.**

C.1.4 Incomplete Shift Share

In shift-share designs where the exposure shares $s_{i,k}$ do not add up to one, a special control must by included: the sum of shares, $S_i = \sum_k s_{i,k}$. This control remedies the bias arising from the correlation between S_i and the error.

C.1.5 A Checklist for the Shift-Based Approach

1. Thinking about what endogeneity bias is being addressed.
2. Bridge the gap between the observed and ideal shifts. Control for $\sum_k s_{ik} q_k$: shift-share aggregates of the industry-level confounders
3. Include the "incomplete share" control.
4. Lag shares to the beginning of the natural experiment.
5. Report descriptive statistics for shifts, such as mean and std. of z_i and g_k.
6. Implement balance tests for shifts in addition to the instrument.
7. Use correct standard errors.

Chapter D

Computational Economics

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