## Practice questions

1. Let X be an Exponential(λ) random variable. Find the PDF of the random variables for

   (a) $Y = X^2$
   (b) $Z = e^{-\lambda X}$.

2. We define the Manhattan distance between two points $A(x_A, y_A)$ and $B(x_B, y_B)$ in a two-dimensional space as $|x_A - x_B| + |y_A - y_B|$, denoted by $d_M(A, B)$. A point is chosen uniformly at random inside a square with each side of length $\sqrt{2}$. The vertices of the square are on the x or y axis and the centre of the square is at the origin (see the figure below). Let X be the Manhattan distance from the point to the centre of the square (the origin O).
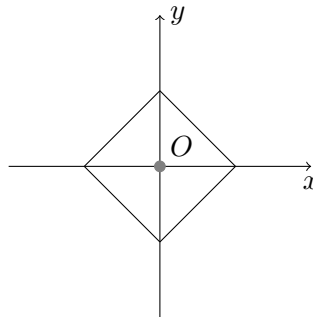
   (a) What is the CDF of X
   (b) What is the PDF of X
   (c) What is the expected value of X
   (d) What is the variance of X

3. The arrival times of the 200 ENGG 2760A / ESTR 2018 students to class are normal random variables with a mean value of 2:25pm and a **standard deviation** of 5 minutes.

   (**Note:** You can use the computer software or calculator to obtain the CDF of the normal random variable. (e.g. Python function `scipy.stats.norm.cdf`))

   (a) What is the expected number of students that have arrived by 2:30pm?
   (b) Assuming students' arrivals are independent, what is the probability that everyone has made it by 2:45pm?

4. The body temperatures of a healthy person and an infected person are Normal($36.8, 0.5$) and Normal($37.8, 1.0$) random variables, respectively. About 1% of the population is infected. What is the conditional probability that I am infected given that my temperature is t? For which values of t am I more likely to be infected than not? (**Hint:** Bayes' rule)

5. Bob's arrival time at a meeting with Alice is X hours past noon (noon is 12:00), where X is a random variable with PDF

$$f(x) = \begin{cases} cx + \frac{1}{2}, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) What is the probability that Bob arrives by 12.30?

(c) What is the expected hour of Bob's arrival?

(d) Given that Bob hasn't arrived by 12.30, what is the probability that he arrives by 12.45?

(e) Given that Bob hasn't arrived by 12.30, what is the expected hour of Bob's arrival?

Additional ESTR 2018 questions

6. Raindrops hit your head at a rate of 1 per second. What is the PDF of the time at which the second raindrop hits you? How about the third one? (**Hint:** convolution)