

Practice questions

1. Bob tosses three fair coins. Given that at least one is a head, what is the probability that there are more heads than tails in the final outcome?
2. Alice tosses a six-sided dice, then she tosses R fair coins, where R is roll of the die. Let M be the event that all the coin tosses came out tails, and Y_i be the event that the roll of the die is i . Calculate (a) $P(M \mid Y_3)$ (b) $P(M)$ (c) $P(Y_3 \mid M)$.
3. There are 5 red balls and 2 blue balls. Each ball is randomly placed in one of two bins.
 - (a) Find the probabilities that the first bin contains k balls for $k \in \{0, 1, 2, 3\}$.
 - (b) Suppose the first bin contains 3 balls, what is the probability that they are all red balls?
4. A bag contains three fair coins and four bias coins, and tossing a bias coin results in a head with probability $3/4$. Alice randomly choose a coin and toss them. Suppose she gets a head, what is the probability that Alice gets a fair coin?

Additional ESTR 2018 questions

5. If Alice flips 10 coins and Bob flips 9 coins, what is the probability that Alice gets more heads than Bob? (*Hint*: Use conditioning.)
6. Benford's law is a probability model over the sample space $\{1, 2, \dots, 9\}$ with $P(\{d\}) = \log_{10}(d+1) - \log_{10} d$. It describes the probability of the leading (most significant) digit in real-life numerical data like accounting records. Benford's law predicts, for example, that the leading digit is a 1 about 30% of the time. Test this hypothesis on some data sets of your choice.