

Practice questions

1. There are 5 cards in a box, one of which has the number '1' on it, one has the number '2' on it, and the other three cards have the number '3' on it. Assuming that the probability of each card being drawn is equal, now randomly draw a card from the box. Please calculate: 1) What is the expectation of the number on the card? 2) What is the variance of the number on the card?
2. Roll a fair six-sided dice 100 times. What is the expected number of times T that four consecutive number '3' occur? (For the case where the "consecutive number '3'" is more than four times, we consider it as multiple "four consecutive number '3'" situations. For example, in sequence "1123333345", we think it contains two cases because the "four consecutive number '3'" case can either start at the 4th or 5th place of the sequence.)
3. A box contains 10 red and 8 blue balls. Two balls in the box are drawn randomly without replacement. If the two balls are in different colors, you win \$1; if they are in the same color, you lose \$1. Suppose you have \$0 at the beginning. Calculate the following problems after you draw two balls from the box (The money in your hand can be negative):
 - (a) Calculate the probability that you have more than \$0.
 - (b) What is the expectation of the money you win?
 - (c) What is the variance of the money you win?
4. Roll a fair six-sided dice twice. Let X and Y be the minimum and maximum of the two rolls, respectively. Please calculate the following questions:
 - (a) $P((Y - X) = 3)$.
 - (b) The joint PMF of X and Y and their marginal PMFs.
 - (c) The expected value of $X + Y$.
5. Throw 50 balls to 30 bins. Each ball is equally likely to land in any of the bins and the throw of each ball is independent. (If the value of final result is hard to calculate, you can write an expression equivalent to the result without calculating the value. For example, you can write $\sum_{i=1}^{50} i$ instead of its final value 5050.)
 - (a) What is the probability that there exists at least one empty box?
 - (b) What is the expected number of bins that receive exactly one ball?
 - (c) What is the expected number of balls that are not alone in their bin?
6. Suppose you will have a snack with the probability of 50% on any given day between Monday and Saturday. And you will have a snack on Sunday *if and only if* you didn't have one in any of the previous six days. Please calculate: 1) What is the expected number of times that you will have a snack in one week? 2) What is the variance of the number of times that you will have a snack in one week?

Additional ESTR 2018 questions

7. Eight boys and eight girls are randomly seated at a round table. What is the expected number of boys that are seated between two girls?