## Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

- 1. A coin is tossed 10 times. What is the probability that
  - (a) we get exactly 4 heads?
  - (b) we get at least 3 heads?
- 2. A standard 52-card deck comprises 13 ranks in each of the four suits. Alice draws three cards without replacement. What is the probability that three cards have the same suit?
- 3. A six-sided die is rolled three times.
  - (a) What is the probability that the face values are all different?
  - (b) Which is more likely: the sum is even or the sum is odd?
- 4. Alice, Bob, and Charlie each toss a 6-sided die. What is the probability that Charlie's face value is strictly larger than both Alice's and Bob's?

## Additional ESTR 2018 questions

Feel free to use any means at your disposal (mathematical analysis, computer experiments, material from external sources) to tackle these questions. Sample answers will not be provided. The additional questions can serve as a starting point for the ESTR 2018 final projects.

- 5. A particle sits in one of seven equally spaced slots along a circle. At each step the particle is equally likely to move into one of the two adjacent slots. Let p(t) be the probability that the particle is back to where it started after t steps.
  - (a) Derive a formula for p(t).
  - (b) Which value  $p_{\infty}$  does p(t) should converge to when t goes to infinity?
  - (c) Is p(t) increasing with t, decreasing with t, or neither?
  - (d) What is the smallest t for which  $0.1 \le p(t) \le 0.2$ ? More generally, what can you say about the smallest t for which  $p_{\infty} \varepsilon \le p(t) \le p_{\infty} + \varepsilon$ ?