Practice questions

1. The joint PDF of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} b(2x+y)y, & 0 \le x \le 2, 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate the following questions:

   (a) The value of b.

   (b) The conditional PDF $f_{Y|X}(y|x)$.

2. The joint PDF of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} 6, & x^2 \le y \le x, \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate the following questions:

   (a) What is the marginal PDF $f_X(x)$ and $f_Y(y)$.

   (b) What is the probability that $X < 2Y$.

3. Alice and Bob agree to meet. Alice's arrival time A is uniform between 12:00 and 12:50 and Bob's arrival time B is uniform between 12:30 and 13:00. Let E be the event "Alice and Bob arrive within 15 minutes of one another". Assuming A and B are independent, what is $\mathrm{P}(E)$?

4. Raindrops hit the ground at a rate of 2 drops per second. An observatory has a raindrop sensing equipment. A signal is received by the computer with a maximum delay of 0.5 second after sensing a raindrop, with all delays equally likely. Assume T is the time of the first raindrop and S is the time of signal reception. Find

   (a) The joint PDF of T and S.

   (b) The marginal PDF of S.

   (c) The conditional PDF of T given S.

5. Here is a way to solve Buffon's needle problem without calculus. Recall that an ℓ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.

   (a) Let A be the number of lines that the needle hits. Let B be the number of times that a polygon of perimeter ℓ hits a line. Show that $\mathrm{E}[A] = \mathrm{E}[B]$. (**Hint:** Use linearity of expectation.)

   (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter ℓ hits a line.

   (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (**Hint:** The number of hits is a Bernoulli random variable.)