

Practice questions

1. Alice rolls three 3-sided dice. Calculate the PMFs and the expected values of
 - (a) The maximum of the three rolls.
 - (b) The minimum of the three rolls.
 - (c) The sum of the three rolls.
2. You flip two bias coins. The probabilities of obtaining head for the two coins are $2/3$ and $3/4$ respectively. If they both come out with the same result, you stop. If not, you try again until they do. Let F be the total number of coin flips you performed. For example if the outcome is THHTHH then $F = 6$. If the outcome is THTT then $F = 4$. What is the PMF (probability mass function) of F ?
3. Suppose the number of school bus arriving at the Sir Run Run Shaw Hall in any time interval is a Poisson random variable, with a rate of 1 bus in 5 minutes.
 - (a) What is the probability that no bus arrives in an interval of 30 minutes?
 - (b) What is the probability that there are at least 5 buses in an interval of 10 minutes?
4. You go to a party with 500 guests.
 - (a) What is the probability that exactly one other guest has the same birthday as you? (For simplicity, exclude birthdays on February 29.) (The result should be rounded to 4 decimal places.)
 - (b) Now model the number of other guests that share your birthday as a $\text{Poisson}(\lambda)$ random variable N . What is the rate λ ? What is the probability that N equals 1? (The result should be rounded to 4 decimal places.)

Additional ESTR 2018 questions

5. The *hot hand paradox* is the belief that if your favorite sports team is on a “winning streak” then it is more likely to win the next game. For example, in this sequence of 38 wins and losses

LLLWWWLWLLLLWLLLLLWLLWWLWLLWLLLWLLWLLLWLLWWWWWWWWWWWWLLLLLW

there are 12 consecutive wins. Was the team on a winning streak?

Do you believe in the hot hand paradox? Please write down the mathematical statement and use the record of your favorite sport team to verify your conjecture.