Practice questions

1. A factory has 200 old widgets, and 500 new widgets in stock. We know that 15% of the old widgets are defective, and 5% of the new ones are defective as well. Alice randomly chooses a widget in the factory. Given that the widget turn out to be defective, what is the probability that it is an old widget?

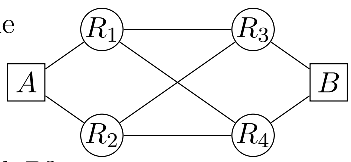
2. Alice usually takes a bus to her company. In summer, it is rainy with probability $\frac{1}{3}$. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{5}$. If it's rainy and there is heavy traffic, Alice arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{10}$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is $\frac{1}{5}$. In a random day in summer:

   (a) What is the probability that it's not raining and there is heavy traffic and Alice is not late?

   (b) What is the probability that Alice is late?

   (c) Given that Alice arrived late at work, what is the probability that it rained that day?

3. Computers A and B are linked through routers R_1 to R_4 as in the picture. Each router fails independently with probability 10%.



   (a) What is the probability there is a connection between A and B?

   (b) Are the events "there is a connection between A and B" and "exactly two routers fail" independent? Justify your answer.

4. In a certain business school, the ratio of the number of full-time students to part-time students is 15:10. At the end of their studies, all the school's 1700 students took a professional examination and 1100 passed. It is known that percentage of the full-time students passing the examination was twice that of the part-time students. A student chosen at random is found to have failed the examination. What is the probability that he was a part-time student?

Additional ESTR 2018 questions

5. Can there be four events E_1, E_2, E_3, E_4 so that every pair E_i, E_j is independent but every triple E_i, E_j, E_k is not (i, j, k are distinct indices)?

   More generally, suppose you are given a set $\mathcal{I}$ consisting of *subsets* of $\{1, \ldots, n\}$. Under which conditions on $\mathcal{I}$ can there exist a sample space Ω and events $E_1, \ldots, E_n$ such that for every set of indices I, the events $E_i \colon i \in I$ are independent when $I \in \mathcal{I}$ and not independent otherwise?