

Practice questions

1. A six person committee is chosen at random from 5 girls and 15 boys. Conditioned on there being at least one girl on the committee, what is the expected number and the variance of boys?
2. Let X and Y be independent random variables with PMFs $P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = 1/4$ and $P(Y = 6) = P(Y = 7) = 1/2$. Let $M = X + Y$ and $N = Y - X$.
 - (a) What is the PMF of M given N ?
 - (b) Are M and N independent? Justify your answer.
 - (c) What is the expectation of M given $N < 2$?
 - (d) What is the expectation of M given $N < 4$?
3. You roll a fair 6-sided die until you get a ordered sequence of $\{3,5,5,1\}$ in 4 recent rounds.
 - (a) What is the probability of you stopping at round n ? Justify you answer.
 - (b) What is the expected value of the number of total rounds?
 - (c) What is the expected value if the number of total rounds is less than or equal to 10?
4. You play 10 rounds of roulette with 1 green, 18 reds, 18 blacks. If you choose green, you may win 3 times of what you bet or lose 3 times of what you bet. For red and black, you may win 1 time of what you bet or lose 1 time of what you bet. You invest \$100 and bet 10% of your balance on each color randomly (with the same probability) in every round. What is your average balance after 10 rounds?
5. Let T be the number of times a 20-sided die is rolled until a 6 appears.
 - (a) What is your average value after 20 rounds?
 - (b) What is the expected value of T conditioned on all rolls producing even numbers?

Additional ESTR 2018 questions

6. Four families consisting of husband, a wife, and a son are randomly seated at a round table. What is the probability that no three members of the same family are seated together?