

### Practice questions

1. Bob tosses three fair coins. Given that at least one is a head, what is the probability that there are more heads than tails in the final outcome?
2. Alice tosses a six-sided dice, then she tosses  $R$  fair coins, where  $R$  is the outcome of the die. Let  $M$  be the event that all the coin tosses came out tails, and  $Y_i$  be the event that the outcome of the die is  $i$ . Calculate (a)  $P(M \mid Y_3)$  (b)  $P(M)$  (c)  $P(Y_3 \mid M)$ .
3. There are 5 red balls and 2 blue balls. Each ball is randomly placed in one of two bins.
  - (a) Find the probabilities that the first bin contains  $k$  balls for  $k \in \{0, 1, 2, 3\}$ .
  - (b) Suppose the first bin contains 3 balls, what is the probability that they are all red balls?
4. A bag contains three fair coins and four bias coins and tossing a bias coin results in a head with probability  $3/4$ . Alice randomly chooses a coin and toss them. Suppose she gets a head, what is the probability that Alice gets a fair coin?

### Additional ESTR 2018 questions

5. If Alice flips 10 coins and Bob flips 9 coins, what is the probability that Alice gets more heads than Bob? (*Hint*: Use conditioning.)
6. Benford's law is a probability model over the sample space  $\{1, 2, \dots, 9\}$  with  $P(\{d\}) = \log_{10}(d+1) - \log_{10} d$ . It describes the probability of the leading (most significant) digit in real-life numerical data like accounting records. Benford's law predicts, for example, that the leading digit is a 1 about 30% of the time. Test this hypothesis on some data sets of your choice.