Coding assignment :

Given data :

- Input data : download
- output data expected by your solution : download

Assignment is to:

[Implementation]

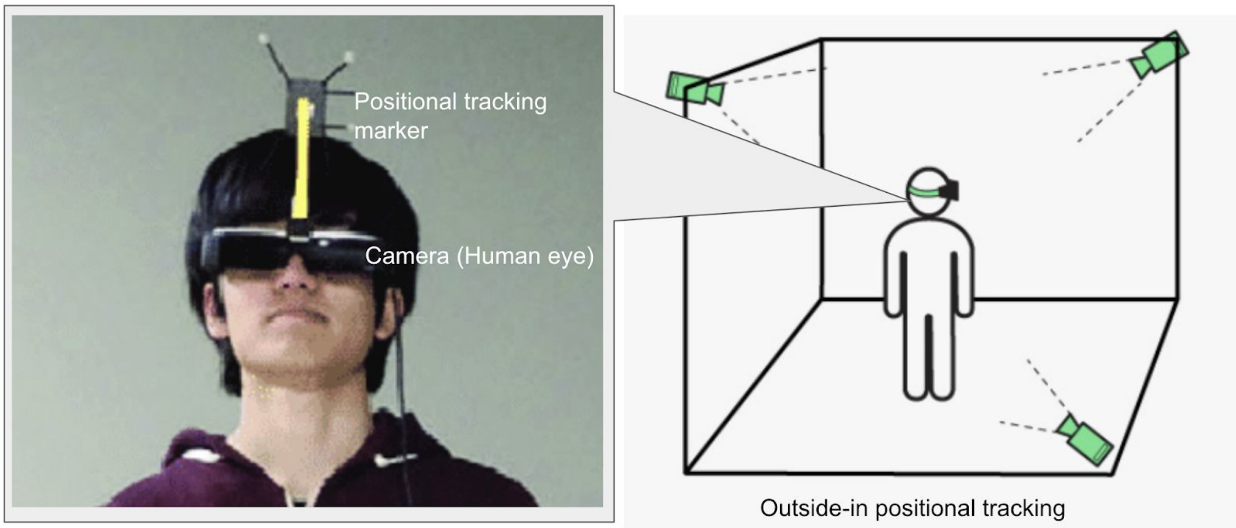
- Find a matrix C.
- Calculate error by comparing with a given output.

[Documentation]

- Report documents

Description :

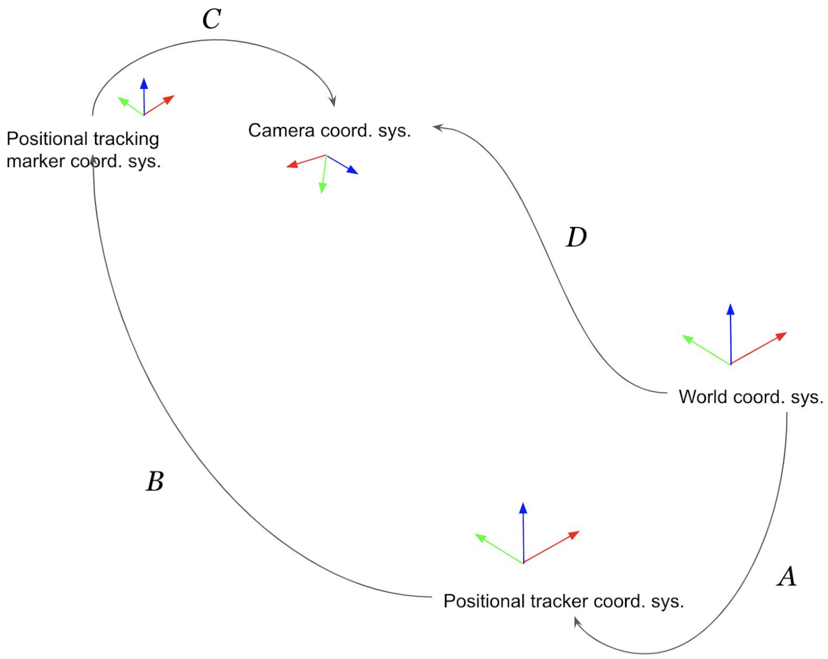
It is important to get camera transformation with respect to world coordinate system in optical see-through head-mounted display as user's head moves.



<Figure 1.>

Given that transform matrix from positional tracking marker to camera coordinate system, C in Figure 2, does not change due to its rigidity, camera coordinate system with respect to world coordinate system can be calculated by Equation 1. In this scenario, user can move its head while collecting sample points data as long as user collects same world coordinate points and its projected image points on its eye.

$$D = CBA \tag{1}$$



<Figure 2.>

Let T_{pose} camera pose in world coordinate system where r_{ij} is a rotation element and t_k is a translation element and T_{proj} camera projection matrix in the image plane coordinate axes.

$$\mathbf{T}_{pose} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\mathbf{T}_{proj} = \begin{bmatrix} f_u & \tau & r_0 & 0 \\ 0 & f_v & c_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

Thus, D in Figure 2 can be calculated by Equation 2 and 3.

$$D = \mathbf{T}_{proj}\mathbf{T}_{pose}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \tag{4}$$

where

$$
\begin{aligned}
a_{11} &= f_u r_{11} + \tau r_{21} + r_0 r_{31} \\
a_{12} &= f_u r_{12} + \tau r_{22} + r_0 r_{32} \\
a_{13} &= f_u r_{13} + \tau r_{23} + r_0 r_{33} \\
a_{14} &= f_u t_1 + \tau t_2 + r_0 t_3 \\
a_{21} &= f_v r_{21} + c_0 r_{31} \\
a_{22} &= f_v r_{22} c_0 r_{32} \\
a_{23} &= f_v r_{23} + c_0 r_{33} \\
a_{24} &= f_v t_2 + c_0 t_3 \\
a_{31} &= r_{31} \\
a_{32} &= r_{32} \\
a_{33} &= r_{33} \\
a_{34} &= t_3
\end{aligned}
$$

Let $P_w = [x_w, y_w, z_w, 1]^T$ homogeneous coordinates in the world coordinate system and $P_I = [u, v, s]^T$ its homogeneous image point.

We can transform world coordinates to marker coordinates by

$$P_M = BAP_w \qquad (5)$$

and can calculate C by using P_M and image coordinate P_I.

Let i^{th} measurement have $P_{M,i} = [x_{M,i}, y_{M,i}, z_{M,i}, 1]^T$ and i^{th} image coordinates $P_{I,i} = [x_i, y_i]^T$. The equation is given by

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} = C \begin{pmatrix} x_{M,i} \\ y_{M,i} \\ z_{M,i} \\ 1 \end{pmatrix} \quad \text{for } i = 1, \cdots, n \qquad (6)$$

where

$$\begin{aligned} x_i &= u_i / w_i \\ y_i &= v_i / w_i \, . \end{aligned} \qquad (7)$$

$$C = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \end{bmatrix} \qquad (8)$$

and we can get equation 9 by equation 6 and 8.

$$\begin{aligned} u_i &= g_{11}x_{M,i} + g_{12}y_{M,i} + g_{13}z_{M,i} + g_{14} \\ v_i &= g_{21}x_{M,i} + g_{22}y_{M,i} + g_{23}z_{M,i} + g_{24} \\ w_i &= g_{31}x_{M,i} + g_{32}y_{M,i} + g_{33}z_{M,i} + g_{34} \end{aligned} \tag{9}$$

By using equation 7, we get

$$\begin{aligned} & x_i(g_{31}x_{M,i} + g_{32}y_{M,i} + g_{33}z_{M,i} + g_{34}) \\ & \quad = g_{11}x_{M,i} + g_{12}y_{M,i} + g_{13}z_{M,i} + g_{14} \\ & y_i(g_{31}x_{M,i} + g_{32}y_{M,i} + g_{33}z_{M,i} + g_{34}) \\ & \quad = g_{21}x_{M,i} + g_{22}y_{M,i} + g_{23}z_{M,i} + g_{24} \end{aligned} \tag{10}$$

This can be rearranged in vector $p = [g_{ij}]^T$ vectorized C (column vector of all elements of C) which can be solved by equation 11.

$$BP = 0 \tag{11}$$

where

$$\mathbf{B} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{M,i} & y_{M,i} & z_{M,i} & 1 & 0 & 0 & 0 & 0 & -x_i x_{M,i} & -x_i y_{M,i} & -x_i z_{M,i} & -x_i \\ 0 & 0 & 0 & 0 & x_{M,i} & y_{M,i} & z_{M,i} & 1 & -y_i x_{M,i} & -y_i y_{M,i} & -y_i z_{M,i} & -y_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{12}$$

The matrix B has $2n$ rows for n data points and 12 columns. We can estimate vector p by minimizing $||Bp||^2$ such that $||p|| = 1$. The solution can be found by finding an eigenvector associated with the smallest eigenvalue. This can be done by finding the singular value decomposition of matrix B given by $B = UDV^T$ and the solution is the column of the matrix V corresponding to the smallest singular value.