# Compilation

T dx = x - other.x; T dy = y - other.y;

```
FLAGS=-g -Wall -Wextra -Wshadow -Wno-unused-result -D_GLIBCXX_DEBUG -fsanitize=address -fsanitize=undefined
\hookrightarrow -fno-sanitize-recover
run_a run_A: clean
        @g++ A.cpp \$(FLAGS) -DJUNCO_DEBUG && ./a.out < z.in</pre>
Template
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
ios::sync_with_stdio(false); cin.tie(nullptr); cout.tie(nullptr);
cout.precision(20); cout << fixed << ans;</pre>
bool is_set(ll x, ll i) {return (x>>i)&1;}
void set_bit(ll &x, ll i) {x |= 111<<i;}</pre>
11 LSB(11 x) {return x & (-x);}
void unset_bit(ll &x, ll i) {x = (x | (111<<i)) ^ (111<<i);}</pre>
stringstream ss;
ss << "Hello world";
while(ss >> s) cout << s << endl;</pre>
ss.clear();
Bitmask
// Iterate over all submasks of a mask. CONSIDER SUBMASK = 0 APART.
for(submask = mask; submask > 0; submask = (submask-1)&mask) {
// With OR and AND you can get XOR.
// (a^b) = (a/b) - (ab).
// (a^b) = (a/b) ^ (ab).
//(a+b) = (a/b) + (a \otimes b).
//(a+b) = (a^b) + 2*(ab).
// Two complement -x = x + 1.
// a^b belongs to [a-b, a+b] and [b-a, a+b].
Point
template<typename T>
class Point {
    public:
    static const int LEFT_TURN = 1;
    static const int RIGHT_TURN = -1;
    T x = 0, y = 0;
    Point() = default;
    Point(T _x, T _y) {
        x = x;
        y = y;
    friend ostream &operator << (ostream &os, Point<T> p) {
        os << "(" << p.x << ", " << p.y << ")";
        return os;
    bool operator == (const Point<T> other) const {
        return x == other.x && y == other.y;
    // Get the (1^{\circ}) bottom (2^{\circ}) left point.
    bool operator < (const Point<T> other) const {
        if(y != other.y) return y < other.y;</pre>
        return x < other.x;
    T euclidean_distance(Point<T> other) {
        T dx = x - other.x;
        T dy = y - other.y;
        return sqrt(dx*dx + dy*dy);
    T euclidean_distance_squared(Point<T> other) {
```

 $\operatorname{time}(\operatorname{NULL})$ 

```
return dx*dx + dy*dy;
}
T manhatan_distance(Point<T> other) {
    return abs(other.x - x) + abs(other.y - y);
// Get the height of the triangle with base b1, b2.
T height_triangle(Point<T> b1, Point<T> b2) {
    if(b1 == b2 || *this == b1 || *this == b2) return 0; // It's not a triangle.
    T a = euclidean_distance(b1);
    T b = b1.euclidean_distance(b2);
    T c = euclidean_distance(b2);
    T d = (c*c-b*b-a*a)/(2*b);
    return sqrt(a*a - d*d);
}
int get_quadrant() {
    if(x > 0 && y >= 0) return 1;
    if(x <= 0 && y > 0) return 2;
    if(x < 0 && y <= 0) return 3;
    if(x >= 0 && y < 0) return 4;
    return 0; // Point (0, 0).
// Relative quadrant respect the point other, not the origin.
int get_relative_quadrant(Point<T> other) {
    Point<T> p(other.x - x, other.y - y);
    return p.get_quadrant();
// Orientation of points *this \rightarrow a \rightarrow b.
int get_orientation(Point<T> a, Point<T> b) {
    T \text{ prod} = (a.x - x)*(b.y - a.y) - (a.y - y)*(b.x - a.x);
    if(prod == 0) return 0;
    return prod > 0? LEFT_TURN : RIGHT_TURN;
// True if a have less angle than b, if *this->a->b is a left turn.
bool angle_cmp(Point<T> a, Point<T> b) {
    if(get_relative_quadrant(a) != get_relative_quadrant(b))
        return get_relative_quadrant(a) < get_relative_quadrant(b);</pre>
    int ori = get_orientation(a, b);
    if(ori == 0) return euclidean_distance_squared(a) < euclidean_distance_squared(b);</pre>
    return ori == LEFT_TURN;
// Anticlockwise sort starting at 1º quadrant, respect to *this point.
void polar_sort(vector<Point<T>> &v) {
    sort(v.begin(), v.end(), [&](Point<T> a, Point<T> b) {return angle_cmp(a, b);});
// Convert v to its convex hull, Do a Graham Scan. O(n \log n).
void convert_convex_hull(vector<Point<T>> &v) {
    if((int)v.size() < 3) return;</pre>
    Point<T> bottom_left = v[0], p2;
    for(auto p : v) bottom_left = min(bottom_left, p);
    bottom_left.polar_sort(v);
    vector<Point<T>> v_input = v; v.clear();
    for(auto p : v_input) {
        while(v.size() >= 2) {
            p2 = v.back(); v.pop_back();
            if(v.back().get_orientation(p2, p) == LEFT_TURN) {
                 v.pb(p2);
                break;
            }
        v.pb(p);
    }
// Constraint: The points have to be in order p0 \rightarrow p1 \rightarrow ... \rightarrow pn, and exist edge pn \rightarrow p0.
static ld get_area_polygon(vector<Point<T>> &v) {
    if(v.size() < 3) return 0;</pre>
    11 i, sum = 0, n = v.size();
    for(i = 0; i < n; i++) {
        sum += v[i].x*v[(i+1)%n].y - v[(i+1)%n].x*v[i].y;
    }
```

```
return abs(sum)/2.0;
    // Rotate p alpha radians (anti clock wise) respect to this point.
    Point<T> rotate(Point<T> p, ld alpha) {
        PointT> q(p.x - x, p.y - y); // p shifted.
        return Point<T>(x + q.x*cos(alpha) - q.y*sin(alpha),
                        y + q.x*sin(alpha) + q.y*cos(alpha));
};
template<typename T>
class Line{
    public:
    bool is_vertical = false; // If vertical, line := x = n.
    T m = 0, n = 0; // y = mx + n.
    Line(T _m, T _n) \{m = _m; n = _n; \}
    Line(Point<T> p1, Point<T> p2) {
        if(p1.x == p2.x) {is_vertical = true; n = p1.x; return;}
        m = (p2.y - p1.y)/(p2.x - p1.x);
        n = m*-p1.x + p1.y;
    friend ostream &operator << (ostream &os, Line<T> 1) {
        if(l.is_vertical) os \ll "x = " \ll l.n;
        else os << "y = " << 1.m << "x + " << 1.n;
        return os:
    Point<T> intersection(Line<T> 1) {
        if(is_vertical && l.is_vertical) return Point<T>(-inf, -inf);
        if(is_vertical) return Point<T>(n, 1.m*n + 1.n);
        if(l.is_vertical) return Point<T>(l.n, m*l.n + n);
        T \text{ new_m} = (1.n-n)/(m-1.m);
        return Point<T>(new_m, m*new_m + n);
};
// Point of intersection of two lines formed by (p1, p2), (p3, p4).
Point<ld> intersection4points(Point<ld> p1, Point<ld> p2, Point<ld> p3, Point<ld> p4) {
    Line<ld> 11(p1, p2), 12(p3, p4);
    return 11.intersection(12);
}
/\!/ Fermat point is the one that minimize the sum to the point to the vertices of a triangle.
pair<Point<ld>, ld> get_fermat_point(Point<ld> p1, Point<ld> p2, Point<ld> p3) {
    Point<ld> fermat = p1, ret;
    ld ans = inf, tans;
    ans = p1.euclidean_distance(p2) + p1.euclidean_distance(p3);
    tans = p2.euclidean_distance(p1) + p2.euclidean_distance(p3);
    if(tans < ans) {ans = tans; fermat = p2;}</pre>
    tans = p3.euclidean_distance(p1) + p3.euclidean_distance(p2);
    if(tans < ans) {ans = tans; fermat = p3;}</pre>
    ret = intersection4points(p3, p2.rotate(p1, PI/3), p1, p2.rotate(p3, PI/3));
    tans = ret.euclidean_distance(p1) + ret.euclidean_distance(p2) + ret.euclidean_distance(p3);
    if(tans < ans) {ans = tans; fermat = ret;}</pre>
    ret = intersection4points(p3, p2.rotate(p1, PI/3), p1, p2.rotate(p3, -PI/3));
    tans = ret.euclidean_distance(p1) + ret.euclidean_distance(p2) + ret.euclidean_distance(p3);
    if(tans < ans) {ans = tans; fermat = ret;}</pre>
    ret = intersection4points(p3, p2.rotate(p1, -PI/3), p1, p2.rotate(p3, PI/3));
    tans = ret.euclidean_distance(p1) + ret.euclidean_distance(p2) + ret.euclidean_distance(p3);
    if(tans < ans) {ans = tans; fermat = ret;}</pre>
    ret = intersection4points(p3, p2.rotate(p1, -PI/3), p1, p2.rotate(p3, -PI/3));
    tans = ret.euclidean_distance(p1) + ret.euclidean_distance(p2) + ret.euclidean_distance(p3);
    if(tans < ans) {ans = tans; fermat = ret;}</pre>
    return mp(fermat, ans);
// Return random float in [0, 1].
ld rand_float() {
    return rand()/(ld)RAND_MAX;
} // Return a point inside a triangle formed by p1, p2, p3.
Point<ld>random_triangle(Point<ld>p1, Point<ld>p2, Point<ld>p3) {
    ld u1 = rand_float(), u2 = rand_float();
    if(u1 + u2 > 1) u1 = 1 - u1, u2 = 1 - u2; // rectangle -> triangle.
    return Point<ld>(p2.x + (p1.x-p2.x)*u1 + (p3.x-p2.x)*u2,
```

```
p2.y + (p1.y-p2.y)*u1 + (p3.y-p2.y)*u2);
```

### **Articulation Point**

```
// v is an AP if removing v from graph it split into more than one component.
// 1- v is the root and v has > 1 child in the DFS.
// 2- v is not the root and has one child u that dont have any back edge.
vector<vi> graph;
class ArticulationPoint{
    int n:
    vi low; // Minimum discover time using a back edge.
    vi discover; // Discover DFS time.
    vi parent;
    int Time = 0;
    void dfs(int u) { // Call dfs(root).
        if(discover[u] != -1) return;
        low[u] = discover[u] = Time++;
        int children = 0;
        for(auto v : graph[u]) {
            if(discover[v] == -1) {
                children++;
                parent[v] = u;
                dfs(v);
                low[u] = min(low[u], low[v]);
                // Every time you set AP[u] = true, the number of components after
                // removing the nodes u or v from the graph increase.
                if(parent[u] == -1 && children > 1) AP[u] = true;
                \label{eq:cover_u} \mbox{if(parent[u] != -1 \&\& low[v] >= discover[u]) AP[u] = true;}
                //if(low[v] > discover[u]) {} // edge u->v is a bridge.
            if(v != parent[u]) low[u] = min(low[u], discover[v]); // Back edge.
        }
    }
    public:
    vector<bool> AP; // True iff i is an Articulation Point.
    ArticulationPoint() {
        n = graph.size();
        low.assign(n, -1);
        discover.assign(n, -1);
        parent.assign(n, -1);
        AP.assign(n, false);
    }
    void get_AP() {
        for(int i = 0; i < n; i++) {
            if(discover[i] == -1) dfs(i);
};
Cycle detection
11 f(11 x) {
    return (x + 1) % 4; // Example.
}
// mu is the first index of the node in the cycle.
// lambda is the length of the cycle.
pll floyd_cycle_detection(11 x0) {
    ll tortoise = f(x0), hare = f(f(x0)), mu = 0, lambda = 1;
    while(tortoise != hare) tortoise = f(tortoise), hare = f(f(hare));
    tortoise = x0:
    while(tortoise != hare) tortoise = f(tortoise), hare = f(hare), mu++;
    hare = f(hare);
    while(tortoise != hare) hare = f(hare), lambda++;
    return mp(mu, lambda);
}
```

```
vector<vi> graph;
                                                            public:
                                                            vi vSorted;
class Toposort{
    vector<bool> visited;
                                                            Toposort(int n) {
    void topo_rec(int u) {
                                                                visited.assign(n, false);
        if(visited[u]) return;
                                                                for(int i = 0; i < n; i++) topo_rec(i);</pre>
                                                                reverse(vSorted.begin(), vSorted.end());
        visited[u] = true;
        for(auto _v : graph[u]) topo_rec(_v);
                                                            }
                                                        };
        vSorted.pb(u);
Flow EdmondKarp
// Edmond Karp implementation of Ford Fulkerson algorithm.
class EdmondKarp{
    vector<vi> graph; // O(V^2) size.
    vector<set<int>>> ady_list; // Convert O(V^2) to O(E).
    int n;
    vi parent;
    bool bfs(int s, int t) {
        vector<bool> visited(n, false);
        queue<int> q;
        q.push(s);
        visited[s] = true;
        while(!q.empty()) {
            int u = q.front(); q.pop();
            for(auto v : ady_list[u]) {
                if(!visited[v] && graph[u][v] > 0) {
                    parent[v] = u;
                    visited[v] = true;
                    if(v == t) return true;
                    q.push(v);
                }
            }
        return false;
    public:
    EdmondKarp(int _n) {
        n = _n;
        graph.assign(n, vi(n, 0));
        ady_list.assign(n, set<int>());
        parent.assign(n, 0);
    }
    void add_edge(int u, int v, int w) { // Add u->v edge.
        graph[u][v] += w; // Multiple edges.
        if(graph[u][v]) ady_list[u].insert(v);
        else ady_list[u].erase(v);
    }
    // Source s to sink t. Complexity O(V*E*E).
    int max_flow(int s, int t) {
        int ans = 0, v;
        int m; // The lowest edge founded in the BFS.
        while(bfs(s, t)) {
            m = INT_MAX;
            for(v = t; v != s; v = parent[v]) {
                m = min(m, graph[parent[v]][v]);
            }
            ans += m;
            for(v = t; v != s; v = parent[v]) {
                add_edge(parent[v], v, -m);
                add_edge(v, parent[v], m);
            }
        }
        return ans;
   }
```

};

## Flow Dinic

```
class Edge {
        public:
         int u, v;
         int cap, flow = 0; // Capacity and current flow.
        Edge(int _u, int _v, int _cap) : u(_u), v(_v), cap(_cap) { }
};
// O(V^2*E). For unit edge capacity O(sqrt(V)*E).
class Dinic{
        vector<Edge> edge;
         vector<vi> graph;
         int n, n_edges = 0;
         int source, sink, inf_flow = INT_MAX;
         vi lvl; // lvl of the node to the source.
         vi ptr; // ptr[u] is the next edge you have to take in order to branch the DFS.
         queue<11> q;
         bool BFS() {
                 while(!q.empty()) {
                          int u = q.front(); q.pop();
                          for(auto el : graph[u]) {
                                   if(lvl[edge[el].v] != -1) continue;
                                   if(edge[el].cap - edge[el].flow <= 0) continue;</pre>
                                   lvl[edge[el].v] = lvl[edge[el].u] + 1;
                                   q.push(edge[el].v);
                          }
                 }
                 return lvl[sink] != -1;
         int dfs(int u, int min_flow) {
                 if(u == sink) return min_flow;
                 int pushed, el;
                  for (;ptr[u] < (int)graph[u].size(); \ ptr[u] ++) \ \{ \ // if \ you \ can \ pick \ ok, \ else \ you \ crop \ that \ edge \ for \ the \ current \ and \ for \ fo
                          el = graph[u][ptr[u]];
                          if(lv1[edge[e1].v] != lv1[edge[e1].u] + 1 || edge[e1].cap - edge[e1].flow <= 0) {
                          pushed = dfs(edge[el].v, min(min_flow, edge[el].cap - edge[el].flow));
                          if(pushed > 0) {
                                   edge[el].flow += pushed;
                                   edge[el^1].flow -= pushed;
                                  return pushed;
                          }
                 }
                 return 0;
        }
        public:
        Dinic(int _n, int _source, int _sink) : n(_n), source(_source), sink(_sink) {
                 graph.assign(_n, vi());
         void add_edge(int u, int v, int flow) { // Add u->v edge.
                 Edge uv(u, v, flow), vu(v, u, 0); // Not multiedge.
                 edge.pb(uv);
                 edge.pb(vu);
                 graph[u].pb(n_edges);
                 graph[v].pb(n_edges+1);
                 n_edges += 2;
         int max_flow() { // It consumes the graph.
                 int flow = 0, pushed;
                 while(true) {
                         lvl.assign(n, -1);
                          lvl[source] = 0;
                         q.push(source);
                          if(!BFS()) break;
                          ptr.assign(n, 0);
                          while(true) {
                                  pushed = dfs(source, inf_flow);
                                   if(!pushed) break;
```

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```
flow += pushed;
            }
        }
        return flow;
    }
};
LCA
const int MAX_N = 1e5+5;
const int MAX_LOG_N = 18;
int parent[MAX_N][MAX_LOG_N]; // Sparse table.
class LCA{ // LCA in O(log n), with O(n log n) preprocess.
    int n;
    vector<vi> graph;
    void dfs_lvl(int u, int p) {
        parent[u][0] = p;
        lvl[u] = lvl[p] + 1;
        for(auto v : graph[u]) {
            if(v == p) continue;
            dfs_lvl(v, u);
        }
   }
   public:
    int lvl[MAX_N];
    LCA() = default;
    LCA(vector<vi> &_graph) {
        int i, j;
        graph = _graph;
        n = graph.size();
        lvl[0] = 0; // The root is 0.
        dfs_lvl(0, 0); // The parent of root is root.
        for(j = 1; j < MAX_LOG_N; j++) {</pre>
            for(i = 0; i < n; i++) {
                parent[i][j] = parent[parent[i][j-1]][j-1];
        }
    }
    int lca(int u, int v) { // O(log n).
        if(lvl[u] > lvl[v]) swap(u, v);
        int i, d = lvl[v] - lvl[u];
        v = get_parent(v, d);
        if(u == v) return u;
        for(i = MAX_LOG_N - 1; i >= 0; i--) {
            if(parent[u][i] != parent[v][i]) {
                u = parent[u][i], v = parent[v][i];
            }
        }
        return parent[u][0];
    }
    int dist(int u, int v) { // distance from u to v O(\log n).
        return lvl[u] + lvl[v] - 2*lvl[lca(u, v)];
    int get_parent(int u, int dst) { // Calculate the dst parent of u.
        dst = max(dst, 0);
        for(int i = 0; i < MAX_LOG_N; i++) {</pre>
            if(is_set(dst, i)) u = parent[u][i];
        return u;
};
SCC Kosaraju
vector<vi> graph;
                                                             vector<bool> visited;
class Kosaraju{ // SCC O(n). x2 times slower than Tarjan.
                                                            void dfs1(int u) {
    vi s; // Stack.
                                                                 visited[u] = true;
    vector<vi> graphT;
                                                                 for(auto v : graph[u])
```

```
if(!visited[v]) dfs1(v);
                                                                 if(!visited[i]) dfs1(i);
    s.pb(u);
                                                            graphT.assign(n, vi());
} // Add to the current component.
                                                            for(i = 0; i < n; i++)
void dfs2(int u) {
                                                                 for(auto v : graph[i])
    visited[u] = true;
                                                                     graphT[v].pb(i);
    for(auto v : graphT[u])
                                                            visited.assign(n, false);
        if(!visited[v]) dfs2(v);
                                                            while(true) {
                                                                 while(!s.empty() && visited[s.back()]) s.pop_back(
    components.back().pb(u);
                                                                 if(s.empty()) break;
                                                                 components.pb(vi());
public:
vector<vi> components;
                                                                 dfs2(s.back());
                                                            }
Kosaraju() {
                                                        }
    int i, n = graph.size();
                                                    };
    visited.assign(n, false);
    for(i = 0; i < n; i++)
```

# Mod operations

```
11 mul(11 a, 11 b) {
    11 ans = 0, neg = (a < 0) \hat{ } (b < 0);
    a = abs(a); b = abs(b);
    while(b) {
        if(b & 1) ans = (ans + a) \% mod;
        b >>= 1;
        a = (a + a) \% mod;
    if(neg) return -ans;
    return ans;
ll elevate(ll a, ll b) { // b >= 0.
    11 \text{ ans} = 1;
    while(b) {
        if (b & 1) ans = ans * a \% mod;
        b >>= 1;
        a = a * a \% mod;
    return ans:
```

# Catalan Numbers // (2nCn)/(n+1).

#### Chinese Remainder

```
class ChineseRemainder{
    11 mul(11 a, 11 b) {
        ll ans = 0, neg = (a < 0) \hat{b} < 0;
        a = abs(a); b = abs(b);
        while(b) {
            if (b & 1) ans = (ans + a)%modulo;
            b >>= 1;
            a = (a + a) \% modulo;
        if(neg) return -ans;
        return ans;
   ll gcdEx(ll a, ll b, ll *x1, ll *y1) {
        if(a == 0) {
           *x1 = 0;
            *y1 = 1;
            return b;
        11 x0, y0, g;
        g = gcdEx(b\%a, a, &x0, &y0);
        *x1 = y0 - (b/a)*x0;
        *y1 = x0;
        return g;
   }
   11 mod(11 x) {
        return ((x%modulo) + modulo)%modulo;
```

```
// ONLY USE WHEN MOD IS PRIME, ELSE USE GCD.
// a^{(mod - 1)} = 1, Euler.
11 inv(11 a) {
    return elevate(((a%mod) + mod)%mod, mod - 2);
}
const int MAX = 1e5 + 10;
//inv_fact is fact^-1
11 fact[MAX], inv_fact[MAX];
void init() {
   int i = 0;
   fact[0] = 1;
   inv_fact[0] = 1;
    for(i = 1; i < MAX; i++) {
        fact[i] = fact[i-1]*i;
        fact[i] %= mod;
        inv_fact[i] = inv(fact[i]);
    }
}
```

 $\operatorname{time}(\operatorname{NULL})$ 

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```
}
    11 n = 0;
    vll c, m; // x == c[i] \mod m[i], m[i] \mod m[i] not need to be coprime.
    void solve_one() { // m[i] <= 1e9</pre>
        11 x, y, g;
        \label{eq:modulo} \mbox{modulo} \ = \ \mbox{m[n-2]*(m[n-1]/\_gcd(m[n-2], m[n-1]));}
        g = gcdEx(m[n-1], m[n-2], &x, &y);
        if((c[n-1]-c[n-2])%g != 0) {n = -1; return;}
        x = c[n-1] + mul(m[n-1], mul(-x, (c[n-1]-c[n-2])/g));
        c.pop_back(); m.pop_back(); n--;
        c[n-1] = mod(x); m[n-1] = modulo;
    }
    public:
    ll modulo;
    void insert(ll _c, ll _m) { // _m > 0.
        n++;
        modulo = _m;
        m.pb(_m);
        c.pb(mod(_c));
    11 solve() {
        while(n > 1) solve_one();
        return n \le 0? -1 : c[0];
};
\mathbf{FFT}
// Fast Fourier Trasnform, to get DFT
\begin{tabular}{ll} // \ {\it Discrete Fourier Transform, point evaluation of} \end{tabular}
// w_n^k (the kth root of unity).
typedef complex<double> cd; // NOT long double, TLE.
class FFT{
    static void convolution(vector<cd> &a) {
        int i = 0, n = a.size(); // n power of 2.
        if(n == 1) return;
        vector<cd> a_even, a_odd;
        for(i = 0; i < n; i++)
             i%2 ? a_odd.pb(a[i]) : a_even.pb(a[i]);
        convolution(a_even);
        convolution(a_odd);
        cd wn = polar((double)1.0, 2*(double)PI/n), w = 1.0;
        for(i = 0; i < n/2; i++) {
             //w = polar(1.0, i*2*(double)PI/n); // Avoids precission error, but slower.
             a[i] = a_even[i] + w*a_odd[i];
             a[i + n/2] = a_{even}[i] - w*a_{odd}[i];
             w = w*wn;
    }
    static void deconvolution(vector<cd> &a) {
        for(auto &el : a) el = conj(el);
        convolution(a);
        for(auto &el : a) el /= (double)a.size();
    public:
    // c[j] is Sum_{0, j} of a[i]*b[j - i].
    static vector<cd> multiply(vector<cd> a, vector<cd> b) {
        int i, n = max(a.size(), b.size());
        if(n == LSB(n)) n--;
        for(i = 30; i >= 0; i--) if(is_set(n, i)) break;
        i += 2; // Next power of two: 3 -> 8.
        n = 1 << i;
        while((int)a.size() < n) a.pb(0.0);</pre>
        while((int)b.size() < n) b.pb(0.0);</pre>
        convolution(a);
        convolution(b); // Don't need to call if a*a.
        vector<cd> ans(n);
        for(i = 0; i < n; i++) ans[i] = a[i]*b[i];
        deconvolution(ans);
```

```
return ans;
    }
    static void show(vector<cd> &v) {
        int i, cont = 0; // Maximum 20 elements.
        for(i = 0; i < min((int)v.size(), 20); i++) {</pre>
             cout << " +" << (v[i].real() > eps ? v[i].real() : 0) << "x^" << cont++;
        } cout << endl;</pre>
};
Formulas
// a/b is truncate function.
ll floor_div(ll a, ll b) {
    return a/b - ((a^b) < 0 \&\& a\%b);
// Sum_{0, n} i = Sum \ of \ i \ in [0, n] = n*(n+1)/2.
11 formula_1(ll _n) {
    return _n*(_n+1)/2;
}
// Sum_{a, b} i = Sum \ of \ i \ in [a, b].
11 formula_2(11 _a, 11 _b) {
    return formula_1(_b) - formula_1(_a-1);
// Sum_{n} i/k = Sum of i/k (floor) in [0, n]. n \ge 0.
11 formula_3(11 _n, 11 _k) {
    11 _{ans} = 0, r = _{n};
    while((r+1)%_k != 0) {_ans += r/_k; r--;}
    return _ans + _k*formula_1(r/_k);
}
// Sum_{0, n} (x + i*d) Arithmetic sum.
11 formula_4(11 x, 11 d, 11 _n) {
    return _n*x + d*formula_1(_n);
}
// Sum_{0, n} i^2 = Sum \ of \ i^2 \ in \ [0, n] = (n(n+1)(2n+1))/6.
11 formula_5(11 _n) {
    return _n*(_n+1)*(2*_n+1)/6;
// Sum_{0, n} i^3 = Sum \ of \ i^3 \ in \ [0, n] = ((n(n+1))/2)^2.
11 formula_6(ll _n) {
    11 ans = formula_1(_n);
    return ans*ans;
}
// Sum_{0}, inf x^{i} = 1/(1-x) if abs(x) < 1, inf abs(x) >= 1.
// Sum_{0, inf} i*x^i = x/(1-x)^2 if abs(x) < 1, inf abs(x) >= 1.
ll elevate(ll a, ll b) { // b >= 0.
    11 \text{ ans} = 1;
    while(b) {
        if(b & 1) ans = ans * a;
        b >>= 1;
        a = a * a;
    }
    return ans;
}
// Sum_{0, n} x^i = Sum \ of \ x^i \ in \ [0, n] = (Last*Ratio - First)/(Ratio - 1). Geometric sum.
11 formula_7(11 r, 11 n) {
    return (elevate(r, n + 1) - 1) / (r - 1);
// Number of digits of num in base 10.
11 formula_8(11 num) { // floor(log10(num)) + 1.
    return log10(num)+1;
}
```

## Extended GCD

```
// a*x1 + b*y1 = g;
                                                             11 x0, y0, g;
ll gcdEx(ll a, ll b, ll *x1, ll *y1) {
                                                             g = gcdEx(b\%a, a, &x0, &y0);
                                                             *x1 = y0 - (b/a)*x0;
    if(a == 0) {
                                                             *y1 = x0;
        *x1 = 0;
        *y1 = 1;
                                                             return g;
        return b;
All inverses
const 11 \mod = 31;
11 inverse[mod];
// Calculates inverse for all i < mod.
void init() {
    inverse[1] = 1;
    for(11 i = 2; i < mod; i++) {
        inverse[i] = -(mod/i)*inverse[mod%i];
        inverse[i] = (inverse[i]%mod + mod) % mod;
    }
}
Linear Sieve
const int MAX_PRIME = 1e6+5;
bool num[MAX_PRIME]; // If num[i] = false => i is prime.
int num_div[MAX_PRIME]; // Number of prime divisors of i.
int min_div[MAX\_PRIME]; // The smallest prime that divide i.
vector<int> prime;
void linear_sieve(){
    int i, j, prime_size = 0;
    min_div[1] = 1;
    for(i = 2; i < MAX_PRIME; ++i){</pre>
        if(num[i] == false) {prime.push_back(i); ++prime_size; num_div[i] = 1; min_div[i] = i;}
        for(j = 0; j < prime_size && i * prime[j] < MAX_PRIME; ++j){</pre>
            num[i * prime[j]] = true;
            num_div[i * prime[j]] = num_div[i] + 1;
            min_div[i * prime[j]] = min(min_div[i], prime[j]);
            if(i % prime[j] == 0) break;
        }
    }
}
bool is_prime(ll n) {
    for(auto el : prime) {
        if(n == el) return true;
        if(n%el == 0) return false;
   }
    return true;
}
vll fact, nfact; // The factors of n and their exponent. n \ge 1.
void factorize(int n) { // Up to MAX_PRIME*MAX_PRIME.
    11 cont, prev_p;
    fact.clear(); nfact.clear();
    for(auto p : prime) {
        if(n < MAX_PRIME) break;</pre>
        if(n\%p == 0) {
            fact.pb(p);
            cont = 0;
            while(n\%p == 0) n \neq p, cont++;
            nfact.pb(cont);
    if(n >= MAX_PRIME)  {
        fact.pb(n);
        nfact.pb(1);
        return;
    while(n != 1) { // When n < MAX\_PRIME, factorization in almost O(1).
```

```
prev_p = min_div[n];
       cont = 0;
       while(n\%prev_p == 0) n /= prev_p, cont++;
       fact.pb(prev_p);
       nfact.pb(cont);
   }
}
Lazy Segment Tree
template<typename T>
class Node { // Only modify this class.
    public:
    int 1 = -1, r = -1; // Interval [1, r].
    T value = 0;
    static const T lazy_default = -inf; // Don't change.
    T lazy = lazy_default;
   Node() = default;
   Node(T _value) {value = _value;}
    // Merge nodes.
   void actualize_update(T x) {
       if(x == -inf) return;
       if(lazy == -inf) lazy = 0;
       lazy += x; // (= SET update), (+= SUM update).
       value += x; // MINMAX query + (= SET update), (+= SUM update).
       // value = (r-l+1)*x; // SUM query + (= SET update), (+= SUM update).
   }
};
template<typename T>
class LazySegmentTree { // Use lazy propagation.
   vector<Node<T>> tree;
    vector<T> v;
    int n;
    // Value is the real value, and lazy is only for its children.
    void push_lazy(int k, int l, int r) {
       if(1 != r) {
           tree[k<<1].actualize_update(tree[k].lazy);</pre>
           tree[k<<1|1].actualize_update(tree[k].lazy);</pre>
           tree[k] = NodeT>(tree[k<<1], tree[k<<1|1]);
           tree[k].1 = 1; tree[k].r = r;
       tree[k].lazy = tree[k].lazy_default;
   }
    void build(int k, int l, int r) {
       if(1 == r) {
           tree[k] = NodeT>(v[1]);
           tree[k].1 = 1; tree[k].r = r;
       }
       int mid = (1 + r) >> 1;
       build(k<<1, 1, mid);
       build(k << 1 | 1, mid+1, r);
       tree[k] = NodeT>(tree[k<<1], tree[k<<1|1]);
       tree[k].1 = 1; tree[k].r = r;
    void update(int k, int l, int r, int ql, int qr, T x) {
       push_lazy(k, l, r);
       if(qr < 1 || r < ql) return;
       if(ql <= l && r <= qr) {
           tree[k].actualize_update(x);
       } else {
           int mid = (1 + r) >> 1;
           update(k<<1, 1, mid, ql, qr, x);
           update(k<<1|1, mid+1, r, ql, qr, x);
       push_lazy(k, 1, r);
   Node<T> query(int k, int l, int r, int ql, int qr) {
```

```
push_lazy(k, l, r);
        if(ql <= 1 && r <= qr) return tree[k];
        int mid = (1 + r) >> 1;
        if(qr <= mid) return query(k<<1, 1, mid, ql, qr);</pre>
        if(mid+1 <= ql) return query(k<<1|1, mid+1, r, ql, qr);
        Node<T> a = query(k<<1, 1, mid, ql, qr);
        Node<T> b = query(k <<1 | 1, mid+1, r, ql, qr);
        return Node<T>(a, b);
    public:
    LazySegmentTree() = default;
    LazySegmentTree(vector<T> _v) {
        v = v;
        n = v.size();
        tree.assign(4*n, {});
        build(1, 0, n-1);
    void update(int ql, int qr, T x) { // [ql, qr].
        if(ql > qr) swap(ql, qr);
        ql = max(ql, 0);
        qr = min(qr, n-1);
        update(1, 0, n-1, ql, qr, x);
    T query(int ql, int qr) { // [ql, qr].
        if(ql > qr) swap(ql, qr);
        ql = max(ql, 0);
        qr = min(qr, n-1);
        Node<T> ans = query(1, 0, n-1, ql, qr);
        return ans.value;
};
Dates
// Change here and date_to_num.
ll is_leap_year(ll y) {
    // if(y\%4 | | (y\%100==0 \& y\%400)) return 0; // Complete leap year.
    if(y\%4 != 0) return 0; // Restricted leap year.
    return 1;
}
ll days_month[12] = {31, 28, 31, 30, 31, 30, 31, 30, 31, 30, 31};
ll days_month_accumulate[12] = {31, 59, 90, 120, 151, 181, 212, 243, 273, 304, 334, 365};
// d 1-index, m 1-index.
11 date_to_num(11 d, 11 m, 11 y) {
    11 sum = d;
    m = 2;
    if(m >= 1) sum += is_leap_year(y);
    if(m >= 0) sum += days_month_accumulate[m];
    if(y >= 0) {
        sum += 365*y;
        // sum += y/4 -y/100 + y/400; // Complete leap year.
        sum += y/4; // Restricted leap year.
    }
    return sum:
// Tiny optimization, binary search the year, month and day.
void num_to_date(ll num, ll &d, ll &m, ll &y) {
    d = 1; m = 1; y = 0; // The date searched is >= this date.
    while(date_to_num(d, m, y) <= num) y++;</pre>
    y--;
    while(date_to_num(d, m, y) <= num) m++;</pre>
    while(date_to_num(d, m, y) <= num) d++;</pre>
    d--;
void cin_date(ll &d, ll &m, ll &y) {
    char c;
    cin >> d >> c >> m >> c >> y;
```

```
}
void cout_date(ll &d, ll &m, ll &y) {
        if(d < 10) cout << "0";
        cout << d << "/";
        if(m < 10) cout << "0";
        cout << m << "/";
        if(y < 10) cout << "000";
        else if(y < 100) cout << "00";
        else if(y < 1000) cout << "0";
        cout << y;</pre>
}
Time
// Read the hour. scanf("%d:%d:%d", &h, &m, &s);
void cin_hour(ll &h, ll &m, ll &s) {
    char c; // Dummy for read ':'.
    cin >> h >> c >> m >> c >> s;
// Prints the hour. printf("%02d:%02d:%02d", h, m, s);
void cout_hour(ll h, ll m, ll s) {
   h \% = 24; h += 24; h \% = 24;
   m \% = 60; m += 60; m \% = 60;
    s \% = 60; s += 60; s \% = 60;
    if(h < 10) cout << "0";
    cout << h << ":";
    if(m < 10) cout << "0";
    cout << m << ":";
    if(s < 10) cout << "0";
    cout << s;</pre>
}
// One day has 60*60*24 = 86400 seconds.
// Converts the hour to number of seconds since 00:00:00.
11 hours_to_seconds(11 h, 11 m, 11 s) {
    return 60*60*h + 60*m + s;
// From sec seconds, get the hour. Just's for one day.
void seconds_to_hours(ll &h, ll &m, ll &s, ll sec) {
    sec %= 86400; sec += 86400; sec %= 86400;
   h = sec / (60*60);
   sec %= 60*60;
   m = sec / 60;
   sec %= 60;
    s = sec;
}
// Convert grades of the clock hand to hours and minutes. qh is grades of hours and qm grades of minutes.
// return mp(-1, -1) if no solution exists.
pair<11, 11> grades_to_hour(ld gh, ld gm) {
    11 h = gh/30, m = gm/6;
    if((ld)30*h + (ld)m/2 != gh || (ld)6*m != gm) return mp(-1, -1);
    return mp(h, m);
}
// Convert hours and minutes to grades of the clock hand, mp(grade of large hour hand, small minute hand).
pair<ld, ld> hour_to_grades(ll h, ll m) {
    return mp((1d)30*h + (1d)m/2, (1d)6*m);
// Convert hours and minutes to grades of the clock hand, mp(grade of large hour hand, small minute hand).
pair<ld, pair<ld, ld>> hour_to_grades(ll h, ll m, ll s) {
    return mp((1d)30*h + (1d)m/2 + (1d)s/120, mp((1d)6*m + (1d)s/10, (1d)6*s));
}
Knapsack
const int MAX_KNAPSACK = 2000005;
11 dp[MAX_KNAPSACK]; // dp[i] is maximum value can get with i weight.
vector<pll> v; // (value, weight).
// Return max value with weight \leftarrow max_knapsack. O(n*max_knapsack).
```

```
// If can repeat elements, iterate O->max_knapsack.
11 knapsack(int max_knapsack) {
    int i, j, n = v.size();
    11 ans = 0, g = v[0].se;
    for(i = 1; i < n; i++) g = \_gcd(g, v[i].se);
    for(i = 0; i < n; i++) v[i].se /= g;
    max_knapsack /= g;
    for(i = 1; i <= max_knapsack; i++) dp[i] = -inf;</pre>
    dp[0] = 0;
    for(i = 0; i < n; i++) {
        for(j = max_knapsack; j >= 0; j--) {
            if(dp[j] != -inf \&\& j+v[i].se < MAX_KNAPSACK)
                dp[j + v[i].se] = max(dp[j + v[i].se], dp[j] + v[i].fi);
    }
    for(i = max_knapsack; i >= 0; i--) ans = max(ans, dp[i]);
    return ans;
}
LIS
vll LIS(vll &v) { // Is >=, but can be transformed to fit >.
    int i, t, n = v.size();
    if(n == 0) return vll();
    vll lis, lis_t(n), ans;
    lis.pb(v[0]); lis_t[0] = 1;
    for(i = 1; i < n; i++) {
        // if(v[i] == lis.back()) continue; // For >.
        if(v[i] >= lis.back())
            {lis.pb(v[i]); lis_t[i] = lis.size(); continue;}
        int pos = upper_bound(lis.begin(), lis.end(), v[i]) - lis.begin();
        // if(pos > 0) & lis[pos - 1] == v[i]) continue; // For >.
        lis[pos] = v[i];
        lis_t[i] = pos+1;
    for(i = n-1, t = lis.size(); i >= 0; i--) {
        if(lis_t[i] == t \&\& (ans.empty() || v[i] <= ans.back()))
            ans.pb(v[i]), t--; //v[i] < ans.back() for >.
    reverse(ans.begin(), ans.end());
    return ans;
}
DSU
class DSU {
                                                                 sz.assign(n, 1);
    int n;
                                                                 for(int i = 0; i < n; ++i) parent[i] = i;</pre>
                                                             }
    vi parent;
                                                            bool is_connected(int a, int b){
    vi rank;
    vi sz; // Size of the component.
                                                                 return find_parent(a) == find_parent(b);
                                                            }
    int find_parent(int a){
        if(parent[a] == a) return a;
                                                             void merge(int a, int b){
        return parent[a] = find_parent(parent[a]);
                                                                 a = find_parent(a);
    }
                                                                 b = find_parent(b);
                                                                 if(a == b) return;
    public:
    int number_components;
                                                                 number_components--;
    DSU() = default;
                                                                 if(rank[a] > rank[b]) parent[b] = a, sz[a] += sz[b];
                                                                 else if(rank[a] < rank[b]) parent[a] = b, sz[b] += sz[</pre>
    DSU(int _n) {
        n = _n;
                                                                 else \{parent[a] = b; rank[b]++, sz[b] += sz[a];\}
                                                             ļ
        number_components = n;
        parent.assign(n, 0);
                                                            int size(int a) {return sz[find_parent(a)];}
        rank.assign(n, 0);
                                                        };
Unordered Set
// Use unordered_set<pii, pair_hash> us or unordered_map<pii, int, pair_hash> um;
```

struct pair\_hash

{

```
template <class T1, class T2>
    size_t operator () (pair<T1, T2> const &pair) const
        size_t h1 = hash<T1>()(pair.first);
        size_t h2 = hash<T2>()(pair.second);
        return (h1 ^ 0b11001001011001101) + (0b011001010011100111 ^ h2);
};
HashSet
// Get random primes: https://primes.utm.edu/nthprime/.
const int MAX = 2*1e5+5;
11 val[MAX];
map<11, 11> m; // Compress elements.
void init() { // CALL ONCE.
    srand(time(0));
    for(int i = 0; i < MAX; i++) val[i] = rand();</pre>
// Contains a set of elements with duplicates.
// a[i] = sum_x\{val[x]\} \% mod p[i].
class HashSet{
    vll p = {1'237'273, 1'806'803, 3'279'209}; // Prime numbers.
    vll a = \{0, 0, 0\};
    int n = 3; // n = p.size();
    public:
    // Insert element x.
    void insert(ll x) {
        if(!m.count(x)) m[x] = m.size();
        for(int i = 0; i < n; i++) a[i] = (a[i] + val[m[x]])%p[i];
    void erase(ll x) {
        for(int i = 0; i < n; i++) a[i] = ((a[i] - val[m[x]])\%p[i] + p[i])\%p[i];
    // Insert all the elements of hs.
    void insert(HashSet hs) {
        for(int i = 0; i < n; i++) a[i] = (a[i] + hs.a[i])%p[i];
    bool operator == (HashSet hs) {
        for(int i = 0; i < n; i++) if(a[i] != hs.a[i]) return false;</pre>
        return true;
    }
};
HashString
// s = a[i], hash = a[0] + b*a[1] + b^2*a[2] + b^n*a[n].
class HashString {
    char initial = '0'; // change initial for range. 'a', 'A', '0'.
    public:
    string s;
    int n, n_p;
    vector<vll> v; // contain the hash for [0..i].
    vll p = {16532849, 91638611, 83157709}; // prime numbers. // 15635513 77781229
    vll base = \{37, 47, 53\}; // base numbers: primes that > alphabet size. // 49 83
    vector<vll> b; // b[i][j] = (b_i^j) \% p_i.
    vector<vll> b_inv; // b_inv[i][j] = (b_i^{\hat{j}})^{-1} \% p_i.
    11 elevate(ll a, ll _b, ll mod){
        11 \text{ ans} = 1;
        while(_b){
            if(_b \& 1) ans = ans*a \% mod;
            _b >>= 1;
            a = a*a \% mod;
        }
        return ans;
    // a^{(mod - 1)} = 1, Euler.
```

```
11 inv(int i, int j){
    if(b_inv[i][j] != -1) return b_inv[i][j];
    return b_inv[i][j] = elevate(b[i][j], p[i] - 2, p[i]);
HashString() = default;
HashString(string &_s) { // Not empty strings.
    s = _s;
    n = _s.length();
    n_p = (int)p.size();
    v.assign(n_p, vll(n, 0));
    b.assign(n_p, vll(n, 0));
    b_inv.assign(n_p, vll(n, -1));
    int i, j;
    for(i = 0; i < n_p; i++) {
        b[i][0] = 1;
        for(j = 1; j < n; j++) {
            b[i][j] = (b[i][j-1]*base[i]) % p[i];
        v[i][0] = s[0]-initial+1;
        for(j = 1; j < n; j++) {
            v[i][j] = (b[i][j]*(s[j]-initial+1) + v[i][j-1]) % p[i];
        }
    }
void add(char c) { // Need something previously added.
    int i;
    s += c;
    n++;
    for(i = 0; i < n_p; i++) {
        b[i].pb((b[i][n-2]*base[i]) % p[i]);
        b_inv[i].pb(-1);
        v[i].pb((b[i][n-1]*(c-initial+1) + v[i][n-2]) % p[i]);
}
void add(string &_s) {
    for(auto c : _s) add(c);
vll getHash(int 1, int r) {
    ll i, ans;
    vll vans;
    for(i = 0; i < n_p; i++) {
        ans = v[i][r];
        if(1 > 0) ans -= v[i][1-1];
        ans *= inv(i, 1);
        ans = ((ans\%p[i])+p[i])\%p[i];
        vans.pb(ans);
    }
    return vans;
// 0(1).
bool operator == (HashString other) {
    if(n != other.n) return false;
    return getHash(0, n-1) == other.getHash(0, n-1);
// return the index of the Longest Comon Prefix, -1 if no Common Prefix.
// O(\log n).
int LCP(HashString other) {
    int 1 = 0, r = min(n, other.n), mid;
    if(s[0] != other.s[0]) return -1;
    if(*this == other) return n-1;
    while(1 + 1 < r) {
        mid = (1 + r) >> 1;
        if(getHash(0, mid) == other.getHash(0, mid)) 1 = mid;
        else r = mid;
    }
    return 1;
bool operator < (HashString other) {</pre>
    int id = LCP(other);
```

```
if(id == -1) return s[0] < other.s[0];</pre>
        if(*this == other) return false;
        if(id == n) return true; // "ho" < "hol"</pre>
        if(id == other.n) return false;
        return s[id+1] < other.s[id+1];</pre>
};
SuffixArray
class SuffixArray {
    public:
    int n;
    string s;
    vi p; // p[i] is the position in the order array of the ith suffix (s[i..n-1]).
    vi c; // c[i] is the equivalence class of the ith suffix. When build, c[p[i]] = i, inverse.
    // dont use lcp[0] = 0.
    vi lcp; // lcp[i] is the longest common prefix in s[p[i-1]..n-1] and s[p[i]..n-1].
    // To get lcp(s[i..n-1], s[j..n-1) is min(lcp[c[i]+1], lcp[c[j]]) (use SegTree).
    void radix_sort(vector<pair<pii, int>> &v) { // O(n).
        vector<pair<pii, int>> v2(n);
        vi freq(n, 0); // first frequency and then the index of the next item.
        int i, sum = 0, temp;
        for(i = 0; i < n; i++) freq[v[i].fi.se]++; // Sort by second component.
        for(i = 0; i < n; i++) {temp = freq[i]; freq[i] = sum; sum += temp;}</pre>
        for(i = 0; i < n; i++) {v2[freq[v[i].fi.se]] = v[i]; freq[v[i].fi.se]++;}
        freq.assign(n, 0); sum = 0;
        for (i = 0; i < n; i++) \ freq[v2[i].fi.fi]++; \ // \ \mathit{Sort by first component}.
        for(i = 0; i < n; i++) {temp = freq[i]; freq[i] = sum; sum += temp;}</pre>
        for(i = 0; i < n; i++) {v[freq[v2[i].fi.fi]] = v2[i]; freq[v2[i].fi.fi]++;}
    SuffixArray() = default;
    SuffixArray(string &_s) {
        s = _s;
        s += \sqrt{s''}; // smaller char to end the string.
        n = s.size();
        int i, k;
        p.assign(n, 0);
        c.assign(n, 0);
        vector<pii> v1(n); // temporal vector to sort.
        for(i = 0; i < n; i++) v1[i] = mp(s[i], i);
        sort(v1.begin(), v1.end());
        for(i = 0; i < n; i++) p[i] = v1[i].se;
        c[p[0]] = 0;
        for(i = 1; i < n; i++) {
            if(v1[i].fi == v1[i - 1].fi) c[p[i]] = c[p[i - 1]];
            else c[p[i]] = c[p[i - 1]] + 1;
        k = 0; // in k+1 iterations sort strings of length 2^{(k+1)}.
        while(c[p[n-1]] != n-1) { // At most ceil(log2(n)).
            vector<pair<pii, int>> v2(n); // temporal vector to sort.
            for(i = 0; i < n; i++) v2[i] = mp(mp(c[i], c[(i + (1 << k)) % n]), i);
            radix_sort(v2);
            for(i = 0; i < n; i++) p[i] = v2[i].se;
            c[p[0]] = 0;
            for(i = 1; i < n; i++) {
                if(v2[i].fi == v2[i - 1].fi) c[p[i]] = c[p[i - 1]];
                else c[p[i]] = c[p[i - 1]] + 1;
            7
            k++;
        }
    void show_suffixes() { // IMPORTANT use this to debug.
        for(int i = 0; i < n; i++) cout << i << " " << p[i] << " " << s.substr(p[i]) << endl;
        if(!lcp.empty()) cout << "LCP: " << lcp << endl;</pre>
    // cmp s with t. return -1 if s < t, 1 if s > t, 0 if s == t.
```

```
int cmp_string(int pos, string &t) {
        for(int i = p[pos], j = 0; j < (int) t.size(); i++, j++) {
            if(s[i] < t[j]) return -1; // i < n because s[n-1] = \footnote{f}.
            if(s[i] > t[j]) return 1;
        }
        return 0:
    // Count the number of times t appears in s.
    int count_substring(string &t) {
        int 1 = -1, r = n, mid, L, R;
        while(1 + 1 < r) { // -1, ..., -1=L, 0, ..., 0, 1=R...1.
            mid = (1 + r) / 2;
            if(cmp_string(mid, t) < 0) 1 = mid;</pre>
            else r = mid;
        }
        L = 1;
        1 = -1; r = n;
        while(1 + 1 < r) {
            mid = (1 + r) / 2;
            if(cmp_string(mid, t) <= 0) 1 = mid;</pre>
            else r = mid;
        }
        R = r;
        return R - L - 1;
    // O(n) build. At most 2n lcp++ and n lcp--;
    void build_lcp() {
        lcp.assign(n, 0);
        for(int i = 0; i < n - 1; i++) {
            if(i > 0) lcp[c[i]] = max(lcp[c[i - 1]] - 1, 0);
            \label{eq:while(s[i + lcp[c[i]]] == s[p[c[i] - 1] + lcp[c[i]]]) lcp[c[i]] ++;} \\
    }
    11 number_substrings() {
        11 \text{ ans} = 0, i;
        for(i = 1; i < n; i++) {
            ans += n - p[i-1] - lcp[i]; // Length of the suffix - lcp with the next suffix.
        ans += n - p[n - 1]; // Plus the last suffix.
        return ans - n; // Remove the '\£' symbol on n substrings.
    }
};
string LCS(string s, string &t) {
    int mx = 0, mxi = 0, i, n2 = t.length();
    string ans = "";
    s += "@" + t; // Concatenate with a special char.
    SuffixArray sa(s);
    sa.build_lcp();
    for(i = 1; i < sa.n; i++) {
        // Suffix of s and before suffix of t.
        if(sa.n - sa.p[i] > n2 + 2 && sa.n - sa.p[i-1] <= n2 + 1) {
            if(sa.lcp[i] > mx) mx = sa.lcp[i], mxi = i;
        // Suffix of t and before suffix of s.
        if(sa.n - sa.p[i] \le n2 + 1 \&\& sa.n - sa.p[i-1] > n2 + 2) {
            if(sa.lcp[i] > mx) mx = sa.lcp[i], mxi = i;
        7
    }
    return sa.s.substr(sa.p[mxi], mx);
}
Z algorithm
// Search the ocurrences of t (pattern to search) in s (the text).
// O(n + m). It increases R at most 2n times and decreases at most n times.
//z[i] is the longest string s[i..i+z[i]-1] that is a prefix = s[0..z[i]-1].
void z_algorithm(string &s, string &t) {
    s = t + "$" + s; // "f" is a char not present in s nor t.
    int n = s.length(), m = t.length(), i, L = 0, R = 0;
```

```
vi z(n, 0);
    // s[L..R] = s[0..R-L], [L, R] is the current window.
    for(i = 1; i < n; i++) {
        if(i > R) { // Old window, recalculate.
            L = R = i;
            while(R < n && s[R] == s[R-L]) R++;
            R--;
            z[i] = R - L + 1;
        } else {
            if(z[i-L] < R - i) z[i] = z[i-L]; // z[i] will fall in the window.
            else { // z[i] can fall outside the window, try to increase the window.
                while (R < n \&\& s[R] == s[R-L]) R++;
                R.--:
                z[i] = R - L + 1;
            }
        }
        if(z[i] == m) { // Match found.}
            //echo("Pattern found at: ", i-m-1);
   }
}
Longest Palindromic Substring
// LPS Longest Palindromic Substring, O(n).
void Manacher(string &str) {
    char ch = '#'; // '#' a char not contained in str.
    string s(1, ch), ans;
    for(auto c : str) {s += c; s += ch;}
    int i, n = s.length(), c = 0, r = 0;
    vi lps(n, 0);
    for(i = 1; i < n; i++) {
        // lps[i] >= it's mirror, but falling in the interval [L..R]. L = c - (R - c).
        if(i < r) lps[i] = min(r - i, lps[c - (i - c)]);
        // Try to increase.
        while(i-lps[i]-1) = 0 \&\& i+lps[i]+1 < n \&\& s[i-lps[i]-1] == s[i+lps[i]+1]) lps[i]++;
        // Update the interval [L..R].
        if(i + lps[i] > r) c = i, r = i + lps[i];
   }
    /\!/ Get the longest palindrome in ans.
    int pos = max_element(lps.begin(), lps.end()) - lps.begin();
    for(i = pos - lps[pos]; i <= pos + lps[pos]; i++) {</pre>
        if(s[i] != ch) ans += s[i];
    //cout << ans.size() << "\n";
}
Aho Corasick
class Node{
    public:
    static const int alpha_size = 26;
    int leaf_cnt = 0; // Number of words ending at current node.
    vi next_letter;
    int p; // Parent of the node.
    char pch; // Parent -pch-> node.
    int suffix_link = -1; // Longest proper suffix of the string ending at node p.
    vi go; // DP for go().
    int count_words_dp = -1; // DP for count_words().
    Node(int _p, char _pch) {
        next_letter.assign(alpha_size, -1);
        p = p;
        pch = _pch;
        go.assign(alpha_size, -1);
}; // O(m*alpha_size) memory, O(m) runtime. m = sum(length(s_i)).
```

class AhoCorasick{

```
vector<Node> t = vector<Node>(1, Node(-1, \$'));
public:
void add_string(string &s) {
    int c, p = 0;
    for(char ch : s) {
        c = ch - 'a';
        if(t[p].next_letter[c] == -1) {
            t[p].next_letter[c] = t.size();
            t.pb(Node(p, ch));
        p = t[p].next_letter[c];
    }
    t[p].leaf_cnt++;
// Get the proper suffix link of v.
// Once called, don't call anymore add_strings.
int get_link(int v) {
    if(t[v].suffix_link == -1) {
        if(v == 0 \mid \mid t[v].p == 0) t[v].suffix_link = 0;
        else t[v].suffix_link = go(get_link(t[v].p), t[v].pch);
    return t[v].suffix_link;
}
// Search for the longest proper suffix of v that has next_letter[c] transition.
int go(int v, char ch) {
    int c = ch - 'a';
    if(t[v].go[c] == -1) {
        if(t[v].next_letter[c] != -1) t[v].go[c] = t[v].next_letter[c];
        else if(v == 0) t[v].go[c] = 0;
        else t[v].go[c] = go(get_link(v), ch);
    return t[v].go[c];
}
// Get the number of words in the automaton that are a suffix of the node \emph{v}.
int count_words(int v) { // with duplicates.
    if(t[v].count_words_dp == -1) {
        t[v].count_words_dp = t[v].leaf_cnt;
        if(v != 0) t[v].count_words_dp += count_words(get_link(v));
    return t[v].count_words_dp;
// Get the number of words in the automaton that are in the text.
int search_num_string(string &text) { // with duplicates.
    int p = 0, ans = count_words(0);
    for(auto ch : text) {
        p = go(p, ch);
        ans += count_words(p);
    }
    return ans;
bool smallest_not_contained_str(int i, int L, int v, string &ans) {
    if(count_words(v) > 0) return false;
    if(i == L) return true;
    for(int j = 0; j < Node::alpha_size; j++) {</pre>
        char ch = j + 'a';
        ans += ch;
        bool ok = smallest_not_contained_str(i+1, L, go(v, ch), ans);
        if(ok) return true;
        ans.pop_back();
    }
    return false;
// Get the smallest string of length L that dont contains any word in the automaton.
string smallest_not_contained_str(int L) {
    string ans = "";
    smallest_not_contained_str(0, L, 0, ans);
    return ans;
}
```

};

# Rope

```
unordered_set<void *> freezed_node;
                                                                     return r->get_char(i - weight);
// List to not double free nodes.
                                                                 }
class Node{
                                                             } // Get the substring [posl, posr].
    Node *1 = nullptr, *r = nullptr;
                                                             void get_substr(ll posl, ll posr, string &ans) {
                                                                 if(weight < posl || posr <= 0) return;</pre>
    11 weight = 0;
    // Number of characters ONLY in left subtree.
                                                                 if(1) 1->get_substr(posl, posr, ans);
    string s = ""; // String in the leaves.
                                                                 int n = posr - posl + 1;
                                                                 if(!l \&\& !r) ans += s.substr(posl-1, n);
    public:
    Node() = default;
                                                                 if(r) r->get_substr(max(posl - weight, 111),
    Node(string _s) {
                                                                 posr - weight, ans);
                                                             }
        s = _s;
        weight = s.length();
                                                        };
                                                         // Is persistent. Since it can self-referenciate, the string
    Node(Node *_1, Node *_r) {
                                                         // can grows very fast, so use ll for indexes.
        1 = _1;
                                                         class Rope{
        r = _r;
                                                             vector<Node*> v; // All functions add the root to v.
                                                             vector<Node*> to_delete; // Temporal list to free all the
        if(l) weight += l->get_length();
                                                             Node* concat(Node *1, Node *r) {return new Node(1, r);}
    "Node() { // Delete it if MLE.
                                                             ~Rope() {
        if(!freezed_node.count(1)) {
            freezed_node.insert(1);
                                                                 for(auto el : to_delete) v.pb(el);
            delete 1;
                                                                 int i, n = v.size();
                                                                 for(i = 0; i < n; i++) {
                                                                     if(freezed_node.count(v[i])) continue;
        if(!freezed_node.count(r)) {
            freezed_node.insert(r);
                                                                     freezed_node.insert(v[i]);
            delete r;
                                                                     delete v[i];
                                                                 }
                                                             } // Show string v[pos].
    void show() { // Print the whole string.
                                                             void show(int pos) {
        if(1) 1->show();
                                                                 v[pos]->show();
        cout << s;</pre>
                                                             } // Get char i of v[pos].
        if(r) r->show();
                                                             char get_char(int pos, ll i) {
                                                                 return v[pos]->get_char(i + 1);
    // void balance() \{\} // TODO. O(n) vs O(\log n).
                                                             } // Get the substring v[pos] [l..r].
    ll get_length() {
                                                             string get_substr(int pos, 11 1, 11 r) {
    // Get length of whole string.
                                                                 string ans = "";
        11 ans = weight;
                                                                 v[pos]->get_substr(l+1, r+1, ans);
        if(r) ans += r->get_length();
                                                                 return ans;
        return ans;
                                                             } // Add a new string to the rope.
    }
                                                             void add(string &s) {
    // Split in [0..i], [i+1..]. Can return
                                                                 v.pb(new Node(s));
    // \ null \ if \ i = -1 \ or \ n - 1.
                                                             } // Concatenate v[posl] with v[posr].
    pair<Node*, Node*> split(ll index) {
                                                             void concat(int posl, int posr) {
                                                                 v.pb(concat(v[posl], v[posr]));
        index++;
                                                             } // Convert v[pos1] to [0..i] + v[pos2] + [i+1..n-1].
        // 1 index. Number of chars, not position.
                                                             void insert(int pos1, int pos2, ll i) {
        pair<Node*, Node*> ret;
        if(index < weight) {</pre>
                                                                 if(i == -1) { // Append to the left.
                                                                     v.pb(concat(v[pos2], v[pos1]));
            if(!1) {
                return mp(new Node(s.substr(0, index)),
                new Node(s.substr(index)));
            }
                                                                 if(i == v[pos1]->get_length() - 1) { // Append to the
            ret = 1->split(index - 1); // -1 convert again.
                                                                     v.pb(concat(v[pos1], v[pos2]));
            return mp(ret.fi, new Node(ret.se, r));
                                                                     return;
        } else if(index > weight) {
            ret = r->split(index - weight - 1);
                                                                 pair<Node*, Node*> ret = v[pos1]->split(i);
            return mp(new Node(1, ret.fi), ret.se);
                                                                 v.pb(concat(concat(ret.fi, v[pos2]), ret.se));
                                                             } // Erase v[pos] [i..j].
        } else {
            if(!r) return mp(this, new Node(""));
                                                             void erase(int pos, ll i, ll j) {
            return mp(1, r);
                                                                 pair<Node*, Node*> ret = v[pos]->split(i-1);
                                                                 pair<Node*, Node*> ret2 = ret.se->split(j-i);
    } // 1 Index str.
                                                                 to_delete.pb(ret.se);
    char get_char(ll i) {
                                                                 to_delete.pb(ret2.fi);
        if(i <= weight) {</pre>
                                                                 v.pb(concat(ret.fi, ret2.se));
            if(!1) return s[i - 1];
                                                             } // Cut v[pos] [i..j] and save it.
                                                             void cut(int pos, ll i, ll j) {
            return 1->get_char(i);
        } else {
                                                                 if(i > j) {
```

```
v.pb(new Node(""));
                                                                 to_delete.pb(ret.se);
            return;
                                                                 to_delete.pb(ret2.se);
        }
                                                                 v.pb(ret2.fi);
        pair<Node*, Node*> ret = v[pos]->split(i-1);
        pair<Node*, Node*> ret2 = ret.se->split(j-i); };
        to_delete.pb(ret.fi);
2 SAT
vector<vi> graph;
class SAT{ // 2SAT, (xi \ or \ xj) and () \dots \ O(n).
    public:
    SAT(int n) {
        graph.assign(2*n, vi());
    int get_pos(int i) {return 2*i;}
    int get_neg(int i) {return 2*i + 1;}
    void add_or(int i, int j) { // Use it with get_pos.
        graph[i^1].pb(j);
        graph[j^1].pb(i);
    }
    void add_value(int i, int val) { // x[i] = val;
        if(val) add_or(get_pos(i), get_pos(i));
        else add_or(get_neg(i), get_neg(i));
    vector <bool> x; // Can add (xi or xi) if you know xi = true.
    bool solve() {
        Kosaraju kosaraju;
        int i, n = graph.size(), n_component = kosaraju.components.size();
        vi el2component(n, 0);
        for(i = 0; i < n_component; i++)</pre>
            for(auto u : kosaraju.components[i]) el2component[u] = i;
        for(i = 0; i < n; i += 2)
            if(el2component[i] == el2component[i + 1]) return false;
        vector<vi> graph2 = graph;
        graph.assign(n_component, vi());
        for(i = 0; i < n; i++)
            for(auto u : graph2[i]) if(el2component[i] != el2component[u])
                graph[el2component[i]].pb(el2component[u]);
        Toposort toposort(n_component);
        x.assign(n/2, false);
        vi component_order(n_component, 0);
        for(i = 0; i < n_component; i++)</pre>
            component_order[toposort.vSorted[i]] = i;
        for(i = 0; i < n; i += 2)
            x[i/2] = component_order[el2component[i]] > component_order[el2component[i + 1]];
        return true;
};
Python Template
from decimal import Decimal, getcontext
import math, sys
input = sys.stdin.readline
# getcontext().prec = 3 # 3 de precision, trunca el resto.
\# n = Decimal(1) / Decimal(3)
# try:
     x = input() # until EOF.
#
     if len(x) == 0:
#
#
         exit(0)
# except:
     exit(0)
\# v = [k \text{ for } k \text{ in } map(int, s.split(' '))]
```

## Mo's algorithm

```
// You can use it when you can extend / shrink interval by 1 element.
// O((Q+N)sqrt(N)), is sqrt decomposition.
class Query{
    static const int BLOCK = 170; // sqrt(n) +-;
    public:
    int 1, r, id;
    Query() = default;
    Query(int _1, int _r, int _id) {
        1 = _1; r = _r; id = _id;
    bool operator < (const Query &other) const {</pre>
        if(1/BLOCK != other.1/BLOCK) {
            return 1/BLOCK < other.1/BLOCK;</pre>
        if(r != other.r) return r < other.r;</pre>
        return 1 < other.1;</pre>
};
// Calculate the num of different numbers in [l, r].
class Mo{
    static const int MAX_FREQ = 1e6+5;
    vector<Query> vquery;
    vll v; // Input vector.
    int currL = 0; // currAns represent answer for the
    int currR = -1;// interval [currL, currR].
    int currAns = 0;
    vi freq;
    void add(int i) {
        freq[v[i]]++;
        if(freq[v[i]] == 1) currAns++;
    void remove(int i) {
        freq[v[i]]--;
        if(freq[v[i]] == 0) currAns--;
    public:
    Mo() = default;
    Mo(vll &_v) {
        v = _v;
    void insert_query(int 1, int r, int id) {
        vquery.pb(Query(1, r, id));
    vi solve() {
        int i, n = vquery.size();
        freq.assign(MAX_FREQ, 0);
        vector<pii> ans(n);
        sort(vquery.begin(), vquery.end());
        for(i = 0; i < n; i++) {
            while(currL < vquery[i].1) {</pre>
                remove(currL);
                currL++;
            while(currL > vquery[i].1) {
                currL--;
                add(currL);
            while(currR < vquery[i].r) {</pre>
                currR++;
                add(currR);
            while(currR > vquery[i].r) {
                remove(currR);
                currR--;
            ans[i] = mp(vquery[i].id, currAns);
        }
```

```
sort(ans.begin(), ans.end());
vi answer(n);
for(i = 0; i < n; i++) {
         answer[i] = ans[i].se;
}
return answer;
}
};</pre>
```