

# ECHO'S NOTEBOOK

```

1 | FLAGS=-Wall -Wextra -Wshadow -Wno-unused-result -D_GLIBCXX_DEBUG -fsanitize=address
   | ↪ -fsanitize=undefined -fno-sanitize-recover
2 |
3 | @g++ A.cpp $(FLAGS) -DJUNCO_DEBUG && ./a.out < z.in
4 |
5 |
6 | // Iterate over all submasks of a mask. CONSIDER SUBMASK = 0 APART.
7 | for(submask = mask; submask > 0; submask = (submask-1)&mask) {}

```

## DP

### LCS

```

1 | int LCS() { // Longest Common Subsequence.
2 |     int ns = s.length(), nt = t.length(), i, j;
3 |     vector<vi> dp(ns + 1, vi(nt + 1, 0)); // One empty row and column, dp is 1-index
4 |     for(i = 1; i <= ns; i++) {
5 |         for(j = 1; j <= nt; j++) {
6 |             if(s[i-1] == t[j-1]) dp[i][j] = dp[i-1][j-1] + 1;
7 |             else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
8 |         }
9 |     }
10 |     return dp[ns][nt]; // Length.
11 | }

```

### LIS

```

1 | vll v_LIS(vll &v) {
2 |     int i, j, n = v.size();
3 |     vll lis, lis_time(n), ans;
4 |     if(!n) return ans;
5 |     lis.pb(v[0]); lis_time[0] = 1;
6 |     for(i = 1; i < n; i++) {
7 |         if(v[i] > lis.back()) {lis.pb(v[i]); lis_time[i] = lis.size(); continue;}
8 |         int pos = upper_bound(lis.begin(), lis.end(), v[i]) - lis.begin();
9 |         // if(pos > 0 && lis[pos-1] == v[i]) continue; // USE IF YOU WANT STRICTLY
   |         ↪ INCREASING.
10 |         lis[pos] = v[i];
11 |         lis_time[i] = pos+1;
12 |     }
13 |     j = lis.size();
14 |     for(i = n-1; i >= 0; i--) {
15 |         if(lis_time[i] == j && (ans.empty() || v[i] <= ans.back())) {ans.pb(v[i]); j--;} //
   |         ↪ <= or <.
16 |     }
17 |     reverse(ans.begin(), ans.end());
18 |     return ans;
19 | }

```

## IO

```

1 | ios::sync_with_stdio(false); cin.tie(nullptr); cout.tie(nullptr);
2 |
3 | stringstream ss;
4 | ss << "Hello world";
5 | ss.str("Hello world");
6 | while(ss >> s) cout << s << endl;
7 | ss.clear();

```

## Geometry

```

1  template<typename T>
2  class Point {
3      public:
4          static const int LEFT_TURN = 1;
5          static const int RIGHT_TURN = -1;
6          T x = 0, y = 0;
7          Point() = default;
8          Point(T _x, T _y) {
9              x = _x;
10             y = _y;
11         }
12         friend ostream &operator << (ostream &os, Point<T> &p) {
13             os << "(" << p.x << " " << p.y << ")";
14             return os;
15         }
16         bool operator == (const Point<T> other) const {
17             return x == other.x && y == other.y;
18         }
19         // Get the (1°) bottom (2°) left point.
20         bool operator < (const Point<T> other) const {
21             if(y != other.y) return y < other.y;
22             return x < other.x;
23         }
24         T euclidean_distance(Point<T> other) {
25             T dx = x - other.x;
26             T dy = y - other.y;
27             return sqrt(dx*dx + dy*dy);
28         }
29         T euclidean_distance_squared(Point<T> other) {
30             T dx = x - other.x;
31             T dy = y - other.y;
32             return dx*dx + dy*dy;
33         }
34         T manhatan_distance(Point<T> other) {
35             return abs(other.x - x) + abs(other.y - y);
36         }
37         // Get the height of the triangle with base b1, b2.
38         T height_triangle(Point<T> b1, Point<T> b2) {
39             if(b1 == b2 || *this == b1 || *this == b2) return 0; // It's not a triangle.
40             T a = euclidean_distance(b1);
41             T b = b1.euclidean_distance(b2);
42             T c = euclidean_distance(b2);
43             T d = (c*c-b*b-a*a)/(2*b);
44             return sqrt(a*a - d*d);
45         }
46         int get_quadrant() {
47             if(x > 0 && y >= 0) return 1;
48             if(x <= 0 && y > 0) return 2;
49             if(x < 0 && y <= 0) return 3;
50             if(x >= 0 && y < 0) return 4;
51             return 0; // Point (0, 0).
52         }
53         // Relative quadrant respect the point other, not the origin.
54         int get_relative_quadrant(Point<T> other) {
55             Point<T> p(other.x - x, other.y - y);
56             return p.get_quadrant();
57         }
58
59
60

```

```

61 // Orientation of points *this -> a -> b.
62 int get_orientation(Point<T> a, Point<T> b) {
63     T prod = (a.x - x)*(b.y - a.y) - (a.y - y)*(b.x - a.x);
64     if(prod == 0) return 0;
65     return prod > 0? LEFT_TURN : RIGHT_TURN;
66 }
67 // True if a have less angle than b, if *this->a->b is a left turn.
68 bool angle_cmp(Point<T> a, Point<T> b) {
69     if(get_relative_quadrant(a) != get_relative_quadrant(b))
70         return get_relative_quadrant(a) < get_relative_quadrant(b);
71     int ori = get_orientation(a, b);
72     if(ori == 0) return euclidean_distance_squared(a) < euclidean_distance_squared(b);
73     return ori == LEFT_TURN;
74 }
75 // Anticlockwise sort starting at 1° quadrant, respect to *this point.
76 void polar_sort(vector<Point<T>> &v) {
77     sort(v.begin(), v.end(), [&](Point<T> a, Point<T> b) {return angle_cmp(a, b);});
78 }
79 // Convert v to its convex hull, Do a Graham Scan. O(n log n).
80 void convert_convex_hull(vector<Point<T>> &v) {
81     if(v.size() < 3) return;
82     Point<T> bottom_left = v[0], p2;
83     for(auto p : v) bottom_left = min(bottom_left, p);
84     bottom_left.polar_sort(v);
85     vector<Point<T>> v_input = v; v.clear();
86     for(auto p : v_input) {
87         while(v.size() >= 2) {
88             p2 = v.back(); v.pop_back();
89             if(v.back().get_orientation(p2, p) == LEFT_TURN) {
90                 v.pb(p2);
91                 break;
92             }
93         }
94         v.pb(p);
95     }
96 }
97 };

```

## Graphs

### UFDS

```

1 void initialize(int n) {
2     for (int i = 0; i < n; ++i) {
3         rankk[i] = 0;
4         parent[i] = i;
5     }
6 }
7 int find(int x) {
8     if (parent[x] == x)
9         return x;
10    else
11        return parent[x] = find(parent[x]);
12 }
13
14 void Union(int a, int b) {
15     int pa = find(a);
16     int pb = find(b);
17     if (pa == pb) {
18         return;
19     }
20     if (rankk[pa] > rankk[pb]) {
21         parent[pb] = pa;
22     } else if (rankk[pa] < rankk[pb]) {
23         parent[pa] = pb;
24     } else {
25         parent[pa] = pb;
26         rankk[pb]++;
27     }
28 }

```

### Articulation points and bridges

```

1 vector<vi> adyList; // Graph
2 vi num, low;       // num and low for DFS
3 int cnt;           // Counter for DFS
4 int root, rchild;  // Root and number of (DFS) children
5 vi artic;          // Contains the articulation points at the end
6 set<pii> bridges;  // Contains the bridges at the end
7
8 void dfs(int nparent, int nnode) {
9     num[nnode] = low[nnode] = cnt++;
10    rchild += (nparent == root);
11
12    for (auto a : adyList[nnode]) {
13        if (num[a] == -1) { // Tree edge
14            dfs(nnode, a);
15            low[nnode] = min(low[nnode], low[a]);
16            if (low[a] >= num[nnode]) {
17                artic[nnode] = true;
18            }
19            if (low[a] > num[nnode]) {
20                bridges.insert((nnode < a) ? mp(nnode, a) : mp(a, nnode));
21            }
22        } else if (a != nparent) { // Back edge
23            low[nnode] = min(low[nnode], num[a]);
24        }
25    }
26 }
27 void findArticulations(int n) {
28     cnt = 0;
29     low = num = vi(n, -1);
30     artic = vi(n, 0);

```

```

31     bridges.clear();
32
33     for (int i = 0; i < n; ++i) {
34         if (num[i] != -1) {
35             continue;
36         }
37         root = i;
38         rchild = 0;
39         dfs(-1, i);
40         artic[root] = rchild > 1; //Special case
41     }
42 }

```

### Bellman Ford's

```

1  for(i = 0; i < n - 1; i++) { // Iterate n - 1 times.
2      for(auto e : edge) {
3          if(dist[e.fi.fi] != inf)
4              dist[e.fi.se] = min(dist[e.fi.se], dist[e.fi.fi] + e.se);
5      }
6  }

```

### Floyd cycle detection

```

1  void floyd_detection() {
2      ll pslow = f(F_0), pfast = f(f(F_0)), iteration = 0;
3      while(pslow != pfast) pslow = f(pslow), pfast = f(f(pfast));
4      pslow = F_0;
5      while(pslow != pfast) pslow = f(pslow), pfast = f(pfast), iteration++;
6      cout << "In " << iteration << " coincide with value: " << pslow << endl;
7      pfast = f(pfast), iteration++;
8      while(pslow != pfast) pfast = f(pfast), iteration++;
9      cout << "In " << iteration << " coincide with value: " << pfast << endl;
10 }

```

### Max Flow: Edmond Karp's

```

1  vector<vector<ll>> adjList;
2  vector<vector<ll>> adjMat;
3
4  void initialize(int n) {
5      adjList = decltype(adjList)(n);
6      adjMat = decltype(adjMat)(n, vector<ll>(n, 0));
7  }
8
9  map<int, int> p;
10 bool bfs(int source, int sink) {
11     queue<int> q;
12     vi visited(adjList.size(), 0);
13     q.push(source);
14     visited[source] = 1;
15     while (!q.empty()) {
16         int u = q.front();
17         q.pop();
18         if (u == sink)
19             return true;
20         for (auto v : adjList[u]) {
21             if (adjMat[u][v] > 0 && !visited[v]) {
22                 visited[v] = true;
23                 q.push(v);
24                 p[v] = u;
25             }
26         }
27     }
28     return false;

```

```

29 }
30 int max_flow(int source, int sink) {
31     ll max_flow = 0;
32     while (bfs(source, sink)) {
33         ll flow = inf;
34         for (int v = sink; v != source; v = p[v]) {
35             flow = min(flow, adjMat[p[v]][v]);
36         }
37         for (int v = sink; v != source; v = p[v]) {
38             adjMat[p[v]][v] -= flow; // Decrease capacity forward edge
39             adjMat[v][p[v]] += flow; // Increase capacity backward edge
40         }
41         max_flow += flow;
42     }
43     return max_flow;
44 }
45 void addedgeUni(int orig, int dest, ll flow) {
46     adjList[orig].pb(dest);
47     adjMat[orig][dest] = flow;
48     adjList[dest].pb(orig); //Add edge for residual flow
49 }
50 void addEdgeBi(int orig, int dest, ll flow) {
51     adjList[orig].pb(dest);
52     adjList[dest].pb(orig);
53     adjMat[orig][dest] = flow;
54     adjMat[dest][orig] = flow;
55 }

```

### Max Flow: Dinic's

```

1 //  $O(V^2 \cdot E)$  max flow algorithm. For bipartite matching  $O(\sqrt{V} \cdot E)$ , always faster than
  ↪ Edmond-Karp.
2 // Creates layer's graph with a BFS and then it tries all possible DFS, branching while
  ↪ the path doesn't reach the sink
3 struct EdgeFlow {
4     ll u, v;
5     ll cap, flow = 0; //capacity and current flow
6     EdgeFlow(ll _u, ll _v, ll _cap) : u(_u), v(_v), cap(_cap) { }
7 };
8
9 struct Dinic {
10     vector<EdgeFlow> edge; //keep the edges
11     vector<vll> graph; //graph[u] is the list of their edges
12     ll n, n_edges = 0;
13     ll source, sink, inf_flow = inf;
14     vll lvl; //lvl of the node to the source
15     vll ptr; //ptr[u] is the next edge you have to take in order to branch the DFS
16     queue<ll> q;
17
18     Dinic(ll _n, ll _source, ll _sink) : n(_n), source(_source), sink(_sink) { //n nodes
19         graph.assign(_n, vll());
20     }
21
22     void add_edge(ll u, ll v, ll flow) { //u->v with cost x
23         EdgeFlow uv(u, v, flow), vu(v, u, 0);
24         edge.pb(uv);
25         edge.pb(vu);
26         graph[u].pb(n_edges);
27         graph[v].pb(n_edges+1);
28         n_edges += 2;
29     }
30
31     bool BFS() {

```

```

32     ll u;
33     while(q.empty() == false) {
34         u = q.front(); q.pop();
35         for(auto e1 : graph[u]) {
36             if(lvl[edge[e1].v] != -1) {
37                 continue;
38             }
39             if(edge[e1].cap - edge[e1].flow <= 0) {
40                 continue;
41             }
42             lvl[edge[e1].v] = lvl[edge[e1].u] + 1;
43             q.push(edge[e1].v);
44         }
45     }
46
47     return lvl[sink] != -1;
48 }
49
50 ll dfs(ll u, ll min_flow) {
51     if(u == sink) return min_flow;
52     ll pushed, el;
53     for(; ptr[u] < (int)graph[u].size(); ptr[u]++) { //if you can pick ok, else you crop
54         ↳ that edge for the current bfs layer
55         el = graph[u][ptr[u]];
56         if(lvl[edge[e1].v] != lvl[edge[e1].u] + 1 || edge[e1].cap - edge[e1].flow <= 0)
57             ↳ {
58                 continue;
59         }
60         pushed = dfs(edge[e1].v, min(min_flow, edge[e1].cap - edge[e1].flow));
61         if(pushed > 0) {
62             edge[e1].flow += pushed;
63             edge[e1^1].flow -= pushed;
64             return pushed;
65         }
66     }
67     return 0;
68 }
69
70 ll max_flow() {
71     ll flow = 0, pushed;
72     while(true) {
73         lvl.assign(n, -1);
74         lvl[source] = 0;
75         q.push(source);
76         if(!BFS()) {
77             break;
78         }
79
80         ptr.assign(n, 0);
81         while(true) {
82             pushed = dfs(source, inf_flow);
83             if(!pushed) break;
84             flow += pushed;
85         }
86     }
87     return flow;
88 }
};

```

## Hungarian Algorithm

```

1  const int MAX_N1 = 1002; //number of workers
2  const int MAX_N2 = 2002; //number of items
3  int cost[MAX_N1][MAX_N2]; //cost matrix, entries >= 0
4  int u[MAX_N1+1], v[MAX_N2+1]; //potentials, always cost[i][j] >= u[i] + v[j]
5  int slack[MAX_N2+1]; //cost[i][j] - u[i] - v[j], always >= 0
6  int prevy[MAX_N2+1]; //edges of the current path: prev[j0] -> yx[prev[j0]] -> j0. Dont need
   ↪ to reset
7  bool used[MAX_N2+1]; //visited array
8  int yx[MAX_N2+1]; //match of the j column
9  int n1=, n2=, INF = INT_MAX-1; //actual size of workers and items.
10
11 //http://e-maxx.ru/algo/assignment_hungary
12 //Solves MINIMUM Assignment. For maximum change cost[i][j] to Max_entry - cost[i][j] and
   ↪ resize the answer.
13 //There are 1..n1 rows and 1..n2 columns, ALWAYS n1 <= n2. Complexity(n1 * n1*n2)
14 //The function use 1-index for variables because it creates a virtual vertex 0
15 int Hungarian() {
16     int i, i0, j, j0, min_j, delta, ans;
17     fill(u, u+n1+1, 0);
18     fill(v, v+n2+1, 0);
19     fill(yx, yx+n2+1, 0);
20     for(i = 1; i <= n1; i++) { //Add row by row to the current matching
21         yx[0] = i; //connect 0 of set 2 with vertex i
22         j0 = 0; //i0 and j0 are the current selected row and column, i and j are just
           ↪ iterators
23         fill(slack, slack+n2 + 1, INF);
24         fill(used, used + n2 + 1, false);
25         do { //while the alternating path is not augmenting path
26             used[j0] = true;
27             delta = INF;
28             i0 = yx[j0];
29             for(j = 1; j <= n2; j++) { //get the delta among all columns not used
30                 if(!used[j]) {
31                     int cur = cost[i0-1][j-1] - u[i0] - v[j];
32                     if(cur < slack[j]) {
33                         slack[j] = cur, prevy[j] = j0;
34                     }
35                     if(slack[j] < delta) { //try if delta == 0 break
36                         delta = slack[j], min_j = j;
37                     }
38                 }
39             }
40             for(j = 0; j <= n2; j++) { //add delta in set 1, subtract delta in set 2
41                 if(used[j]) u[yx[j]] += delta, v[j] -= delta;
42                 else slack[j] -= delta;
43             }
44             j0 = min_j;
45         } while(yx[j0] != 0);
46         do { //invert the augmenting path
47             yx[j0] = yx[prevy[j0]];
48             j0 = prevy[j0];
49         } while(j0);
50     }
51     ans = 0;
52     for(j = 1; j <= n2; j++) { //recover solution. The matched edges are yx[j]-1 -> j-1
53         if(yx[j])
54             ans += cost[yx[j]-1][j-1];
55     }
56     return ans;
57 }
58

```



```

59 // THE ANS IS n1*M_factor - Hungarian().
60 int M_factor; // Change problem finding the minimum cost to maximum cost, that can be
    ↳ solved by Hungarian
61 void min_to_max() { //min in cost[i][j] = max in M - cost[i][j].
62     int i, j;
63     M_factor = 0;
64     for(i = 0; i < n1; i++) {
65         M_factor = max(M_factor, *max_element(cost[i], cost[i]+n2));
66     }
67     for(i = 0; i < n1; i++) {
68         for(j = 0; j < n2; j++) {
69             cost[i][j] = M_factor - cost[i][j];
70         }
71     }
72 }

```

### Floyd - Warshall

```

1 // Try to actualize k times the minimum distance.
2 for(k = 0; k < n; ++k) {
3     for(i = 0; i < n; ++i) {
4         for(j = 0; j < n; ++j) {
5             if(dist[i][k] != INT_MAX && dist[k][j] != INT_MAX)
6                 dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
7         }
8     }
9 }

```

### LCA tree

```

1 const ll LOG_N = 20; //log2(MAX_N) + 4
2 const ll MAX_N = 1e5 + 3; //1e5, example
3 vector<vector<ll>> graph2, graph; //graph2 is the bidirectional and graph is the one you
    ↳ ask LCA
4 //----- LCA in a tree rooted at 0 -----
5 int parent[LOG_N][MAX_N]; //parent[i][j] is the ancestor 2^i of node j. Is a sparse table
6 int level[MAX_N]; //depth of the node in the tree
7
8 //call dfs0(0, 0);
9 void dfs0(int u, int p) {
10     parent[0][u] = p;
11     for(auto v : graph[u]) {
12         if(v == p) continue;
13         level[v] = level[u] + 1;
14         dfs0(v, u);
15     }
16 }
17
18 //O(n log n)
19 void preprocess() {
20     int i, j;
21     dfs0(0, 0);
22     for(i = 1; i < LOG_N; ++i) {
23         for(j = 0; j < MAX_N; ++j) {
24             parent[i][j] = parent[i - 1][parent[i - 1][j]];
25         }
26     }
27 }
28
29 //rise b to the same level as a and continue moving up. O(log n)
30 int lca(int a, int b) {
31     int i;
32
33     if(level[a] > level[b]) swap(a, b);

```

```

34     int d = level[b] - level[a];
35
36     for(i = 0; i < LOG_N; ++i) {
37         if((d >> i) & 1) b = parent[i][b];
38     }
39     if(a == b) return a;
40
41     for(i = LOG_N - 1; i >= 0; --i) {
42         if(parent[i][a] != parent[i][b])
43             a = parent[i][a], b = parent[i][b];
44     }
45
46     return parent[0][a];
47 }
48
49 //distance between two nodes in a tree
50 int dist(int u, int v) {
51     return level[u] + level[v] - 2 * level[lca(u, v)];
52 }
53
54 //call dfs(0, -1) to root a tree at 0. the graph had to be bidirectional
55 vector<bool> visitedd;
56 void dfs(int x, int p) {
57     if(visitedd[x]) return;
58     visitedd[x] = true;
59     if(p != -1) graph[p].pb(x);
60     for(auto el : graph2[x]) {
61         if(el == p) continue;
62         dfs(el, x);
63     }
64 }

```

### Kosaraju

```

1  vector<vi> adyList; // Graph
2  vector<int> visited; // Visited for DFS
3  vector<vi> sccs; // Contains the SCCs at the end
4
5  void dfs(int nnode, vector<int> &v, vector<vi> &adyList) {
6      if (visited[nnode]) {
7          return;
8      }
9      visited[nnode] = true;
10     for (auto a : adyList[nnode]) {
11         dfs(a, v, adyList);
12     }
13     v.push_back(nnode);
14 }
15
16 void Kosaraju(int n) {
17     visited = vi(n, 0);
18     stack<int> s = stack<int>();
19     sccs = vector<vi>();
20
21     vector<int> postorder;
22     for (int i = 0; i < n; ++i) {
23         dfs(i, postorder, adyList);
24     }
25     reverse(all(postorder));
26
27     vector<vi> rAdyList = vector<vi>(n, vi());
28     for (int i = 0; i < n; ++i) {
29         for (auto v : adyList[i]) {

```

```

30         rAdyList[v].push_back(i);
31     }
32 }
33
34 visited = vi(n, 0);
35 vi data;
36 for (auto a : postorder) {
37     if (!visited[a]) {
38         data = vi();
39         dfs(a, data, rAdyList);
40         if (!data.empty())
41             sccs.pb(data);
42     }
43 }
44 }

```

## Mathematics

### Binary operations

```

1  ll elevate(ll a, ll b) { // b >= 0.
2      ll ans = 1;
3      while(b) {
4          if(b & 1) ans = ans * a % mod;
5          b >>= 1;
6          a = a * a % mod;
7      }
8      return ans;
9  }
10
11 // a^(mod - 1) = 1, Euler.
12 ll inv(ll a) {
13     return elevate(((a%mod) + mod)%mod, mod - 2);
14 }
15
16 ll mul(ll a, ll b) {
17     ll ans = 0, neg = (a < 0) ^ (b < 0);
18     a = abs(a); b = abs(b);
19     while(b) {
20         if(b & 1) ans = (ans + a) % mod;
21         b >>= 1;
22         a = (a + a) % mod;
23     }
24     if(neg) return -ans;
25     return ans;
26 }

```

### Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

### Combinatoric numbers

```

1  const int MAX_C = 1+66; // 66 is the for long long, C(66, x)
2  ll Comb[MAX_C][MAX_C];
3
4  void calc() {
5      int i, j;
6      for(i = 0; i < MAX_C; i++) {
7          Comb[i][0] = 1;
8          Comb[0][i] = 1;
9      }

```

```

10     for(i = 1; i < MAX_C; i++) {
11         for(j = 1; j < MAX_C; j++) {
12             if(i+j >= MAX_C) continue;
13             Comb[i][j] = Comb[i-1][j] + Comb[i][j-1];
14         }
15     }
16 }
17
18 ll C(ll i, ll j) {
19     return Comb[i-j][j];
20 }

```

### Chinese Remainder

```

1  const ll MAX = 10;
2  ll a[MAX], p[MAX], n;
3  // Given n x == a[i] mod p[i], find x, or -1 if it doesn't exist.
4  // Let q[i] = (\prod_{i=0}^{n-1} p[j])/p[i].
5  // x will be = \sum_{i=0}^{n-1} a[i]*q[i]*inv(q[i], mod p[i])
6  ll chinese_remainder() {
7      ll i, j, g, ans = 0, inv1, inv2;
8      mod = 1;
9      for(i = 0; i < n; i++) { // If the p[i] are not coprimes, do them coprimes.
10         a[i] %= p[i]; a[i] += p[i]; a[i] %= p[i];
11         for(j = 0; j < i; j++) {
12             g = __gcd(p[i], p[j]);
13             if((a[i]%g + g)%g != (a[j]%g + g)%g) return -1;
14             // Delete the repeated factor at the correct side.
15             if(__gcd(p[i]/g, p[j]) == 1) {p[i] /= g; a[i] %= p[i];}
16             else {p[j] /= g; a[j] %= p[j];}
17         }
18     }
19     // If you have a supermod, take P = min(P, supermod);
20     for(i = 0; i < n; i++) {
21         mod *= p[i];
22     }
23     for(i = 0; i < n; i++) {
24         gcdEx(mod/p[i], p[i], &inv1, &inv2);
25         ans += mul(a[i], mul(mod/p[i], inv1));
26         ans %= mod;
27     }
28     return (ans%mod + mod) % mod;
29 }

```

### Fast Fourier Transform FFT

```

1  typedef complex<double> cd;
2  typedef vector<cd> vcd;
3
4  void show(vcd &e) { //for debug
5      int cont = 0; for(auto el : e) {cout << " + " << (el.real() > eps ? el.real() : 0) <<
6          << "x^" << cont++;} cout << endl;
7  }
8  void convolution(vcd &a) { //insert a_i and get y_i = sum_j(a_j*w_i^j)
9      int i, n = a.size(); //n power of 2
10     if(n == 1) return;
11     vcd a_even, a_odd;
12     for(i = 0; i < n; i++) { //divide part of FFT
13         if(i%2) a_odd.pb(a[i]);
14         else a_even.pb(a[i]);
15     }
16     convolution(a_even); //recursive part
17     convolution(a_odd);
18     cd wn = polar(1.0, 2*(double)PI/n), w = 1.0; //w^n are the n roots of n-unity

```

```

18     //cd w;
19     for(i = 0; i < n/2; i++) {
20         //w = polar(1.0, i*2*(double)PI/n); //avoid precision error, but slower
21         a[i] = a_even[i] + w*a_odd[i]; //A(wn^k) = Aeven(wn/2^k) + wn^k*Aodd(wn/2^k)
22         a[i + n/2] = a_even[i] - w*a_odd[i]; //A(wn^k) = Aeven(wn/2^(k-n/2)) -
           ↪ wn^(k-n/2)+Aodd(wn/2^(k-n/2))
23         w = w*wn;
24     }
25 }
26 void deconvolution(vcd &a) { //insert y_i and get a_i = sum_j(y_j*w_i^-j)/n
27     for(auto &el : a) el = conj(el); //you can conjugate wn and do a[i]/n o can
           ↪ conj(a[i])/n
28     convolution(a); // The coefficients of the polynomial have to be are real
29     for(auto &el : a) el /= (double)a.size();
30 }
31 // Calculate \sum_{i=0}^{n-1} a[i]*b[n-i].
32 vcd FFT(vcd &a, vcd &b) { //multiply polynomial a*b
33     //vcd a = {1.0, 2.0}, b = {3.0}, c; // a and b examples of polynomials to multiply, real
           ↪ coefficients
34     vcd c;
35     if(a.size() < b.size()) swap(a, b);
36     int i, n = a.size();
37     while(n - LSB(n)) n++, a.pb(0.0); //add 0.0's to the next power of two of the next
           ↪ power of two, 3->8
38     n++, a.pb(0.0);
39     while(n - LSB(n)) n++, a.pb(0.0);
40     while((int) b.size() < n) b.pb(0.0); //the grade of a and b equal.
41     convolution(a);
42     convolution(b); //if you want a*a then delete this 2° call
43     for(i = 0; i < n; i++) c.pb(a[i]*b[i]);
44     deconvolution(c);
45     return c;
46 }

```

### Euclides

```

1  ll gcdEx(ll a, ll b, ll *x1, ll *y1) {
2      if(a == 0) {
3          *x1 = 0;
4          *y1 = 1;
5          return b;
6      }
7      ll x0, y0, g;
8      g = gcdEx(b%a, a, &x0, &y0);
9
10     *x1 = y0 - (b/a)*x0;
11     *y1 = x0;
12
13     return g;
14 }

```

### Linear Sieve

```

1  const int MAX_PRIME = 1e6+5;
2  bool num[MAX_PRIME]; // If num[i] = false => i is prime.
3  int num_div[MAX_PRIME]; // Number of divisors of i.
4  int min_div[MAX_PRIME]; // The smallest prime that divide i.
5  vector<int> prime;
6
7  void linear_sieve(){
8      int i, j, prime_size = 0;
9      min_div[1] = 1;
10     for(i = 2; i < MAX_PRIME; ++i){

```

```

11         if(num[i] == false) {prime.push_back(i); ++prime_size; num_div[i] = 1; min_div[i] =
12             ↪ i;}
13
14         for(j = 0; j < prime_size && i * prime[j] < MAX_PRIME; ++j){
15             num[i * prime[j]] = true;
16             num_div[i * prime[j]] = num_div[i] + 1;
17             min_div[i * prime[j]] = min(min_div[i], prime[j]);
18             if(i % prime[j] == 0) break;
19         }
20     }
21
22     bool is_prime(ll n) {
23         for(auto el : prime) {
24             if(n == el) return true;
25             if(n%el == 0) return false;
26         }
27         return true;
28     }
29
30     vll fact, nfact; // The factors of n and their exponent.
31     void factorize(int n) { // Up to MAX_PRIME*MAX_PRIME.
32         ll cont, prev_p;
33         fact.clear(); nfact.clear();
34         for(auto p : prime) {
35             if(n < MAX_PRIME) break;
36             if(n%p == 0) {
37                 fact.pb(p);
38                 cont = 0;
39                 while(n%p == 0) n /= p, cont++;
40                 nfact.pb(cont);
41             }
42         }
43         if(n >= MAX_PRIME) {
44             fact.pb(n);
45             nfact.pb(1);
46             return;
47         }
48         while(n != 1) { // When n < MAX_PRIME, factorization in almost O(1).
49             prev_p = min_div[n];
50             cont = 0;
51             while(n%prev_p == 0) n /= prev_p, cont++;
52             fact.pb(prev_p);
53             nfact.pb(cont);
54         }
55     }

```

### BIT Fenweick tree

```

1  template<typename T>
2  class BIT{
3      vector<T> bit;
4      int n;
5      public:
6      BIT(int _n) {
7          n = _n;
8          bit.assign(n+1, 0);
9      }
10     BIT(vector<T> v) {
11         n = v.size();
12         bit.assign(n+1, 0);
13         for(int i = 0; i < n; i++) update(i, v[i]);
14     }

```

```
15 // Point update.
16 void update(int i, T dx) {
17     for(i++; i < n+1; i += LSB(i)) bit[i] += dx;
18 }
19 // query [0, r].
20 T query(int r) {
21     T ans = 0;
22     for(r++; r > 0; r -= LSB(r)) ans += bit[r];
23     return ans;
24 }
25 // query [l, r].
26 T query(int l, int r) {
27     return query(r) - query(l-1);
28 }
29 // k-th smallest element inserted.
30 int k_element(ll k) { // k > 0 (1-indexed).
31     int l = 0, r = n+1, mid;
32     if(query(0) >= k) return 0;
33     while(l + 1 < r) {
34         mid = (l + r)/2;
35         if(query(mid) >= k) r = mid;
36         else l = mid;
37     }
38     return r;
39 }
40 };
```

## Strings

### KMP

```

1 // Knuth-Morris-Pratt. Search the occurrences of t (pattern to search) in s (the text).
2 // O(n). It increases j at most n times and decreases at most n times.
3 void KMP(string &s, string &t) {
4     int n = s.length(), m = t.length(), i, j, len = 0;
5     // Longest proper prefix that is also suffix.
6     // s[0..lps[i]-1] == s[i-lps[i]+1..i].
7     vi lps(m, 0);
8     for(i = 1; i < m; i++) {
9         if(t[i] == t[len]) {
10             len++;
11             lps[i] = len;
12         } else if(len > 0) {
13             len = lps[len - 1];
14             i--;
15         }
16     }
17     for(i = 0, j = 0; i < n; i++) {
18         if(s[i] == t[j]) {
19             j++;
20             if(j == m) {
21                 echo("Pattern found at:", i-j+1);
22                 // You will math at least lps[j-1] chars.
23                 j = lps[j - 1];
24             }
25         } else if(j > 0) {
26             j = lps[j - 1];
27             i--;
28         }
29     }
30 }

```

### Longes Palindromic Substring

```

1 // LPS Longest Palindromic Substring, O(n).
2 void Manacher(string &str) {
3     char ch = '#'; // '#' a char not contained in str.
4     string s(1, ch), ans;
5     for(auto c : str) {s += c; s += ch;}
6     int i, n = s.length(), c = 0, r = 0;
7     vi lps(n, 0);
8     for(i = 1; i < n; i++) {
9         // lps[i] >= it's mirror, but falling in the interval [L..R]. L = c - (R - c).
10        if(i < r) lps[i] = min(r - i, lps[c - (i - c)]);
11        // Try to increase.
12        while(i-lps[i]-1 >= 0 && i+lps[i]+1 < n && s[i-lps[i]-1] == s[i+lps[i]+1])
13            ↪ lps[i]++;
14        // Update the interval [L..R].
15        if(i + lps[i] > r) c = i, r = i + lps[i];
16    }
17    // Get the longest palindrome in ans.
18    int pos = max_element(lps.begin(), lps.end()) - lps.begin();
19    for(i = pos - lps[pos]; i <= pos + lps[pos]; i++) {
20        if(s[i] != ch) ans += s[i];
21    }
22    //cout << ans.size() << "\n";
23 }

```



## Z-algorithm

```

1 // Search the occurrences of t (pattern to search) in s (the text).
2 // O(n + m). It increases R at most 2n times and decreases at most n times.
3 // z[i] is the longest string s[i..i+z[i]-1] that is a prefix = s[0..z[i]-1].
4 void z_algorithm(string &s, string &t) {
5     s = t + "$" + s; // "$" is a char not present in s nor t.
6     int n = s.length(), m = t.length(), i, L = 0, R = 0;
7     vi z(n, 0);
8     // s[L..R] = s[0..R-L], [L, R] is the current window.
9     for(i = 1; i < n; i++) {
10         if(i > R) { // Old window, recalculate.
11             L = R = i;
12             while(R < n && s[R] == s[R-L]) R++;
13             R--;
14             z[i] = R - L + 1;
15         } else {
16             if(z[i-L] < R - i) z[i] = z[i-L]; // z[i] will fall in the window.
17             else { // z[i] can fall outside the window, try to increase the window.
18                 L = i;
19                 while(R < n && s[R] == s[R-L]) R++;
20                 R--;
21                 z[i] = R - L + 1;
22             }
23         }
24         if(z[i] == m) { // Match found.
25             //echo("Pattern found at: ", i-m-1);
26         }
27     }
28 }

```

## Suffix-Automaton

```

1 #define next _42_
2 //Suffix Automaton, save a directed acyclic graph and a suffix link tree with all the
  ⇨ suffix of a word
3 struct state {
4     //length of the longest string in the equivalence classes
5     int len;
6     //suffix link
7     int link = -1;
8     map<char, int> next;
9     state(int _len) {
10         len = _len;
11     }
12 };
13
14 vector<state> t = {{0}};
15 int t_size = 1, last = 0;
16
17 //add a character to the automaton
18 //last is the state of the last char c added, p is the head of the automaton
19 //q is the state to duplicate
20 void sa_extend(char c) {
21     int p = last, q;
22     t.pb({t[last].len + 1});
23     last = t_size; t_size++;
24     //add c to the previous suffixes
25     while(p != -1 && t[p].next.find(c) == t[p].next.end()) {
26         t[p].next[c] = last;
27         p = t[p].link;
28     }
29     //first time of c in the string
30     if(p == -1) {

```

```

31         t[last].link = 0;
32         return;
33     }
34     q = t[p].next[c];
35     if(t[p].len + 1 == t[q].len) {
36         t[last].link = q;
37         return;
38     }
39     //clone state q
40     t.pb({t[p].len + 1});
41     t_size++;
42     t[t_size - 1].next = t[q].next;
43     t[t_size - 1].link = t[q].link;
44
45     //add links of last and q
46     t[last].link = t_size - 1;
47     t[q].link = t_size - 1;
48
49     //point the last suffixes to q cloned
50     while(p != -1 && t[p].next.find(c) != t[p].next.end()) {
51         t[p].next[c] = t_size - 1;
52         p = t[p].link;
53     }
54 }
55
56 //O(s.length()) to create the automaton. Be careful adding any char once called another
57 ↪ function
58 void sa_ini(string &s) {
59     for(char c : s) sa_extend(c);
60 }
61
62 //A path from root to a terminal node is a suffix of the automaton string
63 vector<bool> terminal;
64 void sa_terminal() {
65     int p = last;
66     if(terminal.empty() == false) return; //previously calculated
67     terminal.assign(t_size, false);
68     while(p != -1) {
69         terminal[p] = true;
70         p = t[p].link;
71     }
72 }
73
74 //true if w is a substring of the automaton string
75 //Also s is the longest prefix of w that is in s
76 //w is a suffix if the last p is a terminal state
77 bool sa_is_substr(string &w) {
78     int p = 0; //string s;
79     for(char ch : w) {
80         if(t[p].next.find(ch) == t[p].next.end()) return false;
81         p = t[p].next[ch];
82         //s += c;
83     }
84     return true;
85 }
86
87 vll dp_num_substr;
88 ll num_substr_rec(int i) {
89     ll sum = 1;
90     if(dp_num_substr[i] != -1) return dp_num_substr[i];
91     for(auto el : t[i].next) sum += num_substr_rec(el.se);
92     return dp_num_substr[i] = sum;

```

```

92 }
93 //Number of different substrings of the automaton string (Is the number of different paths
   ↳ in the automaton)
94 //For the number of the length of all different substring the recursive formula is
95 // sum of dp_num_substr[i] + dp_num_len_substr[i], the previous answer + 1*number of
   ↳ different substrings
96 ll sa_num_substr() {
97     if(dp_num_substr.empty() == false) return dp_num_substr[0]; //previously calculated
98     dp_num_substr.assign(t_size, -1);
99     num_substr_rec(0);
100     return dp_num_substr[0]; // -1 if you don't want the empty substring
101 }
102
103 //k-th string in the sorted substrings set of the automaton string. It's the k-th path in
   ↳ the graph
104 //k is [0..sa_num_substr()-1]
105 string sa_k_substr(int k) {
106     int p = 0;
107     char prev = '$';
108     string ans = "";
109     if(k > sa_num_substr()) return ans; //not exists that k-th string, error
110     while(k > 0) {
111         prev = '$';
112         for(auto el : t[p].next) {
113             prev = el.fi;
114             if(dp_num_substr[el.se] >= k) break;
115             k -= dp_num_substr[el.se];
116         }
117         if(prev == '$') break; //error
118         ans += prev;
119         p = t[p].next[prev];
120         k--;
121     }
122     return ans;
123 }
124
125 //lexicographically smallest cyclic shift of the string s
126 string sa_small_cyclic_shift(string &s) {
127     int p = 0, cnt = s.length();
128     string ans = "";
129     sa_ini(s + s); //initialize sa with s+s, the ans is greedy the first path with length
   ↳ s.length()
130     while(cnt-- > 0) {
131         auto el = *(t[p].next.begin()); //take greedy the first edge
132         ans += el.fi;
133         p = el.se;
134     }
135
136     return ans;
137 }
138
139 //int sa_num_occurrences(string w); //Better use Aho-Corasick
140
141
142 //Test of the automaton string, the number of the substrings and the substrings, sorted
143 void sa_test1() {
144     ll i, n;
145     sa_ini("test");
146     n = sa_num_substr();
147     cout << n << endl;
148     for(i = 0; i < n; i++)
149         cout << sa_k_substr(i) << endl;

```

150 | }

## Aho-Corasick

```

1 //construct trie  $O(m)$  + automaton  $O(mk)$ ,  $O(mk)$  memory,  $m = \text{sum}(\text{len}(\text{word}_i))$ 
2 #define next asdfa
3 //size of alphabet, 26 lowercase
4 const int k = 26;
5
6 struct vertex{
7     vi next;
8     //number of words ending at current vertex
9     int leaf;
10    //ancestor p and ch is the transition of p->v
11    int p;
12    char pch;
13    //proper suffix link of the vertex
14    int link;
15    vi go;
16    //how many suffixes there are in the tree;
17    int count;
18
19    vertex(int _p, char _pch) {
20        next.assign(k, -1);
21        leaf = 0;
22        this->p = _p;
23        this->pch = _pch;
24        link = -1;
25        go.assign(k, -1);
26        count = -1;
27    }
28 };
29
30 vector<vertex> t = {{-1, '$'}};
31 int t_size = 1;
32
33 //add string to the trie t
34 void add_string(string &s) {
35     int c, p = 0;
36     for(char ch : s) {
37         c = ch - 'a';
38         if(t[p].next[c] == -1) {
39             t.pb({p, ch});
40             t[p].next[c] = t_size++;
41         }
42         p = t[p].next[c];
43     }
44     t[p].leaf++;
45 }
46
47 //Search for any proper suffix of v that has next[c] transition
48 //call go(v, ch) for move the automaton from the vertex v using transition ch
49 int go(int v, char ch);
50
51 //get the proper suffix link of v. Once called, don't call anymore add_strings
52 int get_link(int v) {
53     if(t[v].link == -1) {
54         if(v == 0 || t[v].p == 0) t[v].link = 0;
55         else t[v].link = go(get_link(t[v].p), t[v].pch);
56     }
57     return t[v].link;
58 }
59

```

```

60 int go(int v, char ch) {
61     int c = ch - 'a';
62     if(t[v].go[c] == -1) {
63         if(t[v].next[c] != -1) t[v].go[c] = t[v].next[c];
64         //The root doesn't have next[c]
65         else if(v == 0) t[v].go[c] = 0;
66         else {
67             t[v].go[c] = go(get_link(v), ch);
68         }
69     }
70     return t[v].go[c];
71 }
72
73 //get the count of v
74 int count(int v) {
75     if(t[v].count == -1) {
76         t[v].count = t[v].leaf;
77         if(v != 0) t[v].count += count(get_link(v));
78     }
79     return t[v].count;
80 }
81
82 //search the number of the strings in the automaton that are in the text
83 int search_num_string(string &text) {
84     int p = 0, ans=0;
85
86     for(auto ch : text) {
87         ans += count(p);
88         p = go(p, ch);
89     }
90     ans += count(p);
91     return ans;
92 }

```

## Ad-hoc

```

1  int is_leap_year(int y) {
2      if(y%4 || (y%100==0 && y%400)) return 0;
3      return 1;
4  }
5  int days_month[12] = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
6  int days_month_accumulate[12] = {31, 59, 90, 120, 151, 181, 212, 243, 273, 304, 334, 365};
7
8  // d 1-index, m 1-index.
9  int date_to_num(int d, int m, int y) {
10     m -= 2;
11     int sum = d;
12     if(m >= 1) sum += is_leap_year(y);
13     y--;
14     if(m >= 0) sum += days_month_accumulate[m];
15     if(y >= 0) {
16         sum += 365*y;
17         sum += y/4 - y/100 + y/400;
18     }
19     return sum;
20 }
21
22 int nd, nm, ny; // Tiny optimization, binary search the year, month and day.
23 void num_to_date(int num) {
24     nd = 1; nm = 1; ny = 2020; // The date searched is >= this date.
25     while(date_to_num(nd, nm, ny) <= num) ny++;

```

```

26     ny--;
27     while(nm < 12 && date_to_num(nd, nm, ny) <= num) nm++;
28     nm--;
29     while(date_to_num(nd, nm, ny) <= num) nd++;
30     nd--;
31 }

```

### Hash Set

```

1  const int MAX = 2*1e5+5;
2  ll val[MAX]; // For random numbers and not index use f with random xor.
3
4  void ini() { // CALL ME ONCE.
5      srand(time(0));
6      for(int i = 0; i < MAX; i++) val[i] = rand();
7  }
8
9  // Hash_set contains a set of indices [0..MAX-1] with duplicates.
10 // a[i] = sum_x{val_x} % mod p[i].
11 class Hash_set {
12 public:
13     vll p = {1237273, 1806803, 3279209}; // Prime numbers.
14     vll a = {0, 0, 0};
15     int n = 3; // n = p.size();
16     // Insert index x.
17     void insert(int x) {
18         for(int i = 0; i < n; i++) a[i] = (a[i] + val[x]) % p[i];
19     }
20     // Insert all the elements of hs.
21     void insert (Hash_set hs) {
22         for(int i = 0; i < n; i++) a[i] = (a[i] + hs.a[i]) % p[i];
23     }
24
25     bool operator == (Hash_set hs) {
26         for(int i = 0; i < n; i++) if(a[i] != hs.a[i]) return false;
27         return true;
28     }
29 };

```