Junco de los Heros Valenzuela 23-04-2020

1) a) $1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1\int_0^1 c|x-y|\,dx\,dy =$

$$= c\int_0^1\int_0^x (x-y)\,dy\,dx + c\int_0^1\int_x^1 -(x-y)\,dy\,dx =$$

$(x\geq y)$ $\qquad$ $(x<y)$

$$= c\int_0^1\left[xy-\frac{y^2}{2}\right]_0^x dx + c\int_0^1\left[-xy+\frac{y^2}{2}\right]_x^1 dx =$$

$$= c\int_0^1\left(x^2-\frac{x^2}{2}\right)dx + c\int_0^1\left(-x+\frac{1}{2}+x^2-\frac{x^2}{2}\right)dx =$$

$$= c\int_0^1\frac{x^2}{2}dx + c\int_0^1\left(\frac{x^2}{2}-x+\frac{1}{2}\right)dx = c\left[\frac{x^3}{2\cdot 3}\right]_0^1 + c\left[\frac{x^3}{2\cdot 3}-\frac{x^2}{2}+\frac{x}{2}\right]_0^1 =$$

$$= c\left(\frac{1}{6}+\frac{1}{6}-\frac{1}{2}+\frac{1}{2}\right) = c\cdot\frac{2}{6} \Rightarrow 6=2c \Rightarrow \boxed{c=3}$$

vemos que $f(x,y)=\begin{cases}3|x-y| & \text{si } 0<x<1 \text{ y } 0<y<1\\ 0 & \text{en otro caso}\end{cases}$

cumple que su integral es 1 y que $f(x,y)\geq 0\ \forall x,y.$

b) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \overset{x\in(0,1)}{=} \int_0^1 3\cdot|x-y|\,dy =$

$$= \int_0^x 3\cdot(x-y)\,dy + \int_x^1 -3(x-y)\,dy = \left[3xy-3\frac{y^2}{2}\right]_0^x + \left[-3xy+3\frac{y^2}{2}\right]_x^1 =$$

$$= 3x^2-3\frac{x^2}{2} + -3x+\frac{3}{2}+3x^2-\frac{3x^2}{2} = 6x^2-\frac{6}{2}x^2-3x+\frac{3}{2} =$$

$$= 3x^2-3x+\frac{3}{2}$$

$$f_X(x)=\begin{cases}3x^2-3x+\frac{3}{2} & \text{si } 0<x<1\\ 0 & \text{en otro caso}\end{cases}$$

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$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \overset{y\in(0,1)}{=} \int_0^1 3|x-y|\,dx =$$

$$= \int_0^y -3(x-y)\,d\cancel{x} + \int_y^1 3(x-y)\,dx = \left[-3\frac{x^2}{2}+3yx\right]_0^y + \left[3\frac{x^2}{2}-3yx\right]_y^1 =$$

$$= -\frac{3y^2}{2} + 3y^2 + \frac{3}{2} - 3y - \frac{3y^2}{2} + 3y^2 =$$

$$= 6y^2 - \frac{6y^2}{2} + \frac{3}{2} - 3y = 3y^2 - 3y + \frac{3}{2}$$

$$f_Y(y) = \begin{cases} 3y^2 - 3y + \frac{3}{2} & \text{si } 0<y<1 \\ 0 & \text{en otro caso} \end{cases}$$

c) $$f_{X|Y=1/3}(t) = \frac{f(t,1/3)}{f_Y(1/3)} \overset{0\le t\le 1}{=} \frac{3|t-1/3|}{3\cdot\left(\frac{1}{3}\right)^2 - 3\cdot\frac{1}{3} + \frac{3}{2}} =$$

$$= \frac{3|t-\frac{1}{3}|}{\frac{1}{3} - 1 + \frac{3}{2}} = \frac{6\cdot 3}{5}|t-\tfrac{1}{3}| = \frac{18}{5}|t-\tfrac{1}{3}|$$

$$f_{X|Y=\frac{1}{3}}(t) = \begin{cases} \frac{18}{5}|t-\frac{1}{3}| & \text{si } 0<t<1 \\ 0 & \text{en otro caso} \end{cases}$$