Cálculo I CURSO ACADÉMICO 2014-2015	PRUEBA 1

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Problema 1 *(2pt) Demuestra por inducción que*

$$S(n) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

$S(1) = \frac{1}{2} = \frac{n}{n+1}$ o.k!

Supongamos $S(n) = \frac{n}{n+1} \overset{?}{\Rightarrow} S(n+1) = \frac{n+1}{n+2}$

$$S(n+1) = \frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^{\cancel{2}}}{\cancel{(n+1)}(n+2)}$$

$S(n+1) = \frac{n+1}{n+2}$ o.k!

Problema 2 *(4pt) Calcula los límites*

$$a)\ \lim_{n\to\infty}\left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}\right)^n \qquad b)\ \lim_{n\to\infty}\frac{(1+2+\cdots+n)}{(n+\frac{4n^3-n^4}{n^5})^2}\left(1-\frac{1}{n^2}\right)^{3n^2}.$$

$$a) = \lim_{n\to\infty} (S(n))^n = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\frac{1}{\left(\frac{n+1}{n}\right)^n} = \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)^n} \longrightarrow \frac{1}{e}$$

(b) Como $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$(b) = \lim_{n\to\infty} \underbrace{\frac{\frac{n(n+1)}{2}}{\left(n+\frac{4n^3-n^4}{n^5}\right)^2}}_{\downarrow (*)\ 1/2} \underbrace{\left[\left(1-\frac{1}{n^2}\right)^{n^2}\right]^3}_{\downarrow \left(\frac{1}{e}\right)^3};$$

$$(*)\ \frac{\frac{n^2+n}{2}}{\frac{(n^6+4n^3-n^4)^2}{n^{10}}} = \underbrace{\frac{n^{12}+n^{11}}{2(n^6+4n^3-n^4)^2}}_{\downarrow \frac{1}{2}}$$

Problema 3 *(4pt)*

a) Estudia la convergencia y convergencia absoluta de:

$$\sum_{n\geq 3}(-1)^n\left(\sin\left(\frac{n}{n^2+4}\right)\right)^{\frac{1}{\pi}}.$$

Para estudiar la conv. aplico el criterio de Leibnitz $a_n = \left[\operatorname{sen}\left(\frac{n}{n^2+4}\right)\right]^{\frac{1}{\pi}} =$

$a_3 = \left[\operatorname{sen}\left(\frac{3}{9+4}\right)\right]^{\frac{1}{\pi}} = \left[\operatorname{sen}\left(\frac{3}{13}\right)\right]^{\frac{1}{\pi}}$ $\quad \operatorname{sen}(x)^{\frac{1}{\pi}}$ decreciente en $[0, \frac{\pi}{2}]$

$\frac{n}{n^2+4} = \frac{1}{n+\frac{4}{n}}$ decreciente y $\in [0 \frac{\pi}{2}]$ $\forall n \geq 3$

Luego a_n es monótona decreciente

$a_n > 0$

$\lim_{n\to\infty}\left(\operatorname{sen}\frac{n}{n^2+4}\right)^{\frac{1}{\pi}} = \lim_{n\to\infty}\left(\frac{n}{n^2+4}\right)^{\frac{1}{\pi}} = 0$

$\Rightarrow$ La serie alterna converge

$\sum_{n\geq 3}\operatorname{sen}\left(\frac{n}{n^2+4}\right)^{\frac{1}{\pi}}$ $\quad \lim_{n\to\infty}\frac{\operatorname{sen}\left(\frac{n}{n^2+4}\right)^{\frac{1}{\pi}}}{\left(\frac{1}{n}\right)^{\frac{1}{\pi}}} = 1$ $\quad$ Como $\frac{1}{\pi}<1$ $\sum\left(\frac{1}{n}\right)^{\frac{1}{\pi}}$ diverge

Luego la serie no converge absolutamente.

b) Estudia la convergencia de

$$\sum_{n\geq 2}\frac{\left(\sqrt{n+1}+\frac{1}{\sqrt{n+1}}\right)^{2n}}{\left(\sqrt{n}-\frac{1}{\sqrt{n}}\right)^{2(n+1)}\log(n)}.$$

$$\frac{\left(\sqrt{n+1}+\frac{1}{\sqrt{n+1}}\right)^{2n}}{\left(\sqrt{n}-\frac{1}{\sqrt{n}}\right)^{2(n+1)}}\frac{1}{\log(n)} = \frac{\left(1+\frac{1}{n+1}\right)^{2n}\left(\sqrt{n+1}\right)^{2n}}{\left(1-\frac{1}{n}\right)^{2n}\left(\sqrt{n}\right)^{2n}}\frac{1}{\left(\sqrt{n}-\frac{1}{\sqrt{n}}\right)^2}\frac{1}{\log(n)} =$$

$$= \frac{\left(1+\frac{1}{n+1}\right)^{2(n+1)}(n+1)^n}{\left(1-\frac{1}{n}\right)^{2n}n^n}\frac{1}{\left(1+\frac{1}{n+1}\right)^2}\frac{1}{\left(\sqrt{n}-\frac{1}{\sqrt{n}}\right)^2}\frac{1}{\log(n)} =$$

$$= \underbrace{\frac{\left[\left(1+\frac{1}{n+1}\right)^{n+1}\right]^2}{\left[\left(1-\frac{1}{n}\right)^n\right]^2}\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e}\underbrace{\frac{1}{\left(1+\frac{1}{n+1}\right)^2}}_{\to 1}}_{\to e^5}\underbrace{\frac{1}{\left(\sqrt{n}-\frac{1}{\sqrt{n}}\right)^2}\frac{1}{\log(n)}}_{b_n}$$

e^2 $\quad \left(\frac{1}{e}\right)^2$

Es equivalente estudiar la convergencia de $\sum b_n$; Sea $c_n = \frac{1}{n\log(n)}$

$\lim_{n\to\infty}\frac{b_n}{c_n} = 1$; Es equivalente estudiar la convergencia $\sum c_n$.

Por condensación $\sum 2^n c_{2^n} = \sum \cancel{2^n}\frac{1}{\cancel{2^n}\, n\log(2)} \sim \sum\frac{1}{n}$ diverge.

La serie diverge!!