Ejercicio 01-03-2021

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a) $$\frac{\partial(c\mu M)}{\partial y} = c\,\frac{\partial \mu M}{\partial y} \overset{\mu \text{ es factor integrante}}{=} c\,\frac{\partial \mu N}{\partial x} = \frac{\partial(c\mu N)}{\partial x},$$ así que

$c\mu$ es un factor integrante.

b) $$\frac{\partial(F\mu M)}{\partial y} = \frac{\partial F}{\partial y}\cdot \mu M + F\frac{\partial \mu M}{\partial y} \overset{\nabla F=(M,N),\ \mu \text{ factor integrante}}{=} N\cdot\mu\cdot\frac{\partial F}{\partial x} + F\frac{\partial \mu N}{\partial x} =$$

$$= \frac{\partial(F\mu N)}{\partial x},$$ así que $F\mu$ es factor integrante

c) $$\frac{\partial(g(F)\mu M)}{\partial y} = \frac{\partial g(F)}{\partial y}\mu M + g(F)\frac{\partial \mu M}{\partial y} \overset{\text{regla cadena, } g \text{ es } C^1,\ \mu \text{ factor integrante}}{=}$$

$$= g'(F)\cdot\frac{\partial F}{\partial y}\mu M + g(F)\frac{\partial \mu N}{\partial x} \overset{\text{regla cadena},\ \nabla F=(M,N)}{=} \frac{\partial g(F)}{\partial x}\mu N + g(F)\frac{\partial \mu N}{\partial x} =$$

$$= \frac{\partial\, g(F)\mu N}{\partial x},$$ así que $g(F)$ es factor integrante.

d) Sea G la integral de g: $G = \int g(t)\,dt$

y $F_2 = G(F)$, con $\nabla F = (\mu M, \mu N)$, ya que

μ es factor integrante.

Vemos que G es C^1 ~~[illegible]~~ por el TFC, y usamos

la regla de la cadena

$$\frac{\partial F_2}{\partial x} = \frac{\partial G(F)}{\partial x} = \frac{\partial G(F)}{\partial F}\cdot\frac{\partial F}{\partial x} = g(F)\cdot\mu M$$

$$\frac{\partial F_2}{\partial y} = \frac{\partial G(F)}{\partial y} = \frac{\partial G(F)}{\partial F}\cdot\frac{\partial F}{\partial y} = g(F)\cdot\mu N$$

Por lo que $\nabla F_2(x,y) = (g(F)\mu M,\ g(F)\mu N)$ y

(1) es exacta $\Rightarrow g(F)\mu$ es un factor integrante.