Numerical Solution of the Current Distribution and Input Impedance of a Linear Antenna Based on MoM

Junda Feng

***Abstract*—In this literature, a program of moment method is developed to calculate the current distribution and input impedance of a linear antenna having a length of 0.5λ and a radius of 0.001λ at a frequency of 1 GHz. The effects of the discretization density Δl and the delta-gap width on the numerical solution is examined. The numerical results show good agreement with the results generated by existing commercial software.**

***Index Terms*—Method of Moments (MoM), Antenna Analyses, Parameter Sweep, Matlab.**

I. INTRODUCTION

A. the Moment Methods

Finite difference method (FDM) and finite element method (FEM) are used to solve partial differential equations (PDE), so they are called PDE methods. They have a lot of in common and share three important features:

1) they both solve potential of field in space.
2) their systems are sparse matrices because their fields at one point only interact with the field at neighbouring points.
3) we need to truncate the open space into a finite space by introducing artificial box.

Moment methods [1] are based on integral equations (IE) and are fundamentally different with FDM and FEM. We convolve the input with the impulse response (Green's function) and integrate the convolution in the entire volume to get the field or potential. If we can calculate this integral of Green's function, we can convert the IE to the linear algebraic equations by first discretizing the surface of object into small patches and the contour of the object into small segments and then integrating the entire equation over all the small patches/segments numerically or analytically. The unknowns to be solved are the expansion coefficients of the source, which might be current or charge density. The resulting system matrix will be a full matrix because all the source can contribute to the potential/field at each observation point. Thus we can tell that the MoM is a prefered method for most of open space problems in that we no longer have to discretize the entire large space and our discretization is limited to the surface of the object. To be more specific, with moment method, 3D volumetric problem is reduced to surface problem and 2D surface problem is reduced to 1D contour problem.

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In addition, the MoM code used for calculating this thin wire antenna is actually very versatile. We can analyze any EM problems that involve wires, even the scattering problem by solid body. From the EM point of view, a piece of solid conductor is electromagnetically equivalent to and can be approximated by a mesh as long as the edge size of the mesh is smaller than 0.1λ. Thus, we can use the wires that form the mesh to replace the solid conducting panels in the calculation. The philosophy and methodology are the basis of the very first highly successful Numeric Electromagnetic code (NEC) [2] using the moment methods in the 1990s and earily 2000s, which has been widely used for a variety of problems. Now, with the ever growing computational power and the theoretical advancement in the moment methods, we can write more advanced MoM code to deal with surface problems more accurately.

B. Thin Wire Linear Antenna

The linear antenna is one of the fundamental antennas that have been extensively studied due to its structural simplicity. It is of practical interest in that realistic antennas can be modeled by wires whose radius is significantly smaller than their length as well as the wavelength. Thus, the linear antenna in this course project is formulated based on thin wire approximation. It is well known that a dipole in free space has its first resonance at a length just under $\frac{\lambda}{2}$, which is exactly the wire length in our problem setting.

C. Delta-gap Source Model

In antenna problems, the unknowns of interest are most often the input impedance at a particular feed point for the later impedance matching of the wave guide and the current distribution along the wire for the later calculation of the radiation patterns. Thus, we need a way to model the field induced by the feed system without modeling the system itself. Two feed methods are commonly used in thin wire problems, the delta-gap source model and the magnetic frill model. The delta-gap source model is extensively used in modeling linear antennas in the method of moments (MOM) due to its simple expression form [3]. It treats the feed as if the field due to the feed line exists only in the gap between the antenna terminals, with a value of zero outside. This method typically produces less accurate results for input impedance, though it still performs well in computing radiation patterns [4]. It is not accurate to calculate the input impedance simply because no realistic feed point can be infinitesimally small. The magnetic frill source, as a counterpart of the delta-gap source formulated by surface magnetic current, is usually more accurate and has faster convergence. However, we will only focus on the delta-gap source model in this course project due to its simple implementation of the source model [5][6].

The organization of this course project report is as follows. In Section II, we formulate the thin wire problem according to [7]. In Section III, we give the results of current distribution and input impedance with respect to the variation of the discretization density and the delta-gap width. In addition, we used fast frequency sweep to loop through the frequency in a specific range and compare time and compare the result with the commercial software MININEC Pro [8][9]. The discrete frequency sweep result generated by the Matlab code is comparable to that of the MININEC Pro, which demonstrates the correctness of the code.

II. Formulation

A. Thin wire and Delta-gap source formulation

In the thin wire assumption, the current is mostly flow along the wire, the transverse current is usually small and can be ignored. The thin wire model in our problem is illustrated in Fig. 1. The material of the dipole is perfect electric conductor (PEC) and the dipole itself is in the free space environment. The thin wire has a radius a of 0.001λ and a length of 0.5λ. Suppose the antenna is working at a frequency of 1GHz, the wavelength λ will equal to 0.3 meter. As we can see, when we are not considering the existence of the delta-gap source, the center-fed, half-wavelength dipole antenna made from conducting thin wire is discretized into N equally spaced small segments with a length of Δl. To examine the effect of the delta-gap width, we set the length of the delta-gap segment in the middle of the thin wire where the fed point resides to be Δz.

The delta-gap source model assumes that the impressed electric field in the thin gap between the antenna terminals can be expressed as

$$\boldsymbol{E}^{inc} = \frac{V_0}{\Delta z}\hat{z}$$

where Δz is the width of the gap and V_0 denotes the voltage of the delta-gap source. We can simply set $V_m = V_0 = 1$ at the feed point and $V_m = 0$ elsewhere. To let the delta-gap source be right in the middle, we would like to choose an odd number of discrete segments (even number of nodes and basis/test functions) for simplicity.

B. MoM formulation

The MoM, similar to the FEM, can be considered as special a form of weighted residual method. To make the representation of the IE more familiar to us, we can denote the IE as Equ. (1) in a more abstract level, where the RHS f is the integrated result, the integral operator $\mathcal{L}$ would be the surface integral or contour integral of the Green's function, and ϕ is the unknown inside the IE.

$$\mathcal{L}\phi = f \tag{1}$$

For this particular problem, the unknown ϕ is actually the vector current $\boldsymbol{I}\hat{l}$ flowing along the wire and the operator $\mathcal{L}$ is actually the electric field integral equation (EFIE) operator expressed as Equ. (2). The RHS f should be the incident field $\boldsymbol{E}^{inc}$ divided by the wave impedance in free space Z_0.

$$\mathcal{L}(\boldsymbol{X}) = jk_0 \oiint_{S_o} \left[\boldsymbol{I} + \frac{1}{k_0^2}\nabla\nabla'\cdot\right] \boldsymbol{X}(\boldsymbol{r}')G_0(\boldsymbol{r},\boldsymbol{r}')\,\mathrm{d}S' \tag{2}$$

First, we discretize the unknown function with vector triangular basis function $\boldsymbol{\Lambda}$ and multiply it with unknown expansion coefficients as Equ. (3), substituting which into the Equ. (1) yields Equ. (4).

$$\boldsymbol{I} = \sum_{n=1}^{N-1} I_n \boldsymbol{\Lambda}_n \tag{3}$$

$$\sum_{n=1}^{N-1} I_n \mathcal{L}(\boldsymbol{\Lambda}_n) = f \tag{4}$$

Here, we use triangle functions to solve the thin-wire EFIE in that the current will automatically become zero at both ends of the wire. For this thin conducting wire problem, the divergence operator in the EFIE operator in Equ. (2) will become the derivative of the current along the wire and thus can be evaluated to $\frac{1}{\Delta l}$ on a segment of length Δl. Next, we convert this equation into a matrix equation by testing it also with $\boldsymbol{\Lambda}$ (Galerkin formulation) and integrate the resulting equation over the entire solution domain as follows (Equ. (5)).

$$\sum_{n=1}^{N-1} I_n \langle \boldsymbol{\Lambda}_m, \mathcal{L}(\boldsymbol{\Lambda}_n)\rangle = \langle \boldsymbol{\Lambda}_m, f\rangle \qquad m = 1, 2, \ldots, N \tag{5}$$

Equ. (5) defines a set of linear algebraic equations, which yields the system matrix for the MoM as shown in Equ. (6).

$$\boldsymbol{Z} \cdot \boldsymbol{I} = \boldsymbol{V} \tag{6}$$

where the matrix entries can be written as Equ. (7) and the RHS entries can be written as Equ. (8). Note that the resulting matrix will also be symmetric Toeplitz due to the fact that the specific problem we are dealing with is a linear symmetric geometry of dipole antenna fed at its central segment.

By using numerical integration when the two segments do not overlap, together with

$$k_0 = \omega\sqrt{\mu_0\epsilon_0}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

and three dimensional Green's function in free space

$$G_0(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}$$

Equ. (7) can be rewritten as the decretized form shown in Equ. (9), where $\boldsymbol{r}_p$ and $\boldsymbol{r}'_q$ are quadrature points along the wire and ω_p and ω_q are Gauss quadrature weights at the observation segment and source segment, respectively. Δp and Δq denote the length of the segment at the observation point and the source point respectively.

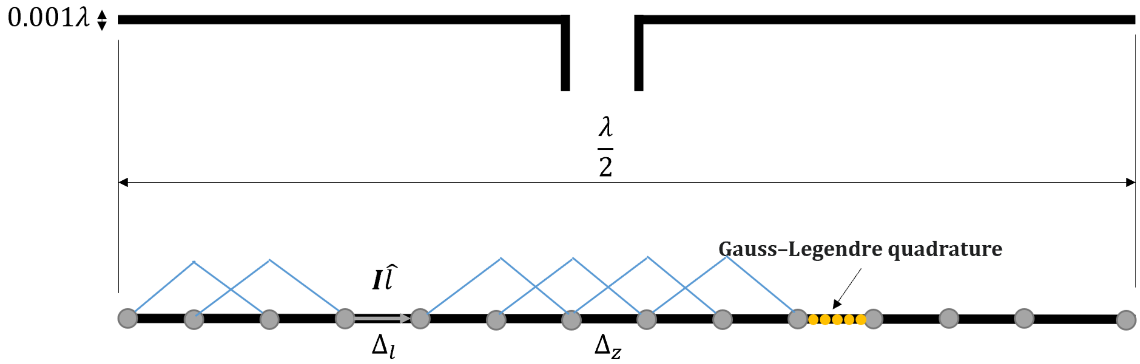

Fig. 1: Schematric illutration of the discretization of filamentary conducting wire carrying current $\boldsymbol{I}\hat{l}$ using five point Gauss-Legendre quadrature.

$$Z_{mn} = jk_0 Z_0 \int_C \Lambda_m(\boldsymbol{r})\hat{l}_m \cdot \int_C \Lambda_n(\boldsymbol{r'})\hat{l}_n G_0(\boldsymbol{r},\boldsymbol{r'})\,\mathrm{d}l'\,\mathrm{d}l - \frac{jZ_0}{k_0}\int_C \frac{\mathrm{d}\Lambda_m(\boldsymbol{r})}{\mathrm{d}l}\int_C \frac{\mathrm{d}\Lambda_n(\boldsymbol{r'})}{\mathrm{d}l'} G_0(\boldsymbol{r},\boldsymbol{r'})\,\mathrm{d}l'\,\mathrm{d}l \tag{7}$$

$$V_m = \int_C \Lambda_m(\boldsymbol{r})\hat{l}_m \cdot \boldsymbol{E}^{inc}\,\mathrm{d}l \tag{8}$$

$$Z_{mn} = \frac{1}{4\pi}\Sigma_{p=1}^{P}\Sigma_{q=1}^{P}\omega_p(\boldsymbol{r}_p)\omega_q(\boldsymbol{r}'_q)\left[j\omega\mu\Lambda_m(\boldsymbol{r}_p)\cdot\Lambda_n(\boldsymbol{r}'_q) \pm \frac{j}{\omega\epsilon\Delta p\Delta q}\right]\frac{e^{-jk_0R_{pq}}}{R_{pq}} \tag{9}$$

$$V_m = \frac{V_0}{2\Delta z}\times\Delta z = \frac{V_0}{2} \tag{10}$$

When the two segments are overlapped, the Green's function and thus the integrand may become singular and we need to evaluate the integral analytically as Equ. (11)

$$\Phi = \frac{\Delta l}{2\pi}\left(\ln\frac{2\Delta l}{a} - 1\right) - \frac{jk_0}{4\pi}(\Delta l)^2 \tag{11}$$

where Δl denotes the segment where the basis triangular function and testing triangular function overlaps. There are other singularity extraction techniques developed in [10][11][12]. When evaluating the triangular function in Equ. (11), we can replace it by the constant sampled at the middle of the particular segment. The resulting RHS vector element can be calculated as Equ. (10).

III. Results

In this section, we first compare the frequency sweep result of input impedance between commercial software and our Matlab code. And then we sweep through the frequency f, segment size $Deltal$ and delta-gap size $Deltaz$ to test their effects to and influence on the numerical solution of current distribution as well as the input impedance. The input impedance is calculated as shown in Equ. (12).

$$Z_{in} = \frac{V_0}{I_m} \tag{12}$$

where I_m denotes the current right at the feed point. As for the current distribution, we will use the interpolation function as shown in Equ. (3) together with the coefficients solved by Equ. (6) to calculate the magnitude and phase of the current along the wire.

A. Frequency Sweep and Comparison with Commercial Software MININEC Pro

MININEC Pro, which uses a method of moments formulation to solve for the currents on a wire, is an antenna analysis program for Windows and Macintosh computers. This software can let the user sweep through the frequency and get the input impedance versus frequency diagram very easily, thus we will validate the correctness of our code by comparing the input impedance - frequency sweep between MININEC Pro and our Matlab code. The results are respectively shown in Fig. 2 and Fig. 3. Both of them sweep through the frequency in from 100 MHz to 5 GHz and the thin wire in both routines are discretized into 99 segments as shown in Fig. 5, where the blue segment is the feed point segment where the delta-gap source resides. Real part (resistance) and imaginary part (reactance) of the input impedance are shown in the figures separately. As we can see, the calculation results of the two are almost identical and hence the correctness of our Matlab code are verified.

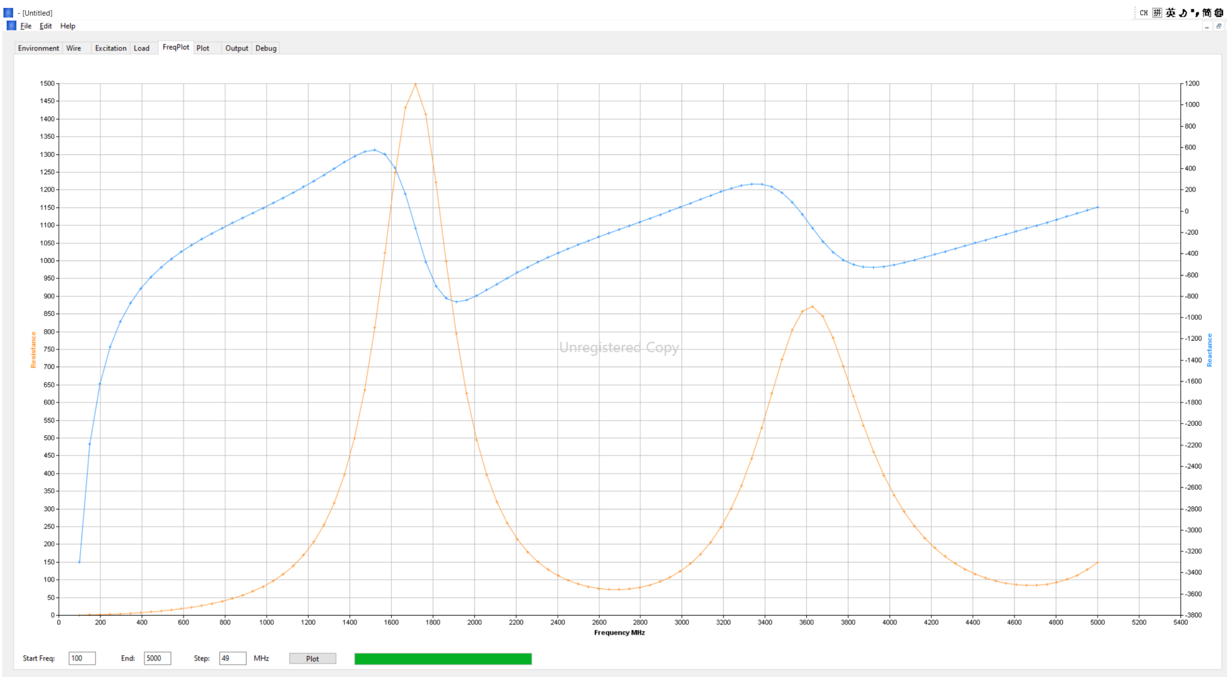

Fig. 2: The frequency sweep result of the input impedance by software MININEC PRO.

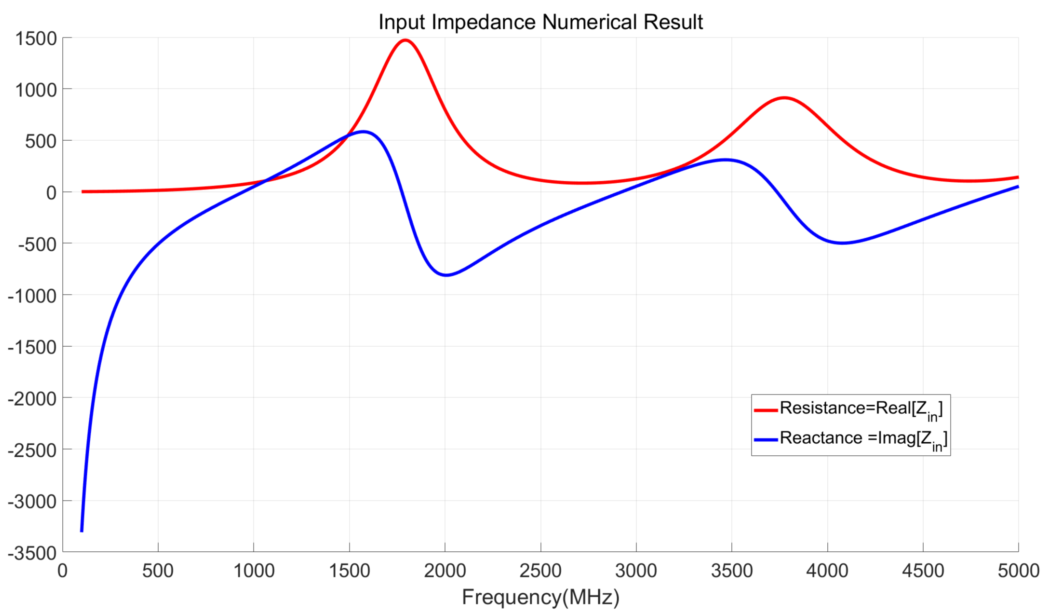

Fig. 3: The frequency sweep result of the input impedance by the Matlab code.

The current distribution along this half-wavelength thin wire linear antenna at 1 GHz is shown in Fig. 4 in magnitude/phase form and visualized in Fig. 6 in a colorful manner by normalization.

Further, similar to the figures in [7], we can also plot the current distribution at integer multiples of the operating frequency $f_0 = 1$GHz. The results for f_0, $2f_0$, $3f_0$, $4f_0$ and $5f_0$ are shown in Fig. 7. The resonances of a half-wavelength dipole antenna are at odd multiples of its fundamental frequency. For the fundamental resonance f_0, the current looks like a cosine function. This accords well with the assumption that the current along the dipole antenna is a cosine function, which has its maximum in the middle (fed point) and the minimum (zero) at both ends. This is due to the fact that when the antenna is shorter than half wavelength, we only have current flowing in one direction. When there is high order resonance at $3f_0$ and $5f_0$, the current will flow in multiple directions, which may cancel the radiation and make the antenna less efficient.

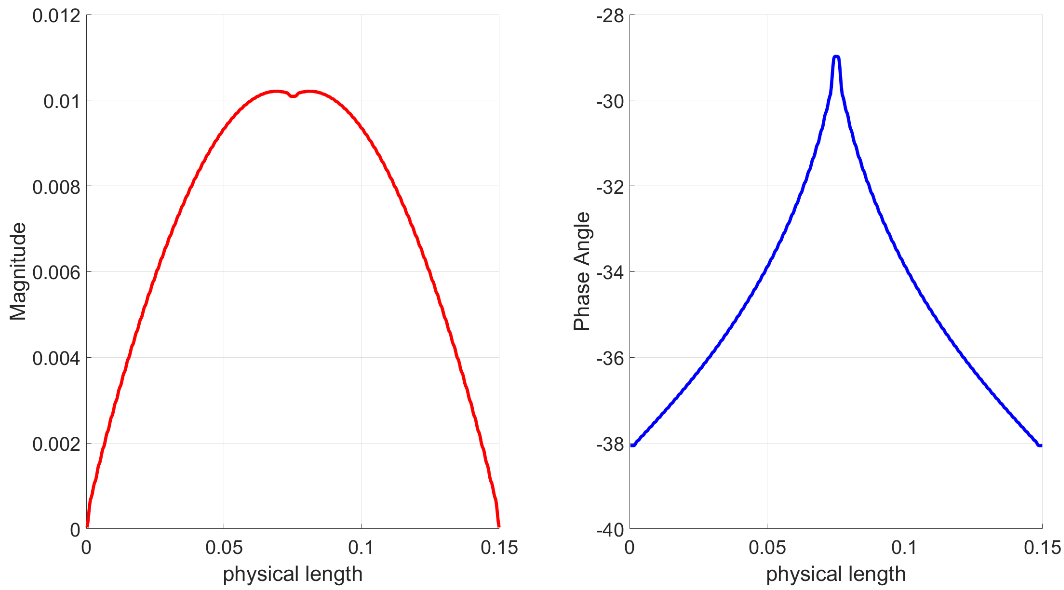

Fig. 4: The current distribution in terms of magnitude and phase at 1 GHz.

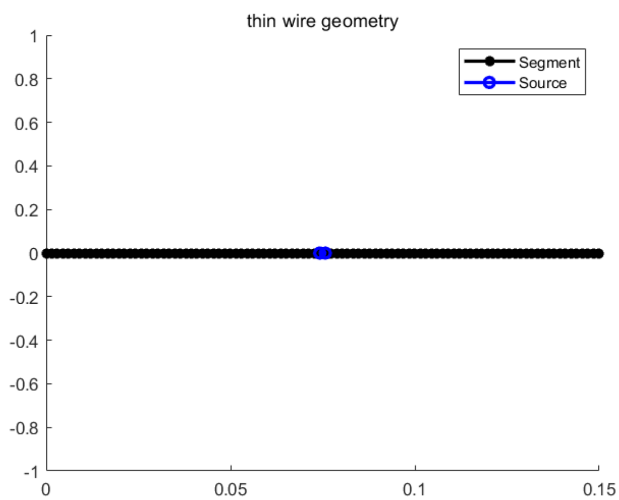

Fig. 5: Discretize the thin wire into 99 segmens.

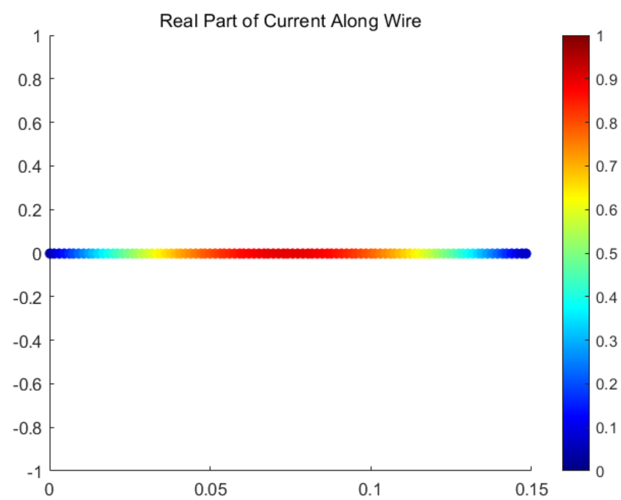

Fig. 6: The normalized current for colorful visualization.

B. Segment Size Sweep

In this part, we are going to show the result of input impedance and current distribution versus different segment sizes by sweeping through different number of total segments that are discretized evenly from coarse to dense.

In Fig. 8, real part and imaginary part of the input impedance calculated at different number of discretization are shown. The segment size ranges from 0.05λ to 0.000833λ. As we can see from the result, when the segment size is large (approximately $\frac{1}{10}$ of the length of half-wavelength linear antenna), the impedance calculated is less accurate than that of the relatively fine discretization. However, when the wire antenna is discretized into over 200 segments, in which case $\Delta l \leq \frac{\lambda}{400}$, the input impedance calculated by the code is unstable and may be even erroneous. This is due to the fact that the original MoM formulation is based on thin wire approximation, which assumes $\Delta l \gg a$. However, this assumption will not hold true when the segment size Δl is smaller and smaller and finally become comparable to the wire radius a.

Based on the sweep result of the segment size in a broad range and previous discussion, we show the current distribution when the number of segments is less than 200 in Fig. 9.

C. Delta-gap Size Sweep

Delta-gap source model has its limitation in its non-vanishing gap size. Because by assumption, the excitation is

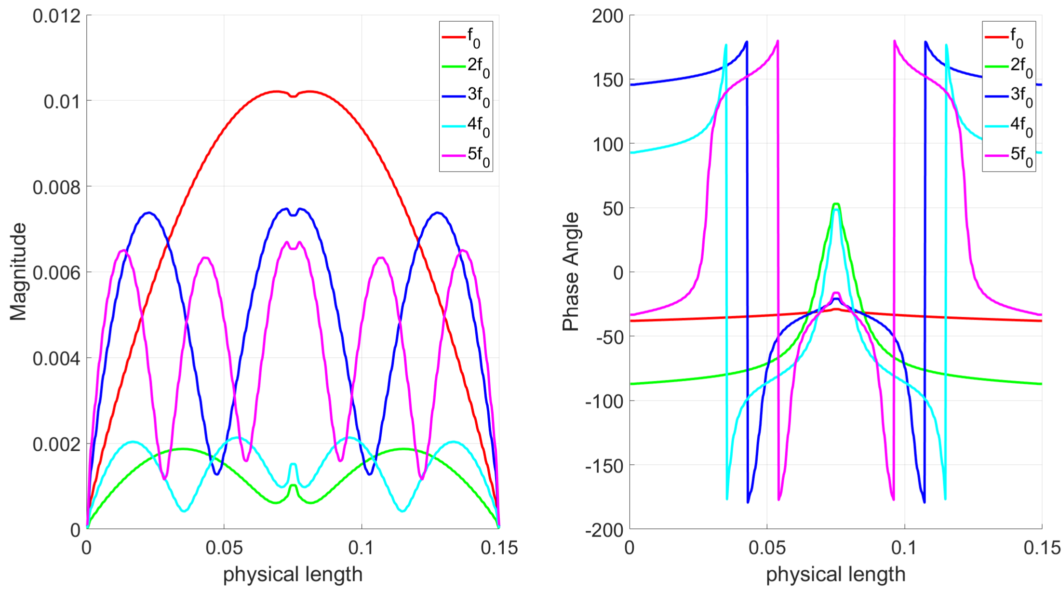

Fig. 7: Current distribution on the linear thin wire antenna at 1 GHz, 2 GHz, 3 GHz, 4 GHz and 5 GHz.

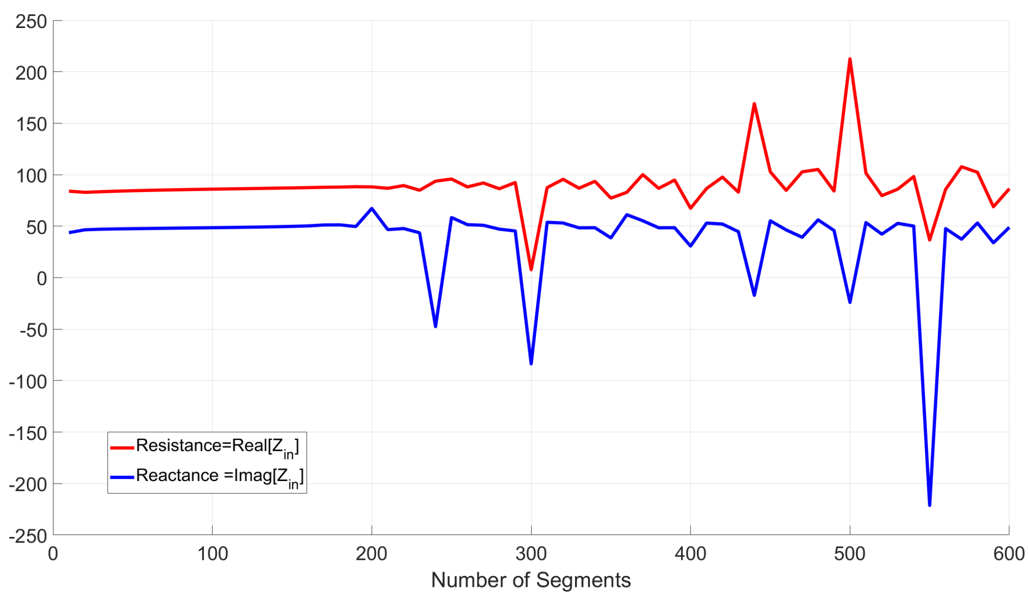

Fig. 8: Sweeping through different segment sizes by increasing the number of segments.

considered as an unphysically infinitesimally small port, with a width of almost zero. Thus, in this part, we examine the impact of the width of the delta-gap source on the impedance calculation and current distribution. The results are shown in Fig. 10 and Fig. 11.

As we can see, when we decrease the delta-gap size from $\frac{3\lambda}{100}$ to $\frac{3\lambda}{10000}$, the calculated input impedance will first stay in a steady level until the segment where the delta-gap source resides become comparable to the wire radius. Again, this is because the erroneously calculated current at the delta-gap segment due to the violation of thin-wire assumption. However, when the delta-gap segment is small but still in a reasonable level as shown in Fig. 11, decreasing the size of the delta-gap segment can increase the accuracy of the resulting current distribution.

IV. Conclusion

In the third project of ECE 540, we developed the MoM codes to calculate the input impedance and current distribution of the thin wire linear antenna formulated in [7]. The correctness of the Matlab code is validated through comparison with the commercial software called MININEC Pro. Also, we swept through the frequency, the segment size and delta-gap width to observe their respective impact on the results we sought.

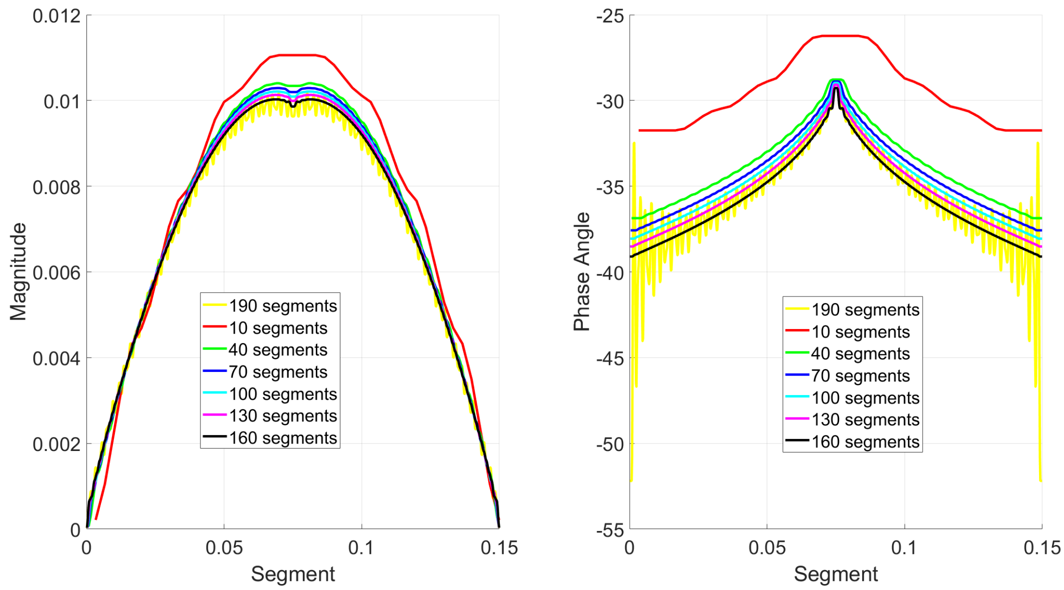

Fig. 9: Current distribution on the linear thin wire antenna discretized into 10, 40, 70, 100, 130, 160 and 190 segments.

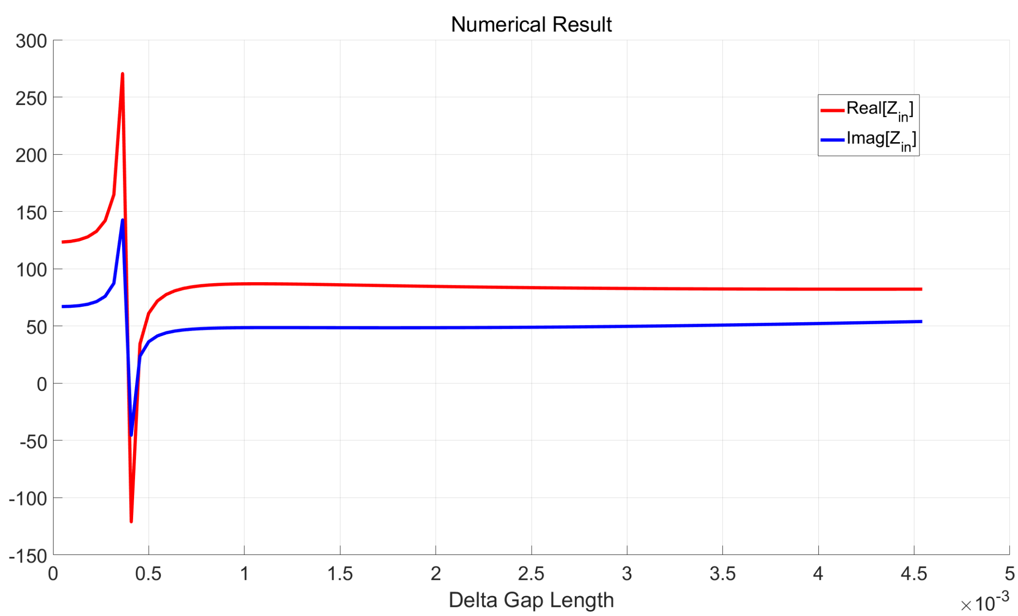

Fig. 10: Sweeping through different delta-gap sizes by decreasing the delta-gap length.

ACKNOWLEDGMENT

The author would like to thank Prof. Jin who delivers this course online during the COVID-19 pandemic and Prof. Peng who offers me the opportunity to study, research and work in the field I love and enjoy. Also, many thanks to Yanan Liu (the TA) who reads and grades the homework and project reports.

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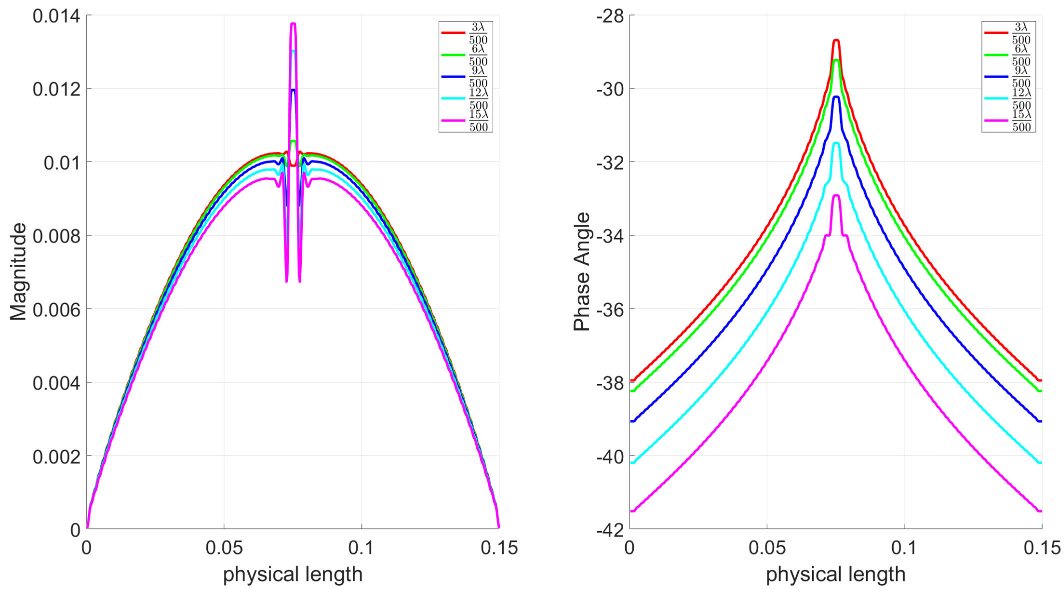

Fig. 11: Current distribution on the linear thin wire antenna with delta-gap sizes of $\frac{3\lambda}{500}$, $\frac{6\lambda}{500}$, $\frac{9\lambda}{500}$, $\frac{12\lambda}{500}$ and $\frac{15\lambda}{500}$.

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