Optimizing Football Game Play Calling

Article *in* Journal of Quantitative Analysis in Sports · January 2009

DOI: 10.2202/1559-0410.1176 · Source: RePEc

Sharif Melouk
University of Alabama

Marcus B. Perry
University of Alabama

Journal of Quantitative Analysis in Sports

Volume 5, *Issue* 2 2009 *Article* 2

2008 Northern California Symposium on Statistics and Operations Research in Sports

Optimizing Football Game Play Calling

Jeremy D. Jordan* Sharif H. Melouk† Marcus B. Perry‡

*Air Force Research Laboratory, jjordan810@gmail.com
†The University of Alabama, smelouk@cba.ua.edu
‡The University of Alabama, mperry@cba.ua.edu

Optimizing Football Game Play Calling

Jeremy D. Jordan, Sharif H. Melouk, and Marcus B. Perry

Abstract

Play calling strategies during football games are extremely important to the success of a team. In the past, coaches and players have subjectively determined the plays to call based on past experiences, personal biases, and various observable factors. This research quantifies these decisions using game theoretic techniques; updating optimal decision policies as new information becomes available during a game. A decision maker changes his perceived optimal strategy based on the information known about the opponent's strategy at the time of the decision. Additionally, utility theory is used to capture the different risk preferences of the decision makers. Furthermore, we use design of experiments and response surface methodology to optimize the risk strategies of each decision maker. By exploring the interaction of two football teams' risk preferences, optimal risk strategies can be suggested in the form of a varying mixed strategy. The techniques presented can be utilized in a precursory analysis to forecast different decisions a coach or player may encounter throughout the game, during a game to optimize each play called, or as a posterior analysis technique to dissect the decisions made and determine the effectiveness of the plays called. The procedures are easily transitioned to rapidly assist football teams or other sports teams in making better decisions through quantitative modeling and statistical analysis. A numerical example is presented to demonstrate the usefulness of the solution approach.

KEYWORDS: game theory, updating optimal decisions, football decision making, utility theory, risk behavior

1 Introduction

The idea of providing a football coach with an optimal decision policy that differs from the perceived optimal play in his mind will likely result in resistance, if not rejection from the majority of professional or college coaches. Therefore, keep in mind the techniques in this manuscript, like most models, are to be used as a decision support tool. As George Box once noted, all models are wrong, some are useful. The model frameworks can be used in pre-game planning, during games for play calling, and as a posterior analysis technique for evaluation of play following a game.

Updating optimal decisions based on new information as it becomes available is of great importance to football coaches. As new information becomes available regarding the possible actions of the defense, the idea of updating an optimal decision policy based on this perception becomes of interest. Optimizing behavior in these situations is of prime interest to sports teams who face these decisions. Further exploring the implications of risk behavior in approach to these situations is of great importance as well. Also, capturing the difference between a perceived optimal strategy and the true optimal strategy will provide insight into the quality of the information perceived.

To illustrate these concepts using a simple example, consider a quarterback attempting to read the defensive formation. If his experience tells him the opponent is in a 3-4 defense, the optimal decision may be to run the fullback between the tackles. If another of the offensive sensors, a coach from the sideline for instance, provides updated information that the defense is in a 3-4 and appears to be blitzing the two middle linebackers, the optimal decision may be a tight end dump. However, if the true defense was a straight 3-4 without a blitz, the perceived optimal decision to throw a tight end dump differs from the true optimal decision which may be a dive. The difference between these two decisions can be thought of as the regret of the decision maker.

Consider a military example of an engagement between a red tank and a blue tank. If the sensors of the red tank indicate that the damage level of the blue tank is a mobility/firepower kill, the perceived optimal decision of the red tank may be to move forward and capture troops. However, if the true damage level of the blue tank was only a mobility kill, the perceived optimal decision of the red tank to move forward is less than optimal. The red tank will actually put itself unnecessarily in harms way. Its true optimal decision may be to actually shoot again. This difference between the true and perceived optimal decisions needs to be accounted for during actual battle or in a simulation model. If the information received and known by a decision maker is imperfect or incomplete, decisions will be affected in proportion to the quality and quantity of the information received. If two entities are engaged in a battle, this information will have some effect on the outcome of this battle.

If perfect information is available, a perfect decision can be made based on that information. If the information received is less than perfect, the effects of this imperfect information on the outcome of the scenario need to be determined. We accomplish this through measuring the difference between the true optimal decision based on perfect information and the perceived optimal decision possibly based on less than perfect information. The outcome of a scenario given one has perfect information is the optimal outcome. The outcome of a scenario, given one has received less than perfect information, is referred to as the perceived optimal decision. The difference between these two values is the degree of loss incurred because of this bad information, or conversely the value of perfect information.

Due to the competitive nature of the aforementioned games, we propose the use of game theory, specifically a two-player zero-sum game where one players loss is the others gain. Additionally, each decision maker approaches the situation in a dissimilar fashion. Thus we use utility theory to capture the individual preferences of the decision makers. After the initial game scenario is constructed, any decision maker can be represented through determining his utility of the reward matrix. This provides flexibility to model any team match-ups that may arise. A decision maker, unsure of the risk behavior with which to approach a situation or who suspects his past risk behavior has resulted in less than desirable effects, would benefit from a study on the effects of risk behavior. Hence, we use response surface methodology to explore the effects of risk behavior on the outcome of the games. We have yet to discover any relevant literature revealing the techniques we have just discussed for updating optimal decisions in any application area.

The paper is organized as follows: section 2 explores the literature on quantitative analysis in football and game theory; section 3 details the process of modeling the football game as a 2-player game theoretic system; in section 4 the model is analyzed and useful analysis techniques are provided; our concluding remarks are in section 5.

2 Literature Overview

Modeling the updating of optimal decisions and measuring the difference between optimal and perceived optimal decisions during decision processes and in simulation models has received minimal research attention. Game theory has been used extensively across many disciplines, however not to date as a

DOI: 10.2202/1559-0410.1176

methodology in a simulation model or during a decision process to update optimal decisions given the information available at the time, nor as a way to measure the difference between a perceived optimal decision and a true optimal decision. In this section, we rummage the literature regarding the updating of decisions and quantitative analysis in football, followed by a synopsis of game theoretic principles.

Quantitative Analysis in Football

Play-by-play quantitative analysis in football is difficult due to the vast number of situations that can arise. Hence, much of the football analysis literature uses quantitative techniques to solve UNIQUE situations that occur. For instance, Sahi and Shubik (1988) use game theory to analyze a simplified sudden death football game, where a team can simply conduct a running play or kick a field goal. Carter and Machol (1978) establish the expected point values of punting, kicking a field goal, and running when an offensive team has passed mid-field and is facing a fourth down and short situation. Furthermore, Carter and Machol (1971) simply computes the expected point values of first down and ten scenarios for various field positions. Its quite difficult to generalize a particular analysis study or strategy study across the many scenarios that occur in a football game, mainly because of the many factors involved and the large number of unique situations that arise. Some of the factors that affect a coaches decisions include field position, down, distance to first down, score, season record, player performance, weather, and time remaining in the game to name a few. The combination of these factors leads to an extraordinary number of possible scenarios. For this reason, most analysts choose to focus on a special situation or small group of similar situations for analysis. This research allows the modeling of any situation that may occur through the use of game theory. Boronico and Newbert (2001) use a similar approach, utilizing game theory and dynamic programming to optimize play selection for first down and goal situations. They use empirical data from past seasons of Monmouth College football games, and the results suggest that their models outperform coaches, also see Boronico and Newbert (1999). However, their analysis considers only specific situations and cannot model all parts of a game. Our methods also include an updating capability when additional information is gained. Additionally, we use utility theory to account for any possible decision maker that may be playing the game, and suggest better risk preferences to use through response surface methodology. Lastly, as noted in Alamar (2006), the expected value of passing has increased in recent years, however the frequency has not increased accordingly. This can be explained through the principles of game theory. As an action gains value, a player may actually use the action less because the opponent also knows the action has gained value (complete information) and will adjust his defense accordingly. A practical application of this phenomenon is seen when a team "establishes the run" to open up their passing game. Numerous professors and authors have stressed the applicability of game theory to football, it has yet to be fully developed due in part to the scarcity of readily available play-by-play data sets.

Updating Decisions

Cooman (2004) talks about updating beliefs based on incomplete information, however only in the context of Bayesian updating and missing information. A technique is presented that allows the decision maker to account for missing data and incomplete information when calculating probabilities. Sandoy and Aven (2006) explores alternative Bayesian updating approaches in the application area of a drilling operation. This type of updating requires a procedure that automatically updates an assessment as new information arrives. Several alternatives are presented and the use of Bayes theorem produces a fast and automatic updating procedure. Also noted is the fact that in many cases, the reception of new information opens the need to rethink the entire process and model. This research will address this exact issue. This paper is the first application using game theory to update optimal decisions.

Other possible areas of application include the allocation of resources, see Bier and Abhichandani (2002), Harris (2004), Sandler and Arce (2003), and Zhuang and Bier (2006). Combat situations provide an excellent setting for the use of game theory for updating optimal decision policies as well, see Berkovitz and Dresher (1959) and Sujit and Ghose (2005).

In conclusion, updating optimal decision policies in American football play calling has not been accomplished to date. Game theory provides an excellent means of increasing the quality of play by play decision making. By updating plays based on the perceived defensive formation and what is known up to this point, a team can readily increase the number of yards gained over time and thus increase winning percentages. Next we cover the general game theoretic concepts needed to understand the methods set forth in this paper.

Game Theoretic Concepts

There are many different types of game theory. The general terminology discussed here is retrieved from Shor (2006). Players take actions according to the rewards they expect to gain from their behavior. In game theory, actions are taken based on a strategy which is simply a distribution across each of their actions or their action set. This strategy is based on the Nash equilibrium, named after John Nash. This is a set of strategies, one for each player, such that no player has incentive to unilaterally change his actions. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if she remained with her current strategy. For games in which players randomize (mixed strategies), the expected or average payoff must be at least as large as that obtainable by any other strategy. In Nash (1951), he proves that every finite non-cooperative game has at least one or more equilibrium points assuming the players are rational, this being the players' good strategy or strategies.

Each player chooses his best strategy assuming that his opponent knows the best strategy for him to follow, thus he maximizes his minimum gain. His opponent chooses the strategy that allows the other to gain the least, attempting to minimize the other players maximum gain. This assumption is fundamental to the theory of games and is called the minimax/maximin principle. Player one will maximize his minimum gain while player two will minimize the maximum gain of player one. This assumes also that each player knows the strategies available to the other players. A general overview of game theory can be found in reference Winston (2004).

3 Football Application

This methodology fits nicely to a football game where the offense is attempting to audible plays based on the observations of the quarterback or coaches. Initially, the game can be set up for each situation of the game. As the quarterback approaches the line, and the coach observes the defense from the sidelines or press box, the defensive formation can be estimated thus eliminating some of the possible defensive setups. This leads to an updated offensive strategy that is based on this perception. The risk behavior of the teams can also be estimated and will change with each play of the game.

3.1 Initial Game Setup

Initially, the plays that are available to each team must be determined for various situations during the game. For instance, when the offense is within 10 yards of the opponents endzone, the long pass is not a possible action to call. Most plays will be available the majority of the game. After the action sets are determined for the offense and defense, the proper statistics must be gathered from past games. Each situation where the offensive action has been used against the defensive formation must be assigned an average number of yards gained from statistical data. Table 1 shows an example of the normal form of a football game after data collection. The defensive formations are designed to limit certain plays, here are the plays that are best defended against by each formation:

- 4-4 Overload - Sweep
- 5-4 Blitz - Middle Run
- 4-4 Zone - Short Pass
- 4-3 Man - Long Pass.

When the offense chooses to run the sweep and the defense chooses the 4-4 overload, past history tells us the average loss for the offense 2.7 yards. Ideally, one could fit a distribution to past data and stochastically play the game, an idea for future research.

Table 1: Initial Football Game

	4-4 Overload	5-4 Blitz	4-4 Zone	4-3 Man
Sweep	-2.7	3.8	4	6.6
Middle Run	3.4	2	5.1	5.9
Short Pass	4	3.9	0.5	7.3
Long Pass	6.1	7	6.3	-3.4

3.2 Automating Risk Behavior

In determining the general risk behavior of a coach, the initial factors that cause a coach to vary his play calling according to the amount of risk he is willing to accept must be expounded. After brainstorming all the possible factors that could affect risk behavior during a football game, design of experiments can be used to determine the most influential factors, the factors that cause the variation in the response variable certainty equivalent. A two-level fractional factorial can be used to weed out the unimportant factors. Suppose that this process revealed the most important factors as down, distance to go for a first down, field position, time left in the game, and score of the game. For simplicity, the scenarios presented herein assume the score and a time left in the game. Let's also assume for the sake of brevity that the remaining factors, down, distance, and field position, only possess two levels, high and low. In reality, there may be four or more levels for each of the original factors, the

DOI: 10.2202/1559-0410.1176

Table 2: Factor Levels

Factor	High(+)	Low(-)
Field Position	≥ 50	< 50
Down	$\geq$ 3rd	$\leq$ 2nd
Distance to Flag	≥ 8	< 8

more the better. Table 2 shows a description of the high and low levels of the three factors.

The risk preference of the decision maker is hereby denoted as ρ. In general, ρ approaching zero from infinity indicates a more risk averse behavior while ρ approaching zero from negative infinity indicates a more risk prone behavior. $\rho = 0$ is undefined for the exponential utility function and $\rho = \infty$ is risk neutral behavior.

Table 3: Example Risk Tolerance Levels

-50	Risk Neutral
-1	Risk Prone
1	Risk Averse
50	Risk Neutral

Table 4 shows a simple setup of a design matrix that allows the risk preference of the decision maker, ρ, to be automated according to down, distance, and field position by inputting a certainty equivalent for each design point. The decision maker is asked to answer a question at each of the design points in Table 4. The question is the certain number of yards willing to be accepted as a trade for the gamble: 50% chance of gaining 3 yards and 50% chance of gaining 10 yards. The responses are given, if the decision maker chooses the expected value, 6.5 in this case, he is considered a risk neutral individual. A value of larger than 6.5 implies risk prone and less than 6.5 indicates a risk averse attitude. The values of the certainty equivalent have been formulated under the assumption that player one is losing by 7 points with 5 minutes or less remaining in the game.

While fitting a model to the input data, it is important to use the certainty equivalent as the response. If the actual value of ρ is used, the model may need cubic terms or higher, which will make the automation process much more difficult and time consuming. After the model has been fit using the certainty equivalent, the values of ρ can be calculated. An accurate model is fit using just main effects and interaction terms. The design matrix $\mathbf{X}$ is given

Table 4: User Risk Survey

Field Position(X_1)	Down(X_2)	Distance(X_3)	Certainty Equivalent(y)
+	+	+	8
+	+	-	6.3
+	-	+	6
+	-	-	5
-	+	+	9.5
-	+	-	8.5
-	-	+	8
-	-	-	7.5

Table 5: Design Matrix for Automating ρ

Intercept	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3
1	1	1	1	1	1	1
1	1	1	-1	1	-1	-1
1	1	-1	1	-1	1	-1
1	1	-1	-1	-1	-1	1
1	-1	1	1	-1	-1	1
1	-1	1	-1	-1	1	-1
1	-1	-1	1	1	-1	-1
1	-1	-1	-1	1	1	1

Table 6: Values for Automating ρ

y	$\hat{y}$	$z_{0.5}$	P	ρ
8	7.975	0.714285714	-0.52	-3.64
6.3	6.325	0.471428571	4.16	29.12
6	6.025	0.428571429	1.76	12.32
5	4.975	0.285714286	0.52	3.64
9.5	9.525	0.928571429	-0.1	-0.7
8.5	8.475	0.785714286	-0.32	-2.24
8	7.975	0.714285714	-0.52	-3.64
7.5	7.525	0.642857143	-0.85	-5.95

in Table 5 and the initial y response vector, fitted values $\hat{y}$, standardized CE $z_{0.5}$, standardized ρ value P, and corresponding ρ are given in Table 6. ρ is computed using the certainty equivalent in Table 4 and techniques for computing ρ in Kirkwood (1997). The values for $\hat{y}$ were obtained using least squares

DOI: 10.2202/1559-0410.1176

techniques which yielded the following model:

$$\begin{aligned}\hat{y} &= \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3 + \theta_{12} X_1 X_2 + \theta_{13} X_1 X_3 + \theta_{23} X_2 X_3 \\ &= 7.35 - 1.025 X_1 + .725 X_2 + .525 X_3 + .1 X_1 X_2 + .15 X_1 X_3 + .15 X_2 X_3.\end{aligned}$$

This may not seem useful at first glance, however when many variables are present and several levels exist for each variable, it is imperative to have a prediction equation. In this case the levels of the variables are categorical, either high or low. When the levels of the factors are continuous, the time of game and the yardline for instance, it is important to have the ability to predict between design points. Again, the above equation is just the certainty equivalent for the situation where player one is losing by 7 points with 5 minutes or less remaining. Ideally, all of the factors will be included in the model and the risk tolerance for every conceivable situation spanning the entire length of the game will be approximated and automated.

Risk behavior can now be automated for any field position, down, and distance to the first down according to the risk preferences of the decision maker, the coach in this football scenario.

3.3 Running the Game

With the data from the initial game setup and the risk preference function of the decision maker, the game commences. With 5 minutes remaining in the game and losing by 7 points, the offense is facing 3rd and 10 on the -38 yardline (a negative sign in front of the yardline implies the offenses own half of the field, whereas no sign implies the defensive half of the field). There are 60 minutes in an entire football game, the number of minutes are thus the time that remains in the game. ρ_1 is the risk tolerance of player one according to the inputs of the situation of the game. All of these inputs combined are considered the situation of the game, $\mathbf{S}$.

$$\begin{aligned}\mathbf{S_1} &= \{Score, TimeLeft, FieldPosition, Down, Distance, \rho_1, \rho_2\} \quad (1) \\ &= \{-7, 5, -38, 3, 10, -.7, -.7\}\end{aligned}$$

shows the situation during the first play of the game. Obviously, the game will normally begin with a score of 0-0 and 0 time elapsed. The game is started here to show the application of the automation of ρ.

The risk tolerance of player one in (1) was found using the risk tolerance function and Table 2. The offensive certainty equivalent in this situation is

$$\begin{aligned}\hat{y} &= 7.35 - 1.025X_1 + .725X_2 + .525X_3 + .1X_1X_2 + .15X_1X_3 + .15X_2X_3 \\ &= 7.35 - 1.025(-1) + .725(1) + .525(1) + .1(-1) + .15(-1) + .15(1) \\ &= 9.525.\end{aligned}$$

This number is converted to ρ using the procedure outlined above. Player one has a risk tolerance of $\rho = -.7$, which is an extreme risk prone approach to the situation. For the purpose of showing the techniques and their usefulness, we assume player 2's risk preference is the same as player 1's. In reality, player 2 would be undergoing the same analysis as player 1. Initially, in the huddle, with lack of prior information about the defensive formation $\boldsymbol{\beta}$ that player two will call, player one chooses from his action set

$$\boldsymbol{\alpha} = [Sweep, MiddleRun, ShortPass, LongPass]$$

based on his perception of the available plays that player two can choose from.

$$\begin{aligned}&\hat{\gamma}^{(0)}_{\mathbf{S_1}} = [\gamma = \{0, 0, .1538, .8462\} \mid \\ &\qquad\qquad \hat{\beta}^{(0)} = \{4-4Overload, 5-4Blitz, 4-4Zone, 4-3Man\},\end{aligned}$$

the initial strategy of player one based on his risk tolerance during the situation and his perception of all the available plays to player two. This shows that because of the risk prone behavior of player one due to the situation, he will call the long pass 85% of the time and the short pass 15% of the time. The strategy of player two is

$$\begin{aligned}\hat{\delta}^{(0)}_{\mathbf{S_1}} &= \{\delta | \hat{\beta}^{(0)}_{\mathbf{S_1}}\} \\ &= \{.8539, 0, 0, .1461\}.\end{aligned} \tag{2}$$

As the quarterback approaches the line of scrimmage, he observes the defense in either a 4-3 man or a 4-4 zone. The perceived actions available to player two are dependent on the quarterback perception:

$$\begin{aligned}\hat{\beta}^{(1)}_{\mathbf{S_1}} &= [\beta | \boldsymbol{\zeta}^{(1)} = \{\zeta_1 = QBObservation\}] \\ &= [4-4Zone, 4-3Man].\end{aligned} \tag{3}$$

His strategy thus updates based on this observation,

$$\begin{aligned}\hat{\gamma}^{(1)}_{\mathbf{S_1}} &= \{\gamma | \hat{\beta}^{(1)}_{\mathbf{S_1}}\} \\ &= \{0, 0, .1933, .8067\}.\end{aligned} \tag{4}$$

DOI: 10.2202/1559-0410.1176

Even though player one knows that player two is defending heavily against the pass by playing the 4-3 Man and the 4-4 Zone, he will still call a pass because he is in a situation where he must get yards and a first down or he will lose the game. Player one now chooses a play according to this distribution and possibly calls an audible to his original play out of the huddle. Based on his knowledge of the actions available to player two and the strategy of player two

$$\hat{\delta}^{(1)}_{\mathbf{S_1}} = \{.8067, .1933\}, \tag{5}$$

he expects to gain

$$\begin{aligned}\hat{\pi}^{(1)}_{\mathbf{S_1}} &= \hat{\gamma}^{(1)}_{\mathbf{S_1}}\, \hat{\mathbf{R}}^{(1)}_{\mathbf{S_1}}\, \hat{\delta}^{(1)'}_{\mathbf{S_1}} \\ &= [0, 0, .1933, .8067] \begin{bmatrix} 4 & 6.6 \\ 5.1 & 5.9 \\ 0.5 & 7.3 \\ 6.3 & -3.4 \end{bmatrix} \begin{bmatrix} .8067 \\ .1933 \end{bmatrix} \\ &= 3.92\end{aligned}$$

yards. This is the amount of yards player one perceives he will gain. The actual yards gained will be dependent on the actual strategy of player two, this is covered in the post game analysis section. Suppose player one calls a long pass, a safety slips and the offense gains 17 yards. The situation now updates to a new play:

$$\mathbf{S_2} = \{\mathbf{-7, 4:45, 45, 1, 10, 12.32, 12.32}\}.$$

Player one now takes a more risk averse attitude towards the situation because he has a few more downs to get ten yards and he has crossed mid-field. In the huddle, player one calls his play based on the situation only without perceived information as to the defense of player two. With only the knowledge of all the plays available to player two,

$$\hat{\gamma}^{(0)}_{\mathbf{S_2}} = \{.0601, .4155, .3129, .2125\} \mid \\ \hat{\beta}^{(0)}_{\mathbf{S_2}} = \{4-4Overload, 5-4Blitz, 4-4Zone, 4-3Man\}, \tag{6}$$

and the defense calls

$$\hat{\delta}^{(0)}_{\mathbf{S_2}} = \{.0910, .4459, .2414, .2217\}. \tag{7}$$

There is a nice distribution across the offensive plays, running up the middle almost half of the time. As the quarterback approaches the line, he observes that the defense is definitely not in the 4-4 zone,

$$\begin{aligned}\hat{\beta}^{(1)}_{\mathbf{S_2}} &= [\beta|\boldsymbol{\zeta}^{(1)} = \{\zeta_1 = QBObservation\}] \\ &= [4-4Overload, 5-4Blitz, 4-3Man].\end{aligned}$$

Thus,

$$\begin{aligned}\hat{\gamma}^{(1)}_{\mathbf{S_2}} &= [\gamma|\hat{\beta}^{(1)}_{\mathbf{S_2}}] \\ &= \{0, 0, .8066, .1934\},\end{aligned}$$

running a short pass the majority of the time. This makes sense because the 4-4 zone defends best against the short pass and player one is perceiving this defense to be unavailable to the defense. The perceived strategy of the defense is

$$\hat{\delta}^{(1)}_{\mathbf{S_2}} = \{.8710, 0, .1290\}.$$

Suppose the coach notices from the press-box that the defense is guarding heavily against the run,

$$\begin{aligned}\hat{\beta}^{(2)}_{\mathbf{S_2}} &= [\beta|\boldsymbol{\zeta}^{(2)} = \{\zeta_1 = QBObservation \cap \zeta_2 = CoachObservation\}] \\ &= [4-4Overload, 5-4Blitz].\end{aligned}$$

Now the perceived optimal strategy is

$$\hat{\gamma}^{(2)}_{\mathbf{S_2}} = \{0, 0, 0, 1\}.$$

It makes sense that the offense would choose a deep pass in the situation where the defense is guarding heavily against the run. The number of yards the offense expects to gain is

$$\begin{aligned}\hat{\pi}^{(2)}_{\mathbf{S_2}} &= \hat{\gamma}^{(2)}_{\mathbf{S_2}}\, \hat{\mathbf{R}}^{(2)}_{\mathbf{S_2}}\, \hat{\delta}^{(2)\prime}_{\mathbf{S_2}} \\ &= [0, 0, 0, 1] \begin{bmatrix} -2.7 & 3.8 \\ 3.4 & 2 \\ 4 & 3.9 \\ 6.1 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= 6.1\end{aligned}$$

Suppose the offense ran the long pass, a lineman missed a block, and the quarterback was sacked for a loss of 6 yards. Now,

$$\mathbf{S_3} = \{\mathbf{-7}, \mathbf{4:15}, \mathbf{-49}, \mathbf{2}, \mathbf{16}, \mathbf{-3.64}, \mathbf{-3.64}\}.$$

The initial strategy of player one is

$$\hat{\gamma}^{(0)}_{\mathbf{S_3}} = \{0, .2854, .3007, .4139\},$$

DOI: 10.2202/1559-0410.1176

and the initial strategy of player two is

$$\hat{\delta}^{(0)}_{\mathbf{S_3}} = \{.3577, 0, .3192, .3231\}.$$

Player one will choose to throw a long or short pass more often in this situation, but still will run up the middle roughly 30% of the time because it is only second down. The defense will not guard against the short run at all because they believe player one to be risk prone, which means player one will not run up the middle. After the quarterback observes the defense to be heavily guarding against the pass,

$$\hat{\beta}^{(1)}_{\mathbf{S_3}} = [4 - 4Zone, 4 - 3Man],$$

the offense calls the play from the updated distribution

$$\hat{\gamma}^{(1)}_{\mathbf{S_3}} = \{0, .8403, 0, .1597\},$$

while the defense is perceived to call

$$\hat{\delta}^{(1)}_{\mathbf{S_3}} = \{.7460, .2537\}.$$

The offense runs up the middle the majority of the time because it observed the defense to be guarding against the pass more heavily. By running this strategy, the offense expects to gain

$$\pi_{\mathbf{S_3}} = 5.0693$$

yards. The offense runs up the middle and gains 11 yards. Thus,

$$\mathbf{S_4} = \{\mathbf{-7, 3 : 57, 40, 3, 5, 29.12, 29.12}\},$$

the offense is using a strategy close to the expected case. This results in the initial strategy coming out of the huddle,

$$\hat{\gamma}^{(0)}_{\mathbf{S_4}} = \{.0567, .3975, .3069, .2390\}.$$

The offense either runs or throws a short pass with the highest probability in this situation and can expect to gain

$$\hat{\pi}_{\mathbf{S_4}} = 3.9546$$

yards. The coach observes that the defense is not in 5-4,

$$\begin{aligned} \hat{\beta}^{(1)}_{\mathbf{S_4}} &= [\beta | \boldsymbol{\zeta}^{(1)} = \{\zeta_1 = CoachObservation\}] \\ &= [4 - 4Overload, 4 - 4Zone, 4 - 3Man]. \end{aligned}$$

The distribution updates,

$$\hat{\gamma}^{(1)}_{\mathbf{S_4}} = \{0, .5415, .2555, .2030\},$$

as well as the perception of the strategy of player two,

$$\hat{\delta}^{(1)}_{\mathbf{S_4}} = \{.6760, .1357, .1883\}.$$

Because the defense is not using the formation that best guards against the run up the middle, the offense chooses this play with greater probability. The quarterback then observes that the defense is not in any type of zone, thus

$$\begin{aligned}\hat{\beta}^{(2)}_{\mathbf{S_4}} &= [\beta|\boldsymbol{\zeta}^{(1)} = \{\zeta_1 = CoachObservation \cap \zeta_2 = QBObservation\}] \\ &= [4-4Overload, 4-3Man].\end{aligned}$$

The perceived optimal decision then becomes

$$\hat{\gamma}^{(2)}_{\mathbf{S_4}} = \{0, 0, .7701, .2299\},$$

with the perceived strategy of player two being

$$\hat{\delta}^{(2)}_{\mathbf{S_4}} = \{.8507, .1493\}.$$

The offense calls an audible according the preceding distribution. While running a short pass, the offense gains just 4 yards, not enough for a first down. So,

$$\mathbf{S_5} = \{\mathbf{-7}, \mathbf{3:39}, \mathbf{36}, \mathbf{4}, \mathbf{1}, \mathbf{29.12}, \mathbf{29.12}\},$$

and the risk strategy remains the same. The initial perceived optimal strategy in this situation is identical to $\hat{\gamma}^{(0)}_{\mathbf{S_4}}$. Upon arrival to the line of scrimmage, the quarterback sees the defense is not in the 4-4 overload, thus

$$\begin{aligned}\hat{\beta}^{(1)}_{\mathbf{S_5}} &= [\beta|\boldsymbol{\zeta}^{(1)} = \{\zeta_1 = QBObservation\}] \\ &= [5-4Blitz, 4-4Zone, 4-3Man],\end{aligned}$$

and

$$\hat{\gamma}^{(1)}_{\mathbf{S_5}} = \{.7970, 0, .0101, .1929\}.$$

Since the defense is not perceived to be overloading in the 4-4,

$$\hat{\delta}^{(1)}_{\mathbf{S_5}} = \{.7286, .0563, .2151\},$$

the sweep is called with high probability while still leaving a chance of calling a long pass to keep the defense on their toes. The coach on the sidelines further notices that the defense is not in a 4-4 of any type so

$$\begin{aligned}\hat{\beta}^{(2)}_{\mathbf{S_5}} &= [\beta|\boldsymbol{\zeta}^{(1)} = \{\zeta_1 = QBObservation \cap \zeta_2 = CoachObservation\}] \\ &= [5-4Blitz, 4-3Man],\end{aligned}$$

DOI: 10.2202/1559-0410.1176

and

$$\hat{\gamma}^{(2)}_{\mathbf{S_5}} = \{0, 0, .7779, .2221\}.$$

The offense abandons the sweep for the short pass as this will yield more yards against the two perceived available defenses. The coach in the pressbox radios down to the head coach exclaiming the defense to be in a 5-4 blitz formation and heavily guarding the run. The perception of the actions available to player two updates

$$\hat{\beta}^{(3)}_{\mathbf{S_5}} = [5 - 4Blitz].$$

The quarterback quickly calls an audible to account for this perception, his optimal strategy being

$$\hat{\gamma}^{(3)}_{\mathbf{S_5}} = \{0, 0, 0, 1\}.$$

The offense calls the long pass, however it is batted down and the defense takes over on downs. This section demonstrated the use of the methodology as applied to a football game. Next, the methods and techniques are presented that allow further optimization of offensive strategies for future situations and corrective action on the strategies used during the game. With this new information, a different outcome may be achieved in future situations.

4 Post-Game Analysis

During the game, it is best to use design of experiments to explore the interactions between ρ_1 and ρ_2, the risk preferences of player 1 and player 2, as this is the most efficient manner to quickly study behavior and outcomes. This is true whenever there are constraints dealing with time or money. In a post game analysis setting with virtually unlimited time, a proper approach would involve exploring each possible situation that may arise. Following the game, the film could be reviewed and a thorough analysis conducted on each combination of offensive and defensive strategies and utilities. This could easily be done using the techniques presented in the game against nature case. However, this section will demonstrate the use of DOE in the post-game setting. This will lead to learning among the team and possibly the determination of a better strategy to play in future game situations.

Each of the five situations will be examined to determine a better strategy for similar future situations and the quality of observations made by the offense. The interaction plots from the possible risk behaviors of player one and player two can be examined to determine the best strategy for player one. The risk behavior of player two is a noise factor, it cannot be controlled. Therefore, player one may choose to select the strategy that is most robust to any changes in the risk behavior of player two.

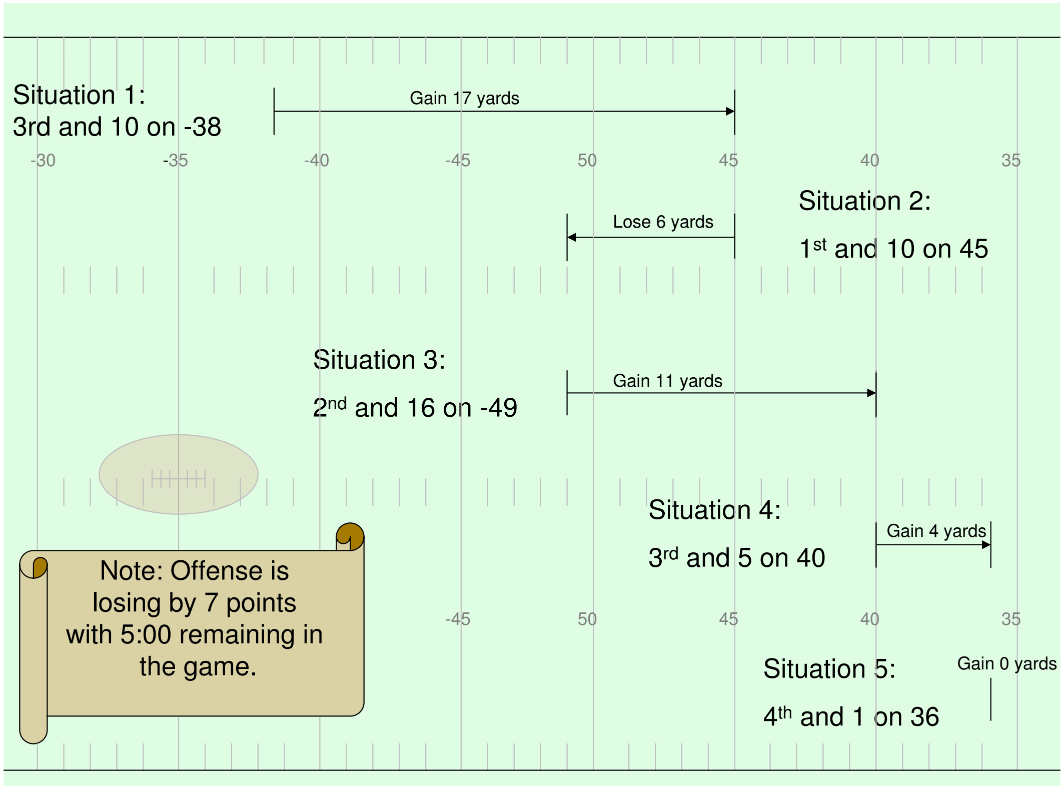

Figure 1: Football Game Flow Chart

4.1 Situation 1

Tending towards a more risk prone risk attitude is the optimal choice when information about the action set available to player two is unavailable. From Figure 2, it appears that the offense was utilizing the best possible strategy in the first situation during the initial game setup. That is, the risk prone approach provides the maximum number of yards gained regardless of the risk strategy of player two. When player two plays the risk averse strategy, player one gains the maximum number of yards by approaching the situation with a risk prone attitude. However, when player two approaches the situation with a risk averse attitude and player one takes a risk averse attitude, player one gains the minimum number of yards. Using the updated information from the quarterback observation, the offense used the poorest possible risk approach with the knowledge it possessed in Equation 3, as shown in Figure 3. The risk prone approach is strictly dominated by both the expected case and the

DOI: 10.2202/1559-0410.1176

risk averse strategy. If the offense used the expected case, they could have

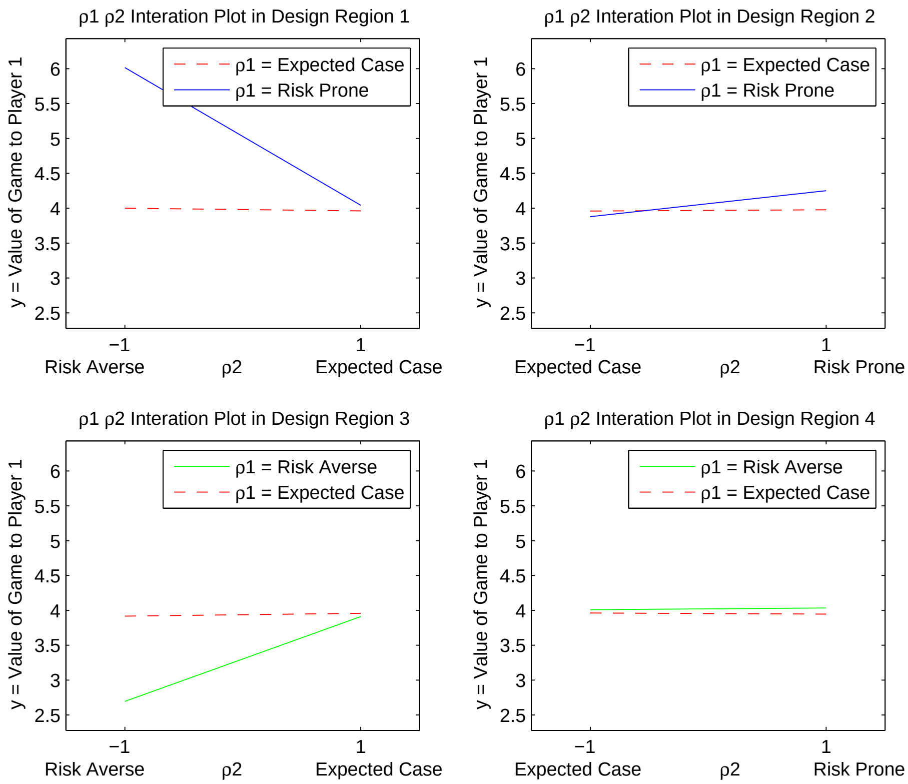

Figure 2: Initial ρ Interaction Plot

expected to gain almost twice the number of yards as with the risk prone approach.

$$\hat{\gamma}_0^{(1)} = \{0, .9238, 0, .0762\}$$

is the best approach for the offense. The offense is advised to alter its risk strategy in future situations resembling situation 1 where the offense perceives the defense to be in 4-4 zone or a 4-3 man. By playing the risk averse or expected case, the offense guarantees a robust risk approach to the situation.

4.2 Situation 2

The initial optimal risk strategy is again that of a risk prone nature as shown in Figure 2. The optimal risk strategy does not change from situation to

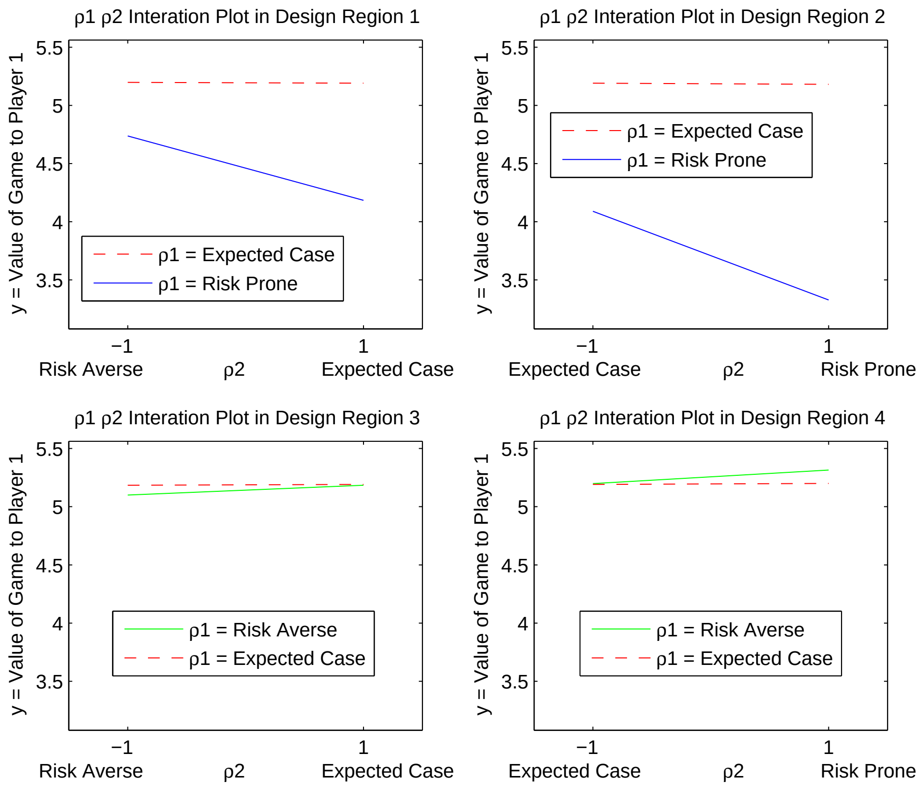

Figure 3: Updated ρ Interaction Plot at S_1

situation when all the actions are available to player two, however it will differ depending on what action set player one perceives player two is choosing from. From Figure 4, player one appears to have chosen a less than optimal risk approach after the update by choosing the more risk averse behavior. When the defense approaches the situation with a risk averse attitude, and the offense also approaches the situation with risk averse attitude, the expected yards gained is only around 4. The offense could have gained the maximum number of yards through the use of a risk prone attitude, however this also introduced the possibility of gaining less yards than the expected case if player two assumed player one to be risk prone. In this situation, since the offense preferred to entertain a risk averse attitude due to its preferences, of equal or better quality risk behavior would have been the expected case. It is robust in that regardless of the risk attitude of player two, player one can expect to gain the same amount of yards.

DOI: 10.2202/1559-0410.1176

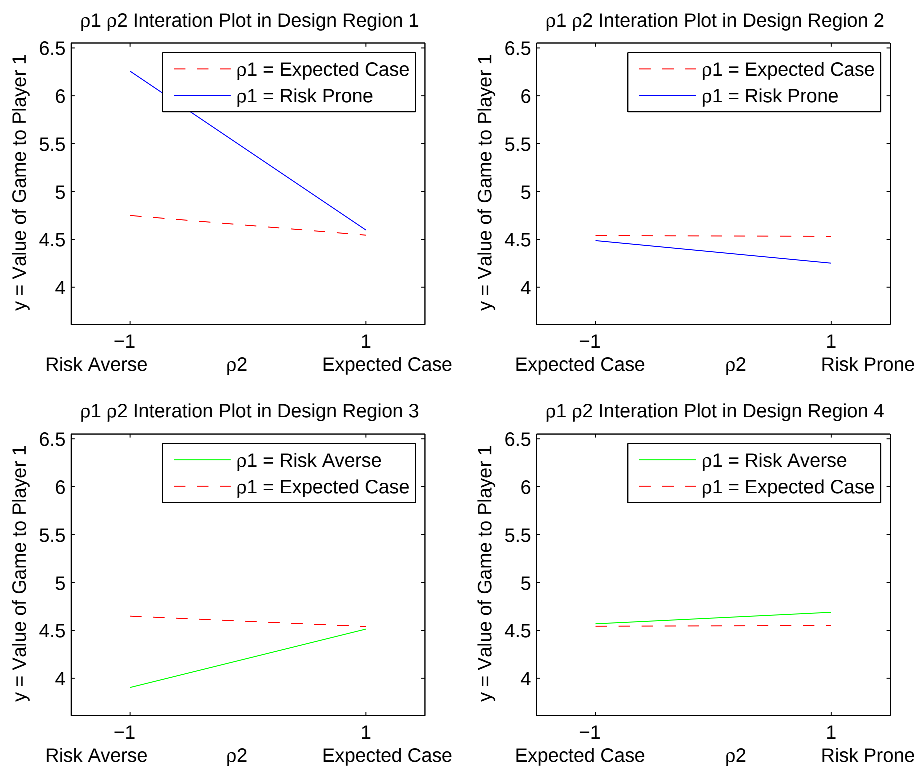

Figure 4: Updated ρ Interaction Plot at S_2

4.3 Situation 3

In this situation, the quarterback perceived the defense to again be in a 4-3 man or a 4-4 zone as in situation 1, thus the optimal risk strategy is still available in Figure 3. The risk strategy of the offense in this situation was not as risk prone as in the first situation, leading to a greater probability of calling the run up the middle. The defensive pressure on the pass caused the offense to take a different approach because they were not in an extreme risk behavior situation as in situation 1. The offense was further from the extreme risk prone approach and could expect to gain near the same amount of yards as the expected case during this situation.

4.4 Situation 4

During situation 4, the first update yields a similar optimal risk strategy approach as that in the initial game setup in Figure 2. After the second update, the interaction plots for the risk strategy are almost identical to that during the second situation at the first update in Figure 4. The offense should have used a more risk prone attitude, possibly gaining more yards than actually achieved during situation 4.

4.5 Situation 5

Initially, the best risk approach to take is the same as the above cases, the risk prone strategy. Upon QB observation that the defense is not in a 4-4 overload, the risk prone approach loses value. In Figure 5,when the defense takes the risk prone approach and the offense takes the risk prone approach, there is a significant loss. Compared with the robustness of the expected case,

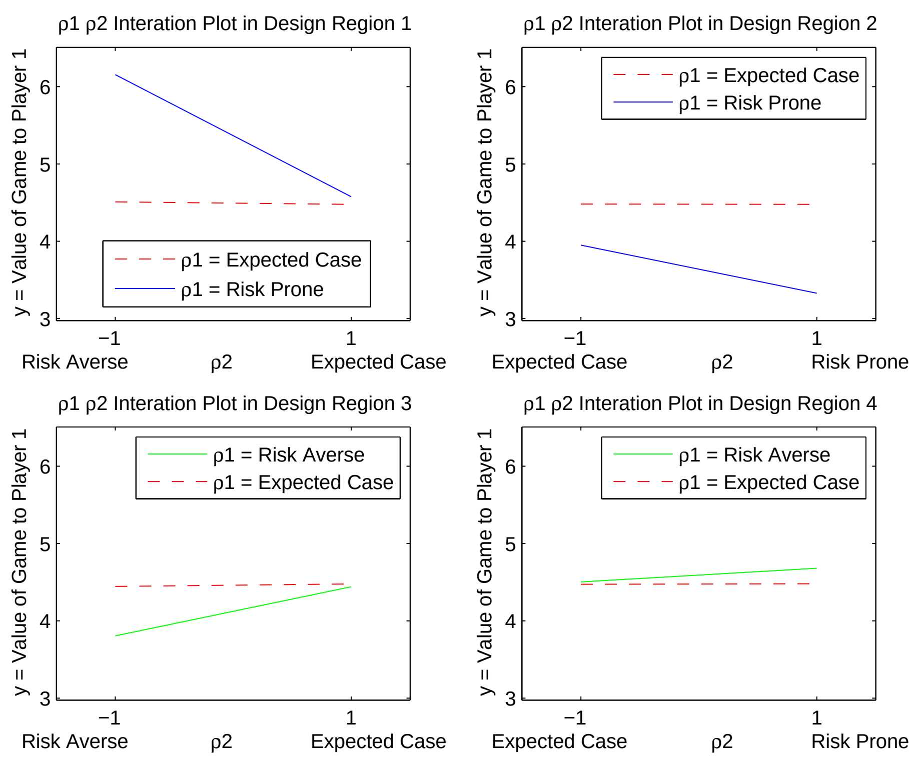

Figure 5: First Update ρ Interaction Plot at S_5

the risk prone strategy may not be the best approach. If the defense plays risk averse, and the offense plays risk prone, the offense can expect to gain more yards than in any other situation. The second update results in similar strategy implications as that shown in Figure 6. The risk averse strategy

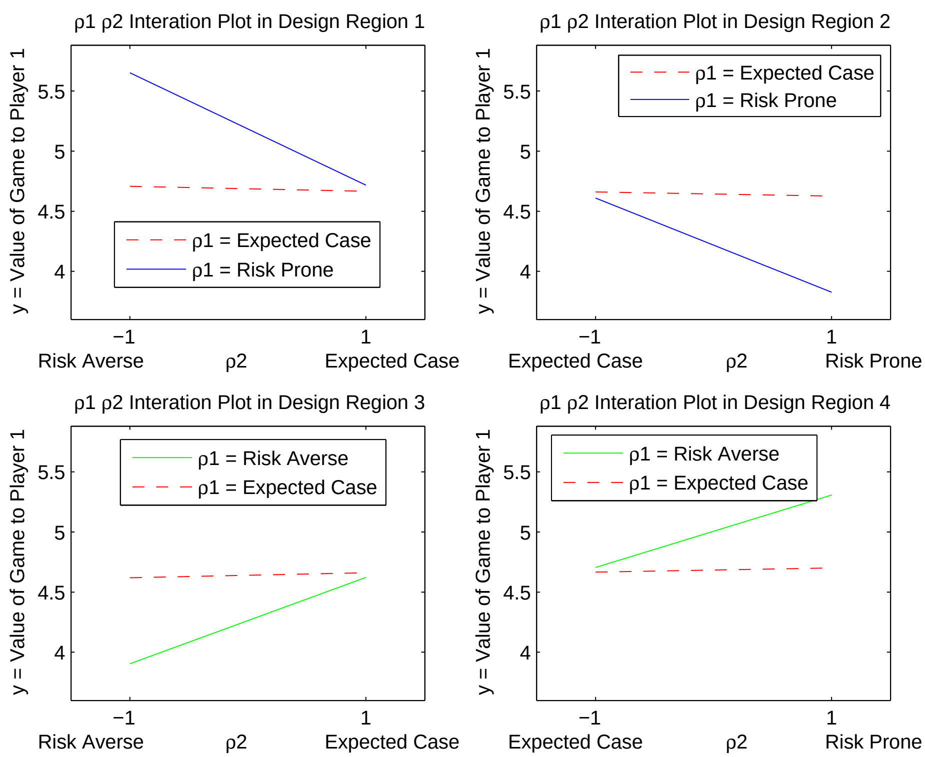

Figure 6: Second Update ρ Interaction Plot at S_5

produces similar results, when the defense takes a risk prone approach to the scenario and the offense is risk averse, the offense can expect to gain more yards. When the defense plays risk averse and the offense is risk averse, the offense can expect to gain less yards.

4.6 Studying Game Film

The coaches may be interested in the quality of their observations during games throughout the season. A way to measure how well they are reading the defense is by obtaining the true optimal decisions from past game tapes and comparing this with the perceived optimal decisions called during the game.

Suppose the true defense in situation 1 was a 4-4 zone. Using the equation to compute the value of the game to player 1 and the perceived optimal strategy after all the updates, $\hat{\gamma}^{(1)}_{\mathbf{S_1}}$, the value of the perceived optimal strategy is

$$\begin{aligned}
\hat{\pi}^{(1)}_{\mathbf{S_1}} &= \hat{\gamma}^{(1)}_{\mathbf{S_1}}\, \mathbf{R}^{(\mathbf{1})}_{\mathbf{S_1}}\, \delta^{(\mathbf{1})'}_{\mathbf{S_1}} \\
&= [0, 0, .1933, .8067] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 5.18,
\end{aligned}$$

while the true value of the game given the offense knew the defensive formation is

$$\begin{aligned}
\pi^{(1)}_{\mathbf{S_1}} &= \gamma^{(1)}_{\mathbf{S_1}}\, \mathbf{R}^{(\mathbf{1})}_{\mathbf{S_1}}\, \delta^{(\mathbf{1})'}_{\mathbf{S_1}} \\
&= [0, 0, 0, 1] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 6.3.
\end{aligned}$$

The true optimal value is calculated using the strategy that the offense would have used had they known the defense was in a 4-4 zone. The difference between these two values is

$$\begin{aligned}
\vec{\pi}^{(1)}_{\mathbf{S_1}} &= \pi^{(1)}_{\mathbf{S_1}} - \hat{\pi}^{(1)}_{\mathbf{S_1}} \\
&= 5.18 - 6.3 \\
&= -1.12
\end{aligned}$$

yards. This is the lost opportunity by the offense for not having perfect information. That is, the offense lost 1.12 yards because they could only perceive the defensive formation in part. To determine the value of the quarterback observation in this scenario, the value of the game at $s = 0$ must be subtracted from the value at $s = 1$. The value at $s = 0$, or the original value of the game taking into consideration the risk strategy of the players $\rho_1 = \rho_2 = -.7$, is

$$\begin{aligned}
\hat{\pi}^{(0)}_{\mathbf{S_1}} &= \hat{\gamma}^{(0)}_{\mathbf{S_1}}\, \mathbf{R}^{(\mathbf{0})}_{\mathbf{S_1}}\, \delta^{(\mathbf{0})'}_{\mathbf{S_1}} \\
&= [0, 0, .1538, .8462] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 5.41.
\end{aligned}$$

Thus the value added by the quarterback is

$$\begin{aligned}
\dot{\pi}^{(1)}_{\mathbf{S_1}} &= \hat{\pi}^{(1)}_{\mathbf{S_1}} - \hat{\pi}^{(0)}_{\mathbf{S_1}} \\
&= 5.18 - 5.41 \\
&= -.23.
\end{aligned}$$

The quarterback observation in this situation, even though correct, actually cost the offense .23 yards of expected gain. Normally, a good observation will add value to the game, this is a rare exception.

During situation 2, suppose the defense was truly in a 5-4 Blitz formation. In this situation, the offense chose to run the deep pass with probability 1 because

$$\hat{\beta}^{(2)}_{\mathbf{S_2}} = [4 - 4Overload, 5 - 4Blitz].$$

In this situation, even though the offense did not have full information about the defensive formation, his strategy was such that

$$\pi^{(2)}_{\mathbf{S_2}} = \hat{\pi}^{(2)}_{\mathbf{S_2}} = 7.$$

The true value of the game given the 5-4 Blitz is equal to the perceived value of the game given the 5-4 Blitz as calculated in Equation 8. Gaining perfect information in this situation would not be of value to the offense. The original strategy by the offense, $\hat{\gamma}^{(0)}$, results in

$$\begin{aligned}
\hat{\pi}^{(0)}_{\mathbf{S_2}} &= \hat{\gamma}^{(0)}_{\mathbf{S_2}}\, \mathbf{R}^{(\mathbf{0})}_{\mathbf{S_2}}\, \delta^{(\mathbf{0})'}_{\mathbf{S_2}} \\
&= [.0601, .4155, .3120, .2125] \begin{bmatrix} 3.8 \\ 3 \\ 3.9 \\ 7 \end{bmatrix} [1] \\
&= 3.76.
\end{aligned}$$

The first observation by the quarterback results in a value of

$$\begin{aligned}
\hat{\pi}^{(1)}_{\mathbf{S_2}} &= \hat{\gamma}^{(1)}_{\mathbf{S_2}}\,\mathbf{R}^{(\mathbf{1})}_{\mathbf{S_2}}\,\delta^{(\mathbf{1})}_{\mathbf{S_2}} \\
&= [0, 0, .8066, .1934] \begin{bmatrix} 3.8 \\ 3 \\ 3.9 \\ 7 \end{bmatrix} [1] \\
&= 4.50.
\end{aligned}$$

Thus the value added by the quarterback is

$$\begin{aligned}
\dot{\pi}^{(1)}_{\mathbf{S_2}} &= \hat{\pi}^{(1)}_{\mathbf{S_2}} - \hat{\pi}^{(0)}_{\mathbf{S_2}} \\
&= 4.5 - 3.76 \\
&= .74.
\end{aligned}$$

The quarterback observation in this situation adds about .75 yards of expected gain by the offense, this is good. The value of the coaches observation is found using the value of the game at s=2

$$\begin{aligned}
\hat{\pi}^{(2)}_{\mathbf{S_2}} &= \hat{\gamma}^{(2)}_{\mathbf{S_2}}\,\mathbf{R}^{(\mathbf{2})}_{\mathbf{S_2}}\,\delta^{(\mathbf{2})'}_{\mathbf{S_2}} \\
&= [0, 0, 0, 1] \begin{bmatrix} 3.8 \\ 3 \\ 3.9 \\ 7 \end{bmatrix} [1] \qquad (8) \\
&= 7.
\end{aligned}$$

The added value of the coaches observation is

$$\begin{aligned}
\dot{\pi}^{(2)}_{\mathbf{S_2}} &= \hat{\pi}^{(2)}_{\mathbf{S_2}} - \hat{\pi}^{(1)}_{\mathbf{S_2}} \\
&= 7 - 4.5 \\
&= 2.5
\end{aligned}$$

yards.

Situation 3 finds the defense truly in the 4-3 Man formation. Recall the offense perceived the actions available to the defense as $\hat{\beta}^{(1)}_{\mathbf{S_3}} = [4-4Zone, 4-3Man]$. The value of the perceived optimal strategy is thus

$$\begin{aligned}
\hat{\pi}^{(1)}_{\mathbf{S_3}} &= \hat{\gamma}^{(1)}_{\mathbf{S_3}}\,\mathbf{R}^{(\mathbf{1})}_{\mathbf{S_3}}\,\delta^{(\mathbf{1})'}_{\mathbf{S_3}} \\
&= [0, .8403, 0, .1597] \begin{bmatrix} 6.6 \\ 5.9 \\ 7.3 \\ -3.4 \end{bmatrix} [1] \\
&= 4.41,
\end{aligned}$$

while the true value of the game is

$$\begin{aligned}
\pi^{(1)}_{\mathbf{S_3}} &= \gamma^{(1)}_{\mathbf{S_3}}\, \mathbf{R}^{(\mathbf{1})}_{\mathbf{S_3}}\, \delta^{(\mathbf{1})'}_{\mathbf{S_3}} \\
&= [0,0,1,0] \begin{bmatrix} 6.6 \\ 5.9 \\ 7.3 \\ -3.4 \end{bmatrix} [1] \\
&= 7.3.
\end{aligned}$$

The difference

$$\begin{aligned}
\vec{\pi}^{(1)}_{\mathbf{S_3}} &= \pi^{(1)}_{\mathbf{S_3}} - \hat{\pi}^{(1)}_{\mathbf{S_3}} \\
&= 7.3 - 4.41 \\
&= 2.89
\end{aligned}$$

yards represents the number of yards the offense could expect to gain had they known the defense was in a 4-3 Man. The value added by the quarterback observation is calculated as in the previous 2 situations.

$$\begin{aligned}
\dot{\pi}^{(1)}_{\mathbf{S_3}} &= \hat{\pi}^{(1)}_{\mathbf{S_3}} - \hat{\pi}^{(0)}_{\mathbf{S_3}} \\
&= 4.41 - 2.47 \\
&= 1.94,
\end{aligned}$$

is the number of yards gained due to the quarterback observation.

During situation 4, the offensive strategy failed to include the true action of the defense as a possible action, the defense was actually in a 4-4 zone. The offense chose to throw the short pass in the situation and only gained a few yards. The perceived optimal decision was based on

$$\hat{\beta}^{(2)}_{\mathbf{S_4}} = [4 - 4Overload, 4 - 3Man].$$

This resulted in a low value of the perceived optimal decision,

$$\begin{aligned}
\hat{\pi}^{(2)}_{\mathbf{S_4}} &= \hat{\gamma}^{(2)}_{\mathbf{S_4}} * \mathbf{R}^{(\mathbf{2})}_{\mathbf{S_4}}\, \delta^{(\mathbf{2})'}_{\mathbf{S_4}} \\
&= [0,0,.7701,.2299] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 1.83,
\end{aligned}$$

while the true value of the game is

$$\begin{aligned}
\pi^{(2)}_{\mathbf{S_4}} &= \gamma^{(2)}_{\mathbf{S_4}}\,\mathbf{R}^{(\mathbf{2})}_{\mathbf{S_4}}\,\delta^{(\mathbf{2})'}_{\mathbf{S_4}} \\
&= [0,0,0,1]\begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix}[1] \\
&= 6.3.
\end{aligned}$$

The difference

$$\begin{aligned}
\vec{\pi}^{(2)}_{\mathbf{S_4}} &= \pi^{(2)}_{\mathbf{S_4}} - \hat{\pi}^{(2)}_{\mathbf{S_4}} \\
&= 6.3 - 1.83 \\
&= 4.47
\end{aligned}$$

yards is significant and probably would have gained the offense a first down had they properly perceived the situation even to some degree. The yards gained by the original coach observation was

$$\begin{aligned}
\dot{\pi}^{(1)}_{\mathbf{S_4}} &= \hat{\pi}^{(1)}_{\mathbf{S_4}} - \hat{\pi}^{(0)}_{\mathbf{S_4}} \\
&= 4.17 - 3.91 \\
&= .26,
\end{aligned}$$

while the added yards of the QB observation was

$$\begin{aligned}
\dot{\pi}^{(2)}_{\mathbf{S_4}} &= \hat{\pi}^{(2)}_{\mathbf{S_4}} - \hat{\pi}^{(1)}_{\mathbf{S_4}} \\
&= 1.83 - 4.17 \\
&= -2.34.
\end{aligned}$$

Individually studying the value added by the observations leads us to conclude the QB observation was in error and cost the offense about 2.34 yards.

During the final situation, the offense perceived the defense to be in 5-4 Blitz formation and decided to call a deep pass. The ball was knocked down and the defense took over on downs. Recall that

$$\hat{\beta}^{(2)}_{\mathbf{S_5}} = [5 - 4Blitz, 4 - 3Man],$$

was correct up to the second update. The true defensive formation was a 4-3 Man, the observation from the press box was in error,

$$\begin{aligned}
\vec{\pi}^{(3)}_{\mathbf{S_5}} &= (-3.4) - 7.3 \\
&= -10.7.
\end{aligned}$$

DOI: 10.2202/1559-0410.1176

The strategy used by the offense resulted in a loss of opportunity of 10.7 yards. With true information, the offense would have surely gained a first down and possibly won the game.

The performance of the offense can be graphed over time to determine the quality of the reads by the quarterback, sideline coach, and pressbox coach collectively and individually. Using the value of perfect information $\vec{\pi}$ at each situation gives a feel for the performance. The difference between the true and perceived optimal decisions can also be thought of as the regret or error that the offense shouldered because of their perception. If the perception by the offense is the defensive formation that the defense actually runs, perfect information has no value. When the offensive perception is in error or incomplete, the offensive regret increases, or the value of obtaining perfect information increases. A high value of perfect information indicates a poor perception by the offense. Looking at Figure 7, the error of the offensive strategy increased over time, indicating that the offensive perception of the defensive formation degraded as the game progressed. This is computed by using the true optimal strategy given the truth in each situation. This is subtracted from the value of the perceived optimal strategy given the truth. For example, during situation 1, the true defensive formation was the 4-4 zone. The true optimal offensive strategy for the 4-4 zone is

$$\gamma_{\mathbf{S_1}} = \{0, 0, 0, 1\}.$$

This results in a true optimal value of the game

$$\begin{aligned}
\pi_{\mathbf{S_1}} &= \gamma_{\mathbf{S_1}} \, \mathbf{R}_{\mathbf{S_1}} \, \delta'_{\mathbf{S_1}} \\
&= [0, 0, 0, 1] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 6.3.
\end{aligned}$$

The perceived value of the game is

$$\begin{aligned}
\hat{\pi}_{\mathbf{S_1}} &= \hat{\gamma}_{\mathbf{S_1}} \, \mathbf{R}_{\mathbf{S_1}} \, \delta'_{\mathbf{S_1}} \\
&= [0, 0, .1933, .8067] \begin{bmatrix} 4 \\ 5.1 \\ .5 \\ 6.3 \end{bmatrix} [1] \\
&= 5.18.
\end{aligned}$$

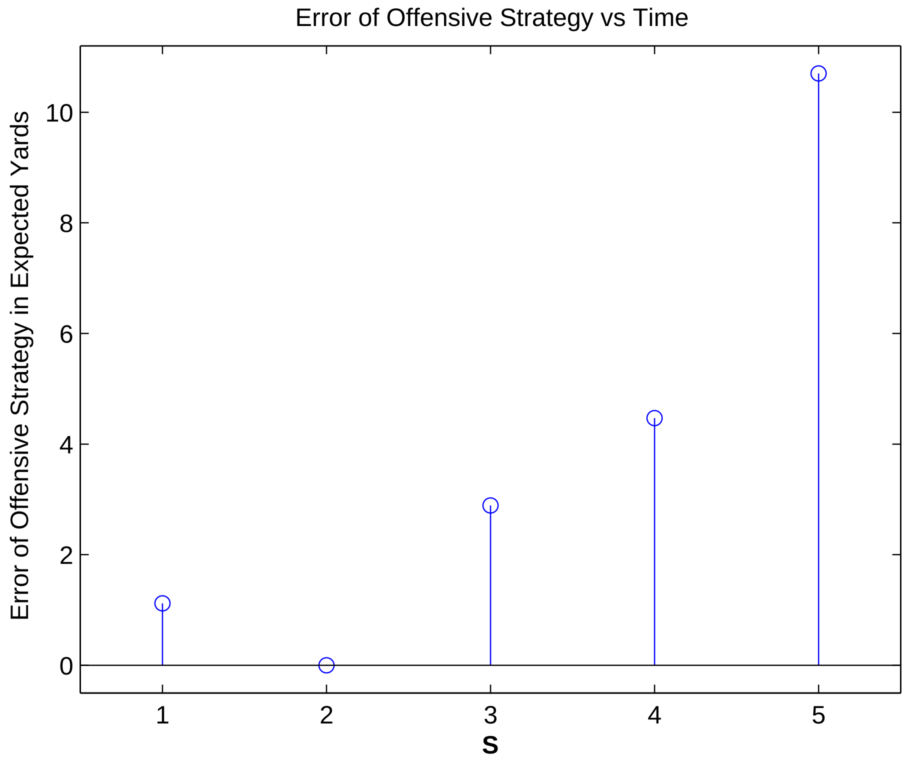

Figure 7: Value of Perfect Information

Subtracting these two values gives the error of the offensive strategy during situation 1, $6.3 - 5.18 = 1.12$. The performance of each individual observer can also be examined over time. The value added in relation to the previous time step or update can be determined. Figure 8, shows the value added to the game by the quarterback over time.

The observation of the quarterback hurt the value of the game during situation 4. The value of all the observations is a better measure of how well the offense is reading the defense, as the observations or sensors are not independent, each observation relies on the previous. Figure 9 shows the value of all the observations over time. During situations 2 and 3, the observations of the offense added a significant number of yards to the expected gain, while during situations 4 and 5, the offensive observations actually impaired their strategy.

These are simple examples, in reality this graph may contain much more insight into the performance of an individual over the course of a season or

DOI: 10.2202/1559-0410.1176

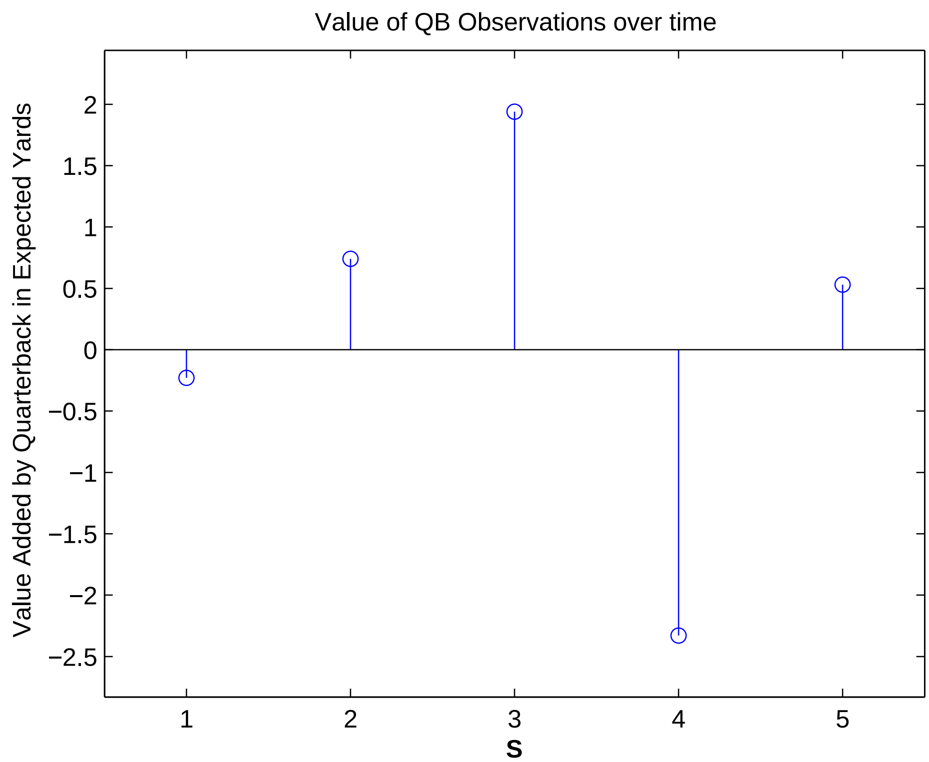

Figure 8: QB Added Value over time

game. This may indicate things such as fatigue during a game, or learning during the game. Over a season, these graphs could show the maturity gained by a junior quarterback, or lack there of. In this way, the performance of individuals or coaches (sensors) can be analyzed over time.

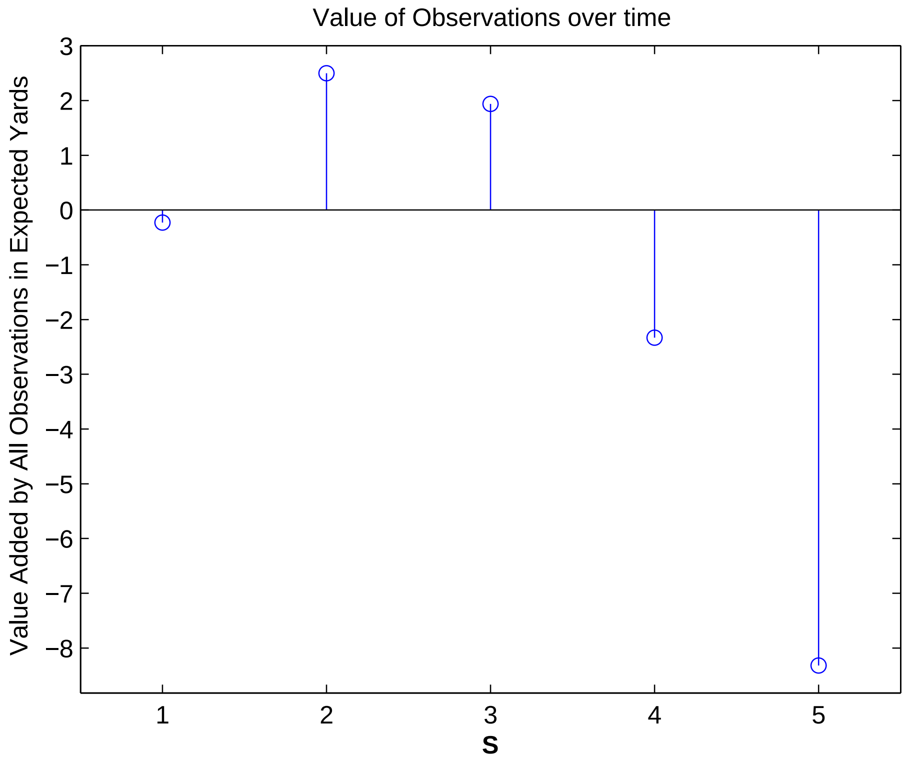

Figure 9: Observations Added Value over time

5 Concluding Remarks

There are numerous strengths in using these techniques. Specifically, it allows a decision maker to update his optimal decision policy based on new information as it arrives. Utility theory allows flexibility in this model by allowing any type of decision maker to be represented. Exploring good risk strategies in approach to each situation further strengthens the quality of the decision.

The techniques discussed here cannot account for all the factors that influence what plays are called. The model should thus be used as a decision support tool for coaches and teams, not a replacement for a coach and his experience. There are 3 major areas in which a coach could utilize the methods. First, the model should be used in planning a strategy for a game. By looking at the implications of the Nash equilibrium strategies, a team can better plan for situations that may arise during the game. Secondly, the algorithm could be used during an actual game to give real time optimal strategies based on what the offense has observed. The computer program developed to produce the numbers in this paper runs very quickly, and could easily be adapted to perform real-time analysis. Finally, as shown in Section 4, the methodology gives insight to decision makers following a game. Player and coach performance can be analyzed, providing a team with added training potential and post-game discussion points.

Although the model does have limitations, we believe game theory provides the most suitable framework for analyzing football play calling. The techniques presented provide a framework for a wide variety of situations to be modeled, as well as open the door for future research using game theory to update optimal decisions.

References

[1] Alamar, B. (2006). The Passing Premium Puzzle. *Journal of Quantitative Analysis in Sports*, **2** (4), Article 5.

[2] Berkovitz, L.D. and Dresher, M. (1959). A Game-Theory Analysis of Tactical Air-War. *Operations Research*, **7** (5) 599-620.

[3] Bier, Vicki M. and Abhichandani, Vinod. (2002). Optimal Allocation of Resources for Defense of Simple Series and Parallel Systems from Determined Adversaries. *Proceedings of the Engineering Foundation Conference on Risk Based Decision Making in Water Resources X*, American Society of Civil Engineers, Santa Barbara, CA.

[4] Boronico, J. and Newbert, S. (1999). Play Calling Strategy in American Football: A Game-Theoretic Stochastic Programming Approach. *Journal of Sports Management*, **13**, 103-113.

[5] Boronico, J. and Newbert, S. (2001). An Empirically Driven Mathematically Modeling Analysis for Play Calling Strategy in American Football. *European Sport Management Quarterly*, **1**, 21-38.

[6] Carter, V. and Machol, R. (1971). Operations Research on Football. *Operations Research*, **19** (2), 541-544.

[7] Carter, V. and Machol, R. (1978). Optimal Strategies on Fourth Down. *Management Science*, **24** (16), 1758-1762.

[8] Harris, Bernard (2004). *Mathematical Methods in Combatting Terrorism. Risk Analysis*, **24** (4), 985-988.

[9] Kirkwood, C. W. (1997). Strategic Decision Making. *Wadsworth Publishing Company*, **6**, pp 64-68.

[10] Krichman, M., Ghose, D., Speyer, J.L., and Shamma, J.S. (2001). Theater Level Campaign Resource Allocation. *Proceedings of the American Control Conference*, **6** 4716-4721.

[11] Nash, J. (1951) "Non-Cooperative Games". *Annals of Mathematics*, **54** (2), 286-295.

[12] Sahi, S. and Shubik, M. (1988). A Model of a Sudden-Death Field-Goal Football Game as a Sequential Duel. *Mathematical Social Sciences*, **15**, 205-215.

[13] Sandoy, Marlene and Aven, Terje. (2006). Real Time Updating of Risk Assessments During a Drilling Operation - Alternative Bayesian Approaches. *International Journal of Reliability, Quality and Safety Engineering*, **13** (1), 85-95.

[14] Sandler, Todd and Arce, Daniel (2003). Terroism and Game Theory. *Simulation Gaming*, **3434**, (3) 319-337.

[15] Shor, Mikhael. Dictionary of Game Theory Terms, Game Theory .net, http://www.gametheory.net/dictionary/ Web accessed: October 15, 2006.

[16] Sujit, P.B. and Ghose, D. (2005). Search by UAVs with Flight Time Constraints using Game Theoretic Models. *Proceedings of the AIAA Guidance, Navigation, and Conference and Exhibit*, **AIAA 2005-6241**.

[17] Thomas, Clayton J. and Deemer, Walter L. (1957). The Role of Operational Gaming in Operations Research. *Operations Research*, **5** (1) 1-27.

[18] Von Neumann, J. and Morgenstern, O. (1944). Theory of Games and Economic Behavior. *Science Editions*, **3**, 1964.

[19] Winston, W. L. (2004). Operations Research: Applications and Algorithms. *Thomson Learning*, **6**.

[20] Zhuang, Jun and Bier, Vicki (2006). Balancing Terrorism and Natural Disasters - Defensive Strategy with Endogenous Attacker Effort. *Submitted to Operations Research*.

DOI: 10.2202/1559-0410.1176