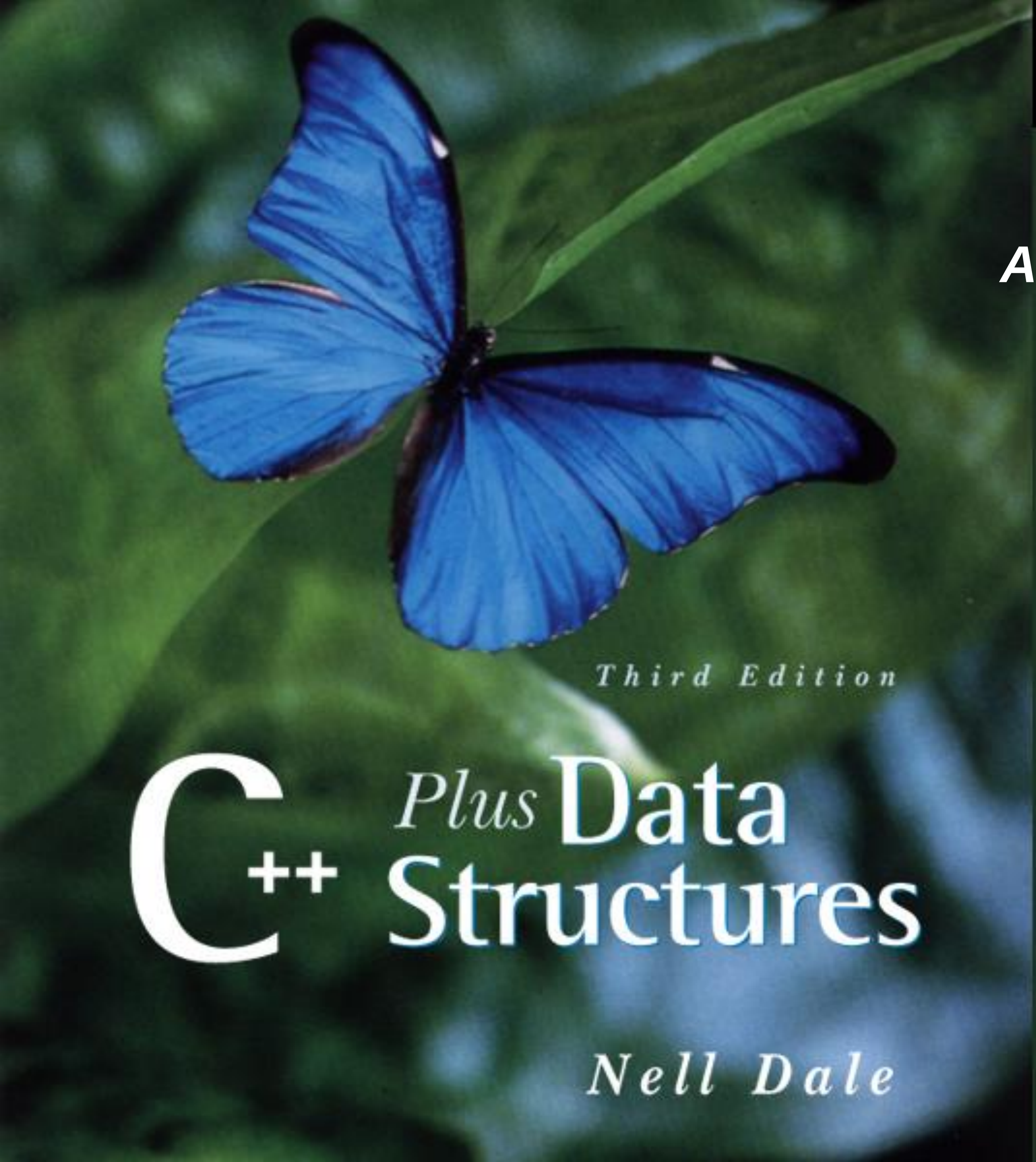


Chapter

3

ADTs Unsorted List and Sorted List

List Definitions

Linear relationship Each element except the first has a unique predecessor, and each element except the last has a unique successor.

Length The number of items in a list; the length can vary over time.

List Definitions

Unsorted list A list in which data items are placed in no particular order; the only relationship between data elements is the list predecessor and successor relationships.

Sorted list A list that is sorted by the value in the key; there is a semantic relationship among the keys of the items in the list.

Key The attributes that are used to determine the logical order of the list.

Abstract Data Type (ADT)

- **A data type whose properties (domain and operations) are specified independently of any particular implementation.**

Data from 3 different levels

- ***Application (or user) level:*** **modeling real-life data in a specific context.**
- ***Logical (or ADT) level:*** **abstract view of the domain and operations.** WHAT
- ***Implementation level:*** **specific representation of the structure to hold the data items, and the coding for operations.** HOW

4 Basic Kinds of ADT Operations

- **Constructor** -- creates a new instance (object) of an ADT.
- **Transformer** -- changes the state of one or more of the data values of an instance.
- **Observer** -- allows us to observe the state of one or more of the data values of an instance without changing them.
- **Iterator** -- allows us to process all the components in a data structure sequentially.

Sorted and Unsorted Lists

UNSORTED LIST	SORTED LIST
Elements are placed into the list in no particular order.	List elements are in an order that is sorted in some way -- either numerically or alphabetically by the elements themselves, or by a component of the element (called a KEY member) .

ADT Unsorted List Operations

Transformers

- MakeEmpty
- InsertItem
- DeleteItem

Observers

- IsFull
- LengthIs
- RetrieveItem

Iterators

- ResetList
- GetNextItem

change state

observe state

process all

What is a Generic Data Type?

What type of data item does the list contain?

A generic data type is a type for which the operations are defined but the types of the items being manipulated are not defined.

One way to simulate such a type for our UnsortedList ADT is via a user-defined class **`ItemType`** with member function **`ComparedTo`** returning an enumerated type value **`LESS`**, **`GREATER`**, or **`EQUAL`**.

```
//  SPECIFICATION FILE              ( unsorted.h )
#include  "ItemType.h"

class  UnsortedType         // declares a class data type
{
public :                    //  8 public member functions

   void    UnsortedType ( ) ;
   bool    IsFull ( )  const ;
   int     LengthIs ( )  const ;  // returns length of list
   void    RetrieveItem ( ItemType&  item, bool&  found ) ;
   void    InsertItem ( ItemType  item ) ;
   void    DeleteItem ( ItemType  item ) ;
   void    ResetList ( );
   void    GetNextItem ( ItemType&  item ) ;

private :                          //  3 private data members

   int        length ;
   ItemType   info[MAX_ITEMS] ;
   int        currentPos ;
} ;
```

Class Constructor

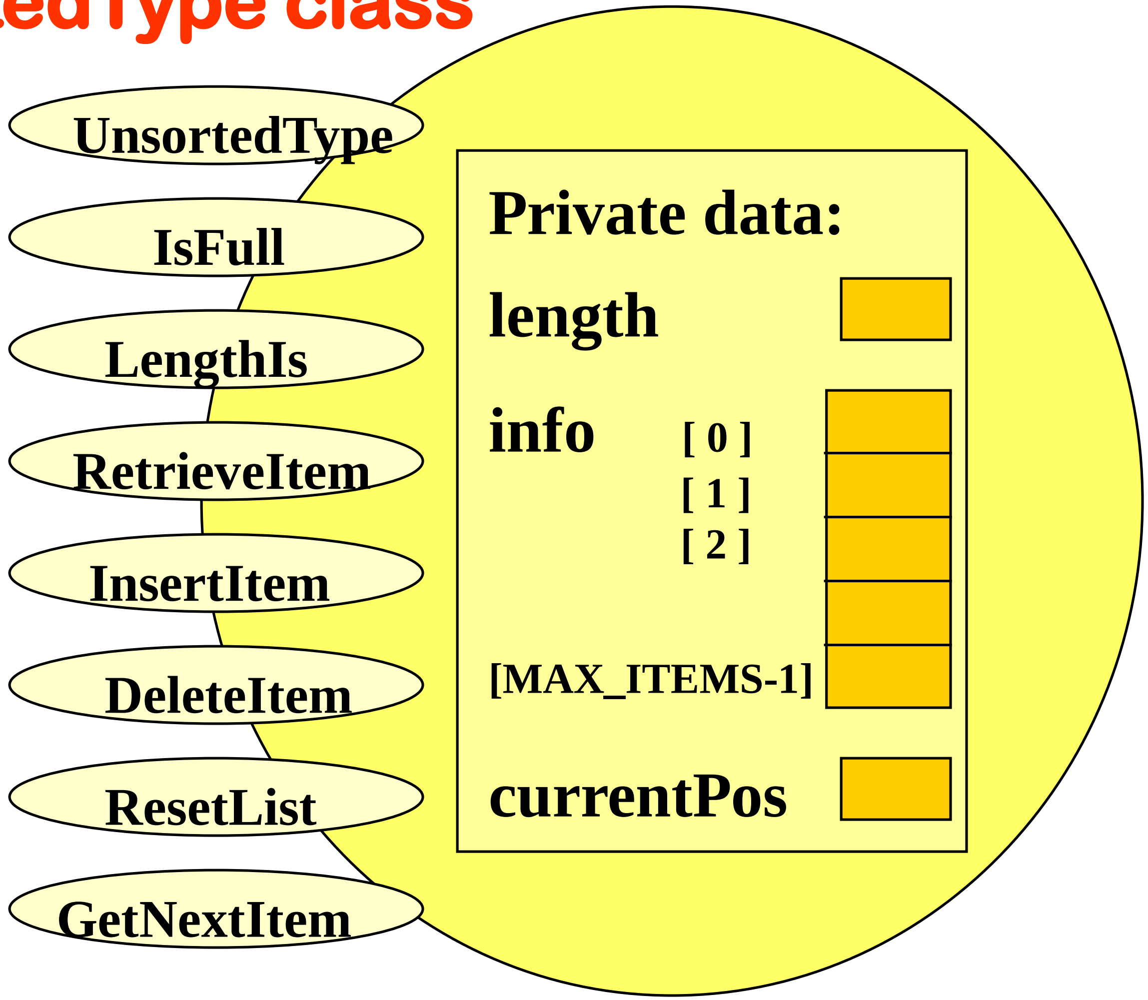
A special member function of a class that is implicitly invoked when a class object is defined.

Class Constructor Rules

1. A constructor cannot return a function value, and has no return value type.
2. A class may have several constructors. The compiler chooses the appropriate constructor by the number and types of parameters used.
3. Constructor parameters are placed in a parameter list in the declaration of the class object.
4. The parameterless constructor is the default constructor.
5. If a class has at least one constructor, and an array of class objects is declared, then one of the constructors must be the default constructor, which is invoked for each element in the array.

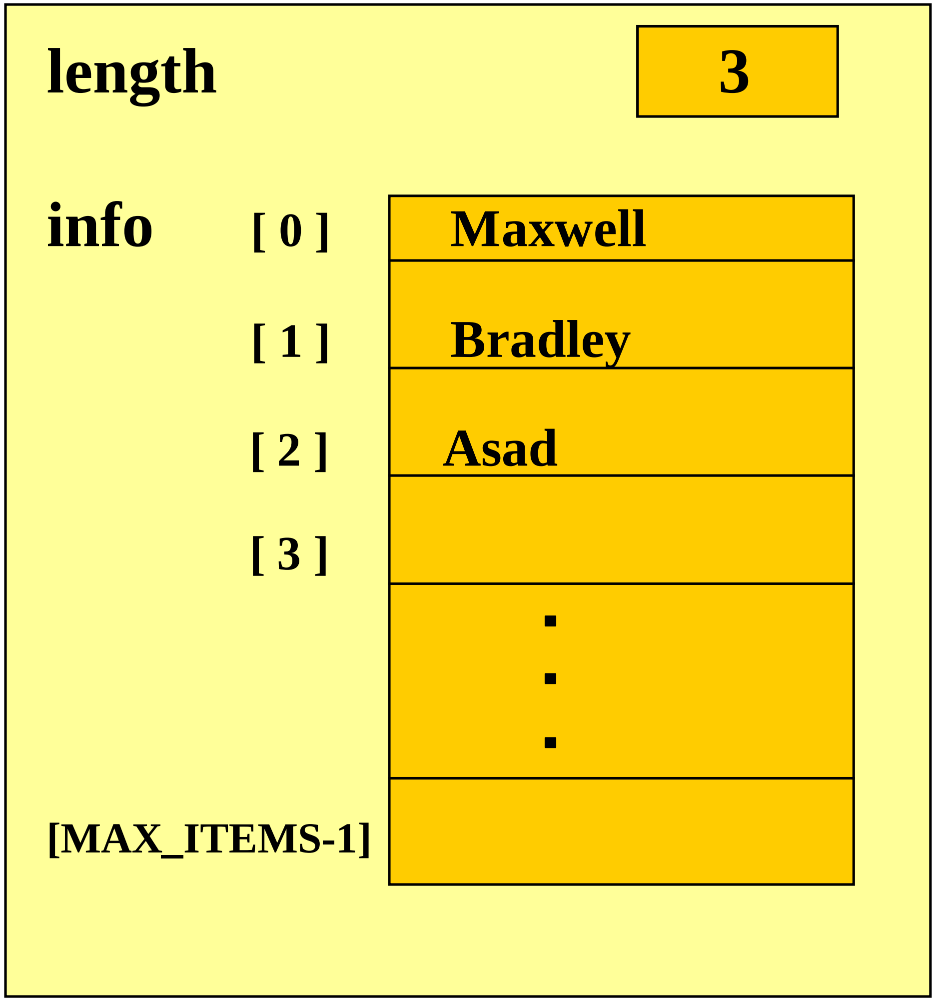
Class Interface Diagram

UnsortedType class



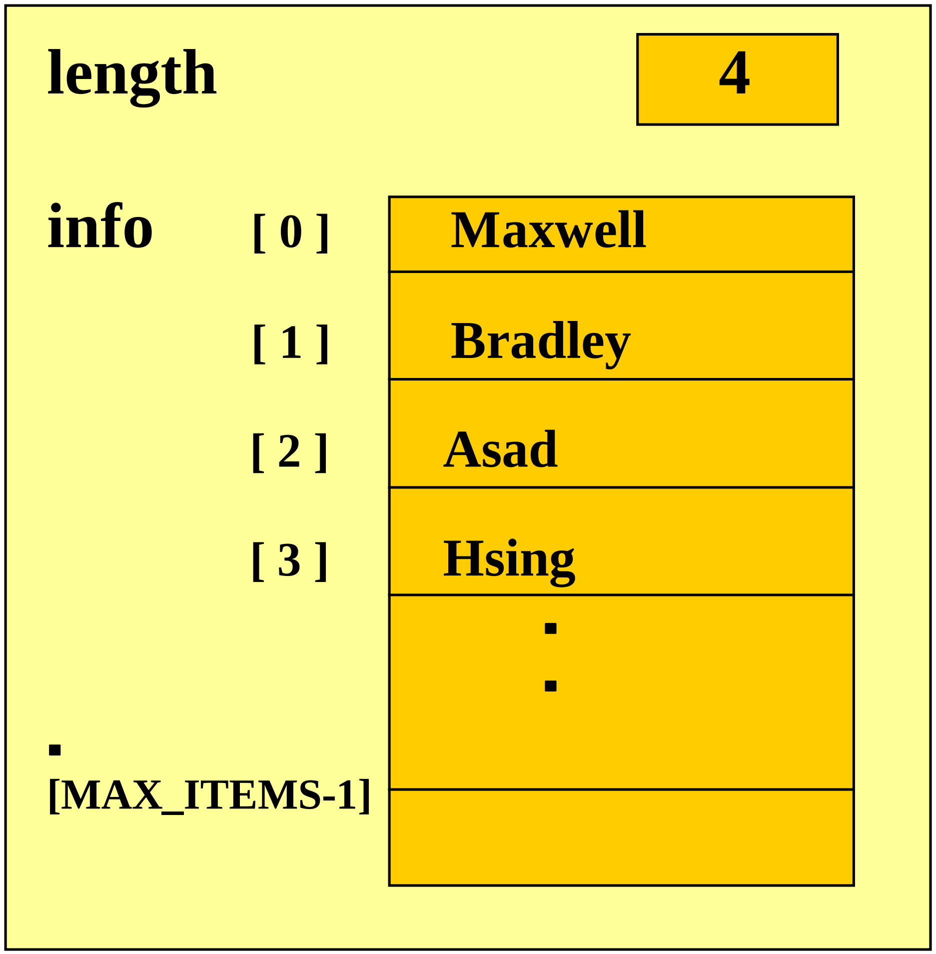
```
// IMPLEMENTATION FILE   ARRAY-BASED LIST  ( unsorted.cpp )
#include "itemtype.h"

void UnsortedType::UnsortedType ( )
//  Pre: None.
// Post: List is empty.
{
   length  =  0 ;
}

void  UnsortedType::InsertItem ( ItemType  item )
//  Pre: List has been initialized. List is not full.
// item is not in list.
// Post:  item is in the list.
{
   info[length] = item ;
   length++ ;
}
```

Before Inserting Hsing into an Unsorted List



The item will be placed into the length location, and length will be incremented.

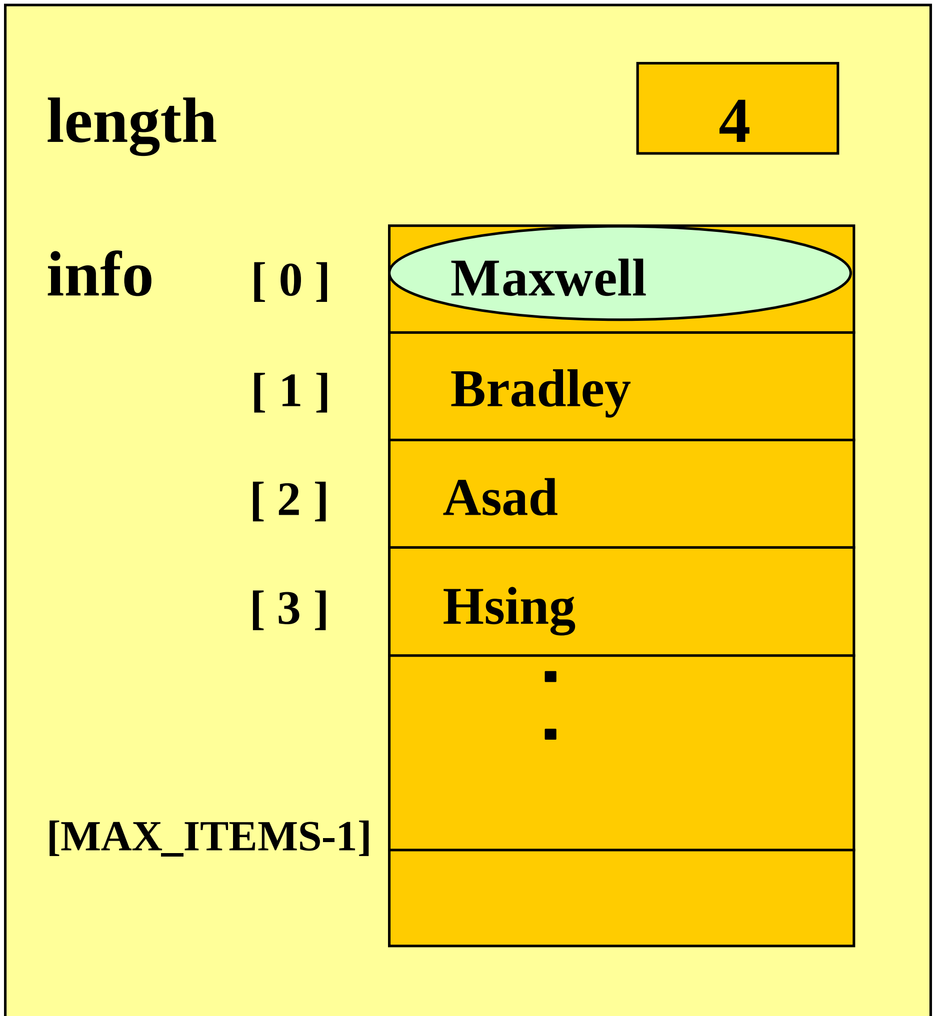
After Inserting Hsing into an Unsorted List



```
int UnsortedType::LengthIs ( )  const
//  Pre: List has been inititalized.
// Post: Function value == ( number of elements in
// list ).
{
   return  length ;
}


bool  UnsortedType::IsFull ( )  const
//  Pre: List has been initialized.
// Post: Function value == ( list is full ).
{
   return ( length == MAX_ITEMS ) ;
}
```

```
void  UnsortedType::RetrieveItem ( ItemType&  item,  bool& found )
//  Pre: Key member of item is initialized.
// Post: If found, item's key matches an element's key in the list
// and a copy of that element has been stored in item;
// otherwise, item is unchanged.
{  bool  moreToSearch ;
   int   location = 0 ;

   found = false ;
   moreToSearch = ( location < length ) ;
   while ( moreToSearch  &&  !found )
   {  switch ( item.ComparedTo( info[location] ) )
      { case   LESS     :
         case  GREATER  : location++ ;
                          moreToSearch = ( location < length ) ;
                          break;
         case  EQUAL    : found = true  ;
                          item = info[ location ] ;
                          break ;
      }
   }
}
```

Retrieving Ivan from an Unsorted List

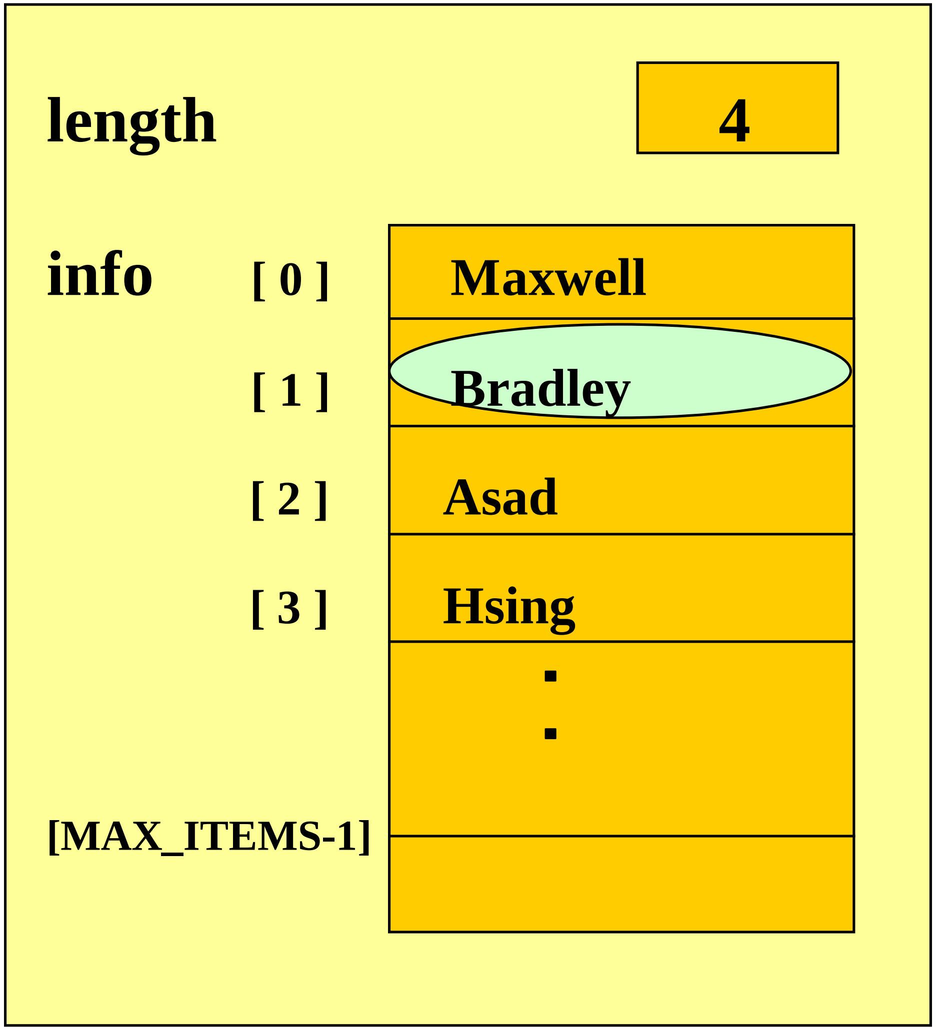


moreToSearch: true

found: false

location: 0

Retrieving Ivan from an Unsorted List



moreToSearch: true

found: false

location: 1

Retrieving Ivan from an Unsorted List

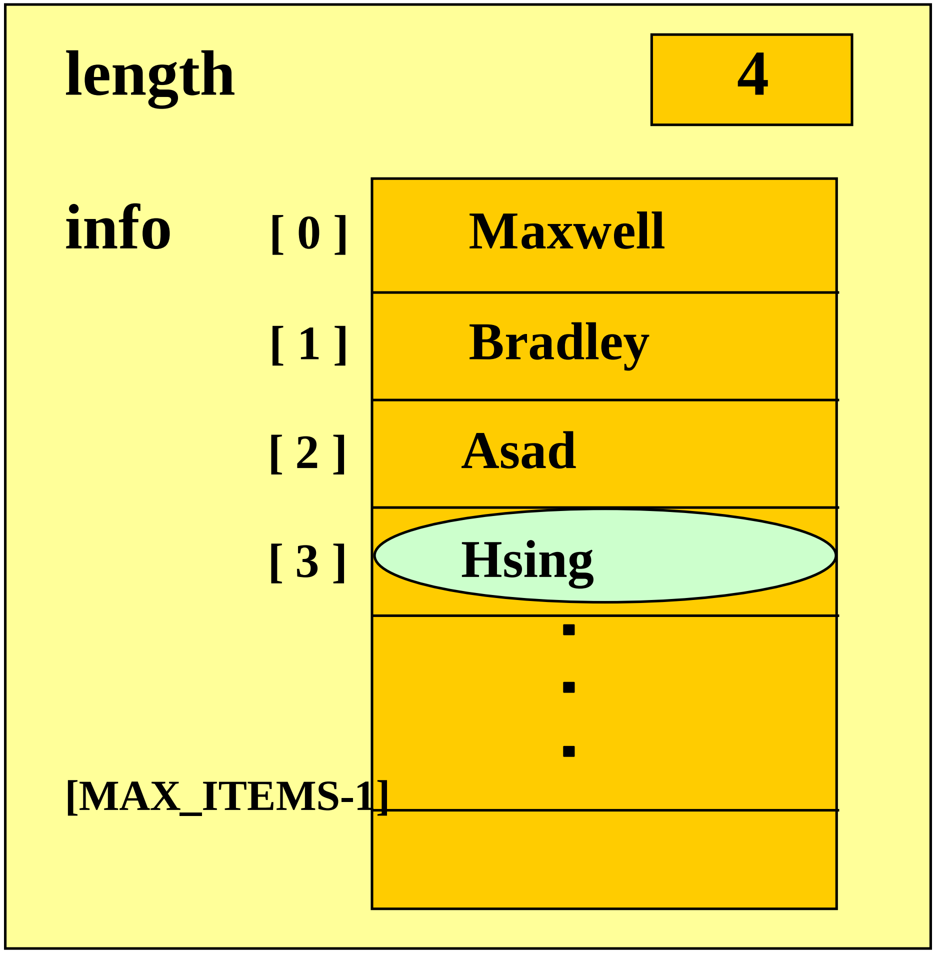
length		4
info	[0]	Maxwell
	[1]	Bradley
	[2]	Asad
	[3]	Hsing
		. . .
	[MAX_ITEMS-1]	

moreToSearch: true

found: false

location: 2

Retrieving Ivan from an Unsorted List



moreToSearch: true

found: false

location: 3

Retrieving Ivan from an Unsorted List

length 4

info

[0]	Maxwell
[1]	Bradley
[2]	Asad
[3]	Hsing
.	.
.	.
.	
[MAX_ITEMS-1]	

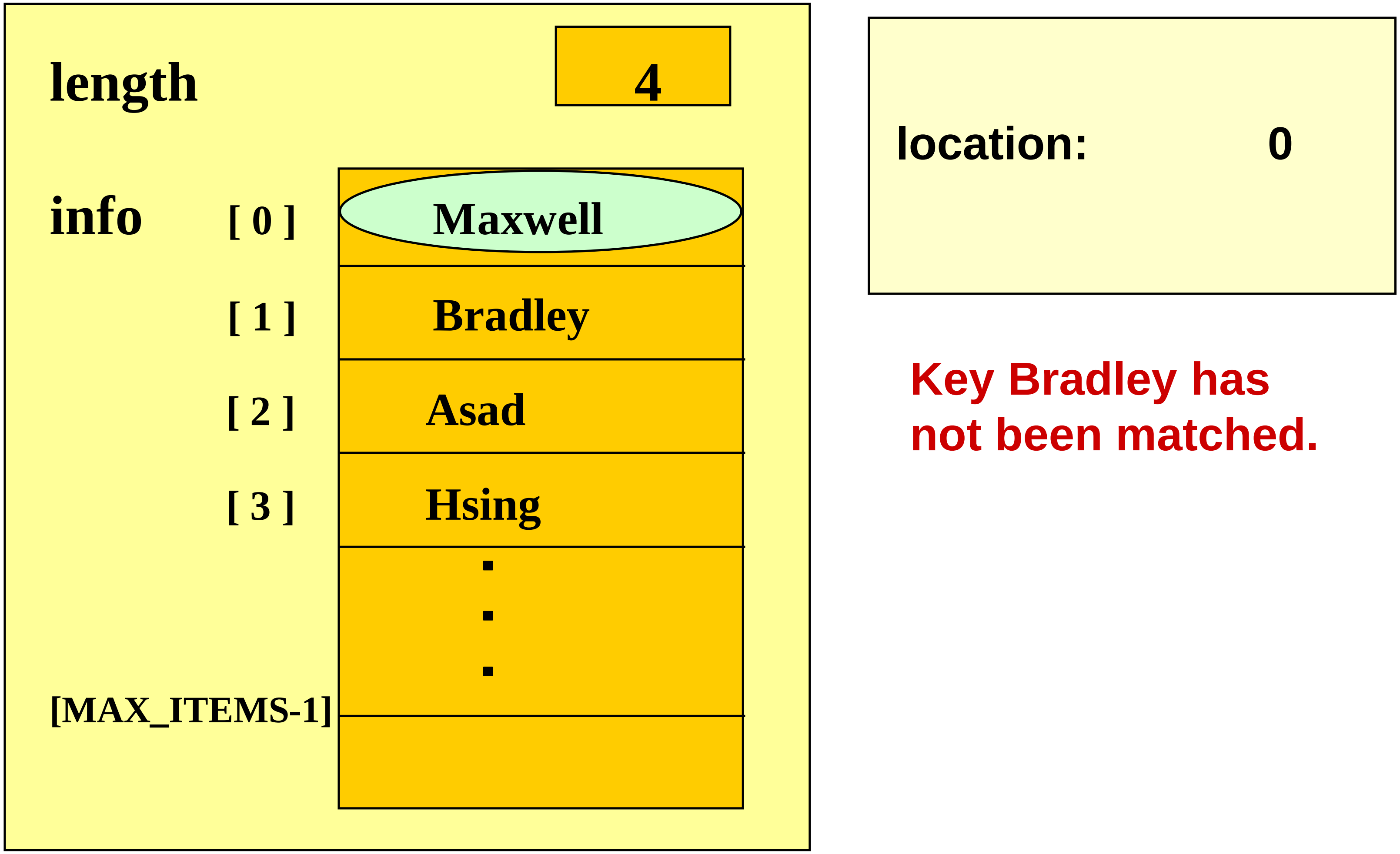
moreToSearch: false

found: false

location: 4

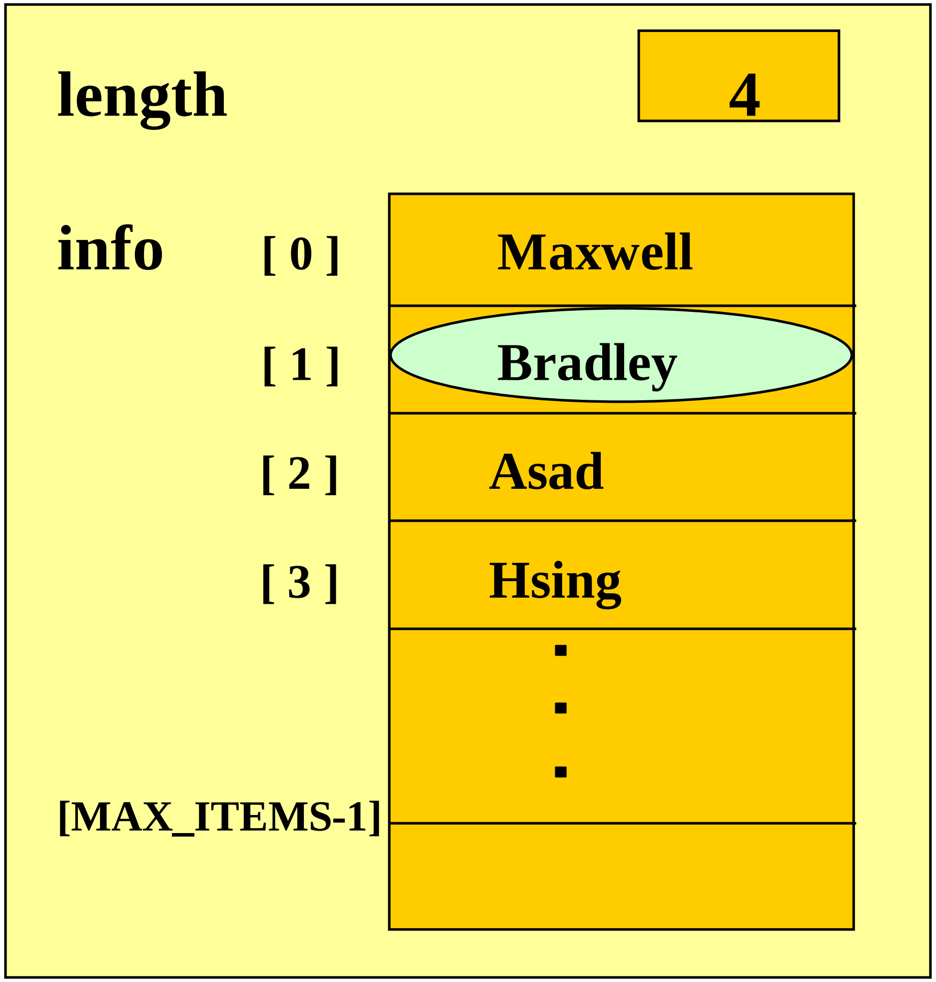
```
void UnsortedType::DeleteItem ( ItemType  item )
//  Pre: item’s key has been initialized.
// An element in the list has a key that matches item’s.
// Post: No element in the list has a key that matches item’s.
{
   int  location  =  0 ;

    while (item.ComparedTo (info [location] )  !=  EQUAL )
        location++;

   // move last element into position where item was located

   info [location] = info [length - 1 ] ;
   length-- ;
}
```

Deleting Bradley from an Unsorted List



Key Bradley has not been matched.

Deleting Bradley from an Unsorted List



location: 1

Key Bradley has been matched.

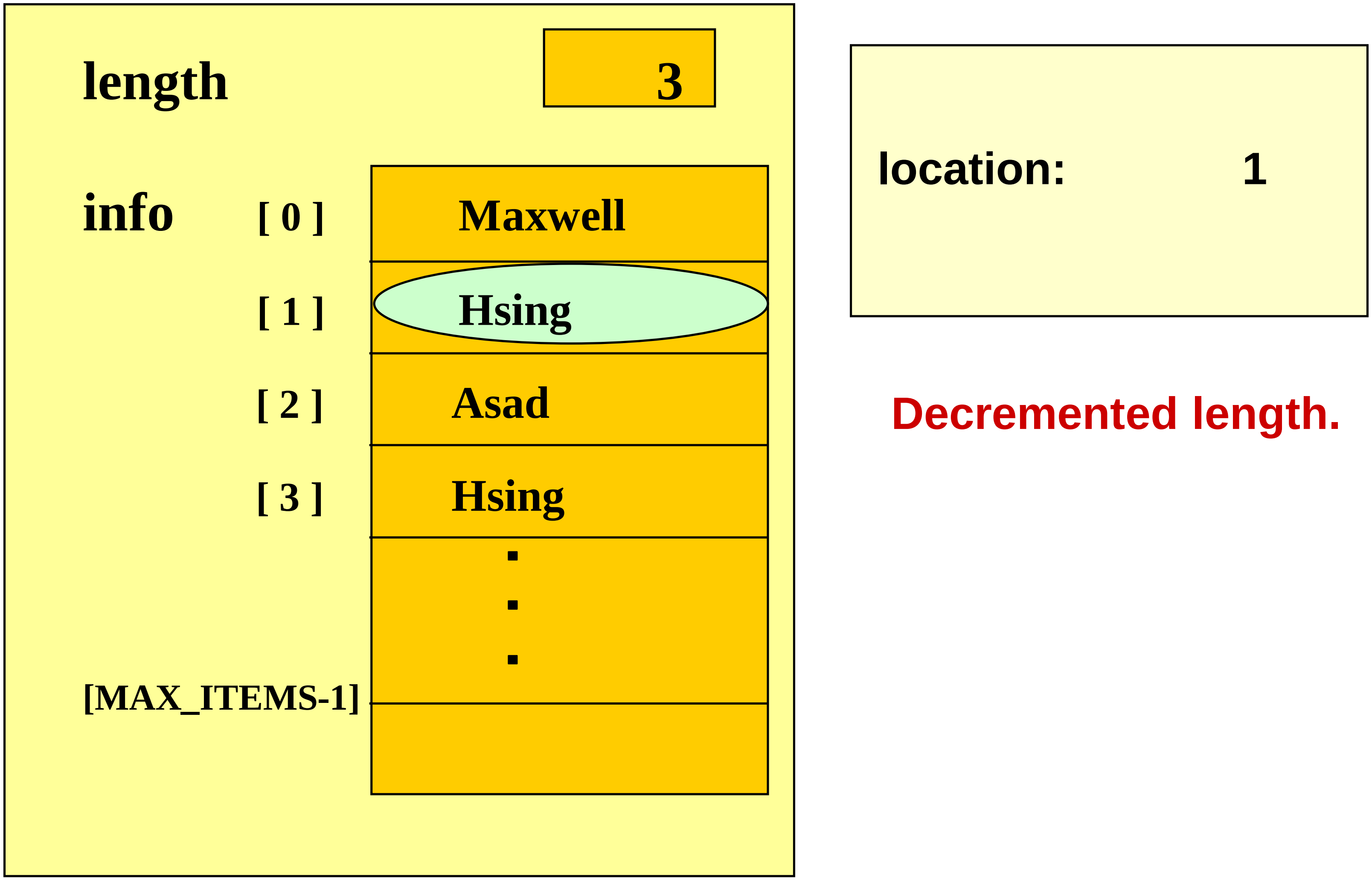
Deleting Bradley from an Unsorted List

length		4
info	[0]	Maxwell
	[1]	Hsing
	[2]	Asad
	[3]	Hsing
		. . .
	[MAX_ITEMS-1]	

location: 1

Placed copy of last list element into the position where the key Bradley was before.

Deleting Bradley from an Unsorted List



Decremented length.

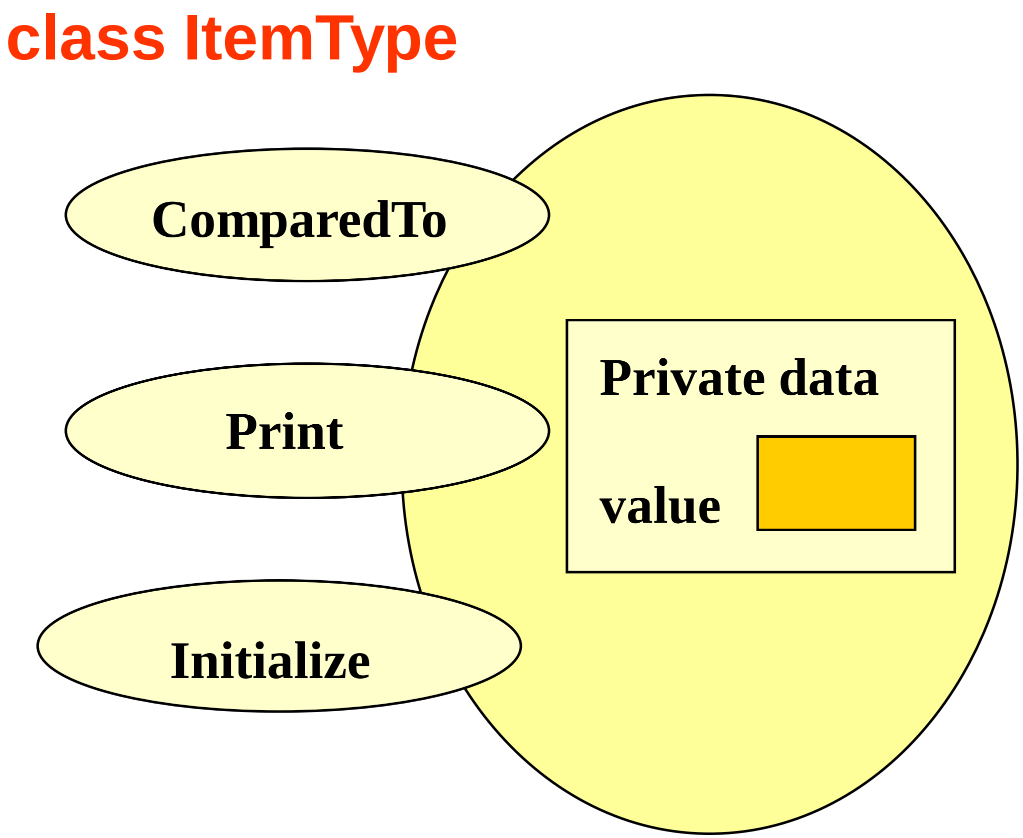
```
void UnsortedType::ResetList ( )
//  Pre: List has been inititalized.
// Post: Current position is prior to first element in list.
{
   currentPos  =  -1 ;
}

void  UnsortedType::GetNextItem ( ItemType&  item )
// Pre: List has been initialized. Current position is defined.
//  Element at current position is not last in list.
// Post: Current position is updated to next position.
// item is a copy of element at current position.
{
   currentPos++  ;
   item = info [currentPos] ;
}
```

Specifying class ItemType

```
//   SPECIFICATION FILE           ( itemtype.h )

const  int  MAX_ITEM = 5 ;
enum  RelationType { LESS, EQUAL, GREATER } ;

class  ItemType                // declares class data type
{
public :                       //  3 public member functions
   RelationType    ComparedTo ( ItemType )  const ;
   void            Print ( )  const ;
   void            Initialize ( int  number )   ;

private :                      //  1 private data member
   int value   ;               // could be any different type
} ;
```

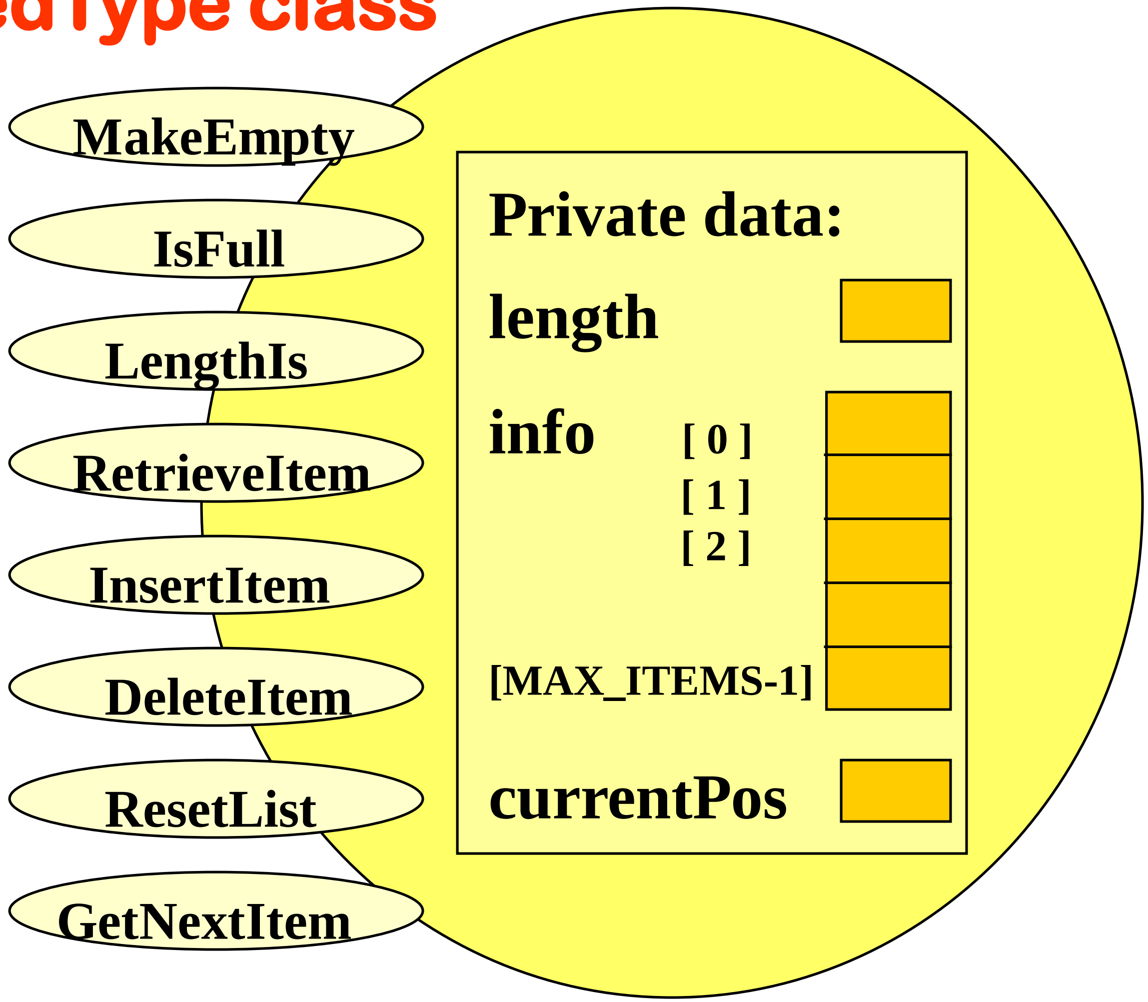
```
//  IMPLEMENTATION FILE              ( itemtype.cpp )
//  Implementation depends on the  data type of value.

#include  "itemtype.h"
#include  <iostream>

RelationType ItemType::ComparedTo (
                           ItemType  otherItem )  const
{
   if  ( value  <  otherItem.value )
          return  LESS ;
   else  if ( value  > otherItem.value )
          return  GREATER ;
   else  return  EQUAL ;
}
void ItemType::Print ( ) const
{
    using namespace std;
   cout  <<  value  <<  endl ;
}
void ItemType::Initialize ( int  number )
{
   value  =  number  ;
}
```

ItemType Class Interface Diagram



Sorted Type Class Interface Diagram

SortedType class



Member functions

Which member function specifications and implementations must change to ensure that any instance of the Sorted List ADT remains sorted at all times?

- **InsertItem**
- **DeleteItem**

InsertItem algorithm for SortedList ADT

- **Find proper location for the new element in the sorted list.**
- **Create space for the new element by moving down all the list elements that will follow it.**
- **Put the new element in the list.**
- **Increment length.**

Implementing SortedType member function InsertItem

```
// IMPLEMENTATION FILE                           (sorted.cpp)

#include "itemtype.h"        // also must appear in client code

void  SortedType :: InsertItem ( ItemType  item )
//  Pre: List has been initialized. List is not full.
//  item is not in list.
//  List is sorted by key member using function ComparedTo.
// Post:  item is in the list.  List is still sorted.
{
             .
             .
             .

}
```

```
void  SortedType :: InsertItem ( ItemType  item )
{  bool  moreToSearch ;
   int   location = 0 ;
                    // find proper location for new element
  moreToSearch = ( location < length ) ;
  while ( moreToSearch  )
  {   switch ( item.ComparedTo( info[location] ) )
      { case   LESS   : moreToSearch = false ;
                          break ;
        case  GREATER : location++ ;
                        moreToSearch = ( location < length ) ;
                        break ;
      }
  }                 //  make room for new element in sorted list
  for  ( int  index = length ;  index  >  location ;  index-- )
         info [ index ] = info [ index - 1 ] ;
  info [ location ] = item ;
  length++ ;
}
```

DeleteItem algorithm for SortedList ADT

- Find the location of the element to be deleted from the sorted list.
- Eliminate space occupied by the item being deleted by moving up all the list elements that follow it.
- Decrement length.

Implementing SortedType member function DeleteItem

```
// IMPLEMENTATION FILE continued          (sorted.cpp)

void  SortedType :: DeleteItem ( ItemType  item )
//  Pre: List has been initialized.
//       Key member of item is initialized.
//  Exactly one element in list has a key matching item's key.
//  List is sorted by key member using function ComparedTo.
// Post:  No item in list has key matching item's key.
// List is still sorted.
{
              .
              .
              .

}
```

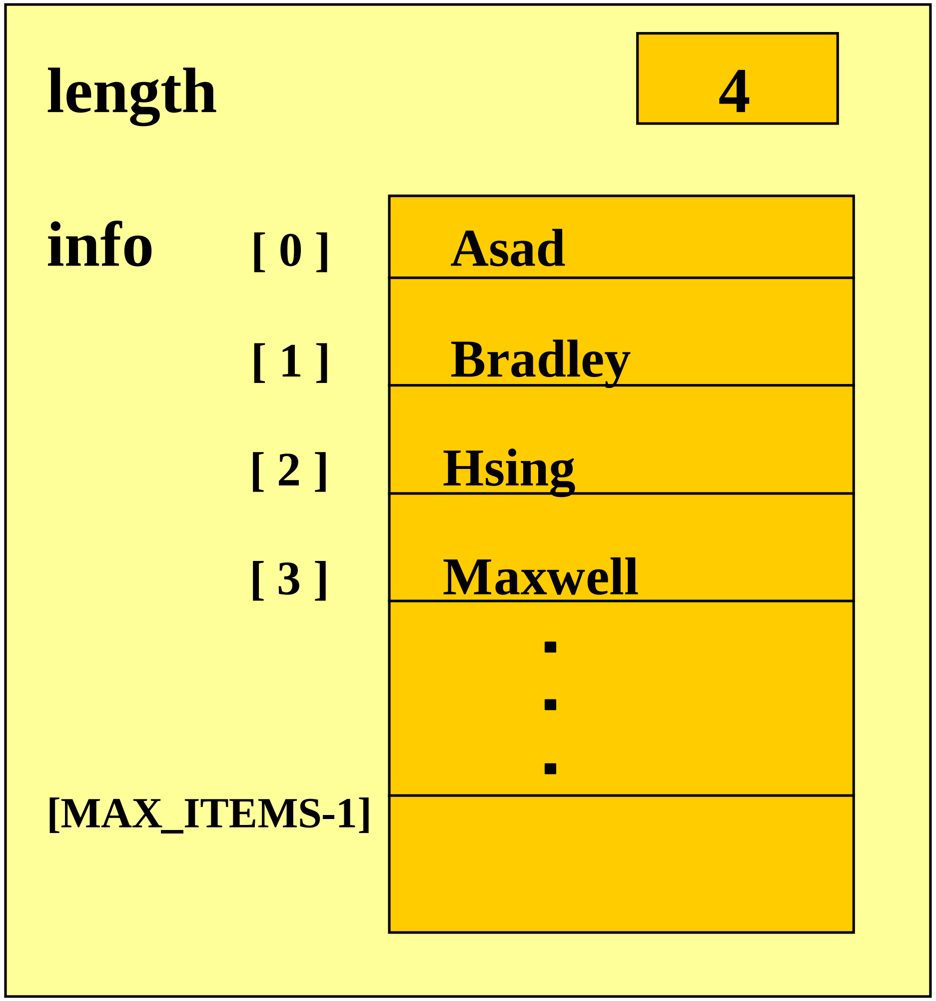
```
void  SortedType :: DeleteItem ( ItemType  item )
{
  int location = 0 ;
               //   find location of element to be deleted

   while ( item.ComparedTo ( info[location] )   !=  EQUAL )
        location++ ;

  // move up elements that follow deleted item in sorted list

   for ( int  index = location + 1 ; index  <  length; index++ )
        info [ index - 1 ] = info [ index ] ;

    length-- ;
}
```

Improving member function RetrieveItem

Recall that with the Unsorted List ADT we examined each list element beginning with `info`[0], until we either found a matching key, or we had examined all the elements in the Unsorted List.

How can the searching algorithm be improved for Sorted List ADT?

Retrieving Eliot from a Sorted List



The sequential search for Eliot can stop when Hsing has been examined.

Binary Search in a Sorted List

- **Examines the element in the middle of the array. Is it the sought item? If so, stop searching. Is the middle element too small? Then start looking in second half of array. Is the middle element too large? Then begin looking in first half of the array.**
- **Repeat the process in the half of the list that should be examined next.**
- **Stop when item is found, or when there is nowhere else to look and item has not been found.**

```
void  SortedType::RetrieveItem ( ItemType&  item,   bool&  found )
//  Pre: Key member of item is initialized.
//  Post: If found, item's key matches an element's key in the list
//  and a copy of that element has been stored in item; otherwise,
//  item is unchanged.
{  int midPoint ;
   int   first  =  0;
   int  last   = length - 1 ;
   bool  moreToSearch  =  ( first  <=  last ) ;

   found = false ;
   while ( moreToSearch  &&  !found )
   {   midPoint  =  ( first + last ) / 2 ;  // INDEX OF MIDDLE ELEMENT
       switch ( item.ComparedTo( info [ midPoint ] ) )
       {
          case   LESS    :    . . .  // LOOK IN FIRST HALF NEXT
          case  GREATER  :    . . .  // LOOK IN SECOND HALF NEXT
          case  EQUAL    :    . . .  // ITEM HAS BEEN FOUND
       }
   }
}
```

Trace of Binary Search

item = 45

info[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
15	26	38	57	62	78	84	91	108	119
first				midPoint					last

LESS → **last = midPoint - 1**

info[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
15	26	38	57	~~62~~	~~78~~	~~84~~	~~91~~	~~108~~	~~119~~
first	midPoint		last						

GREATER → **first = midPoint + 1**

Trace continued

item = 45

~~15~~	~~26~~	38	57	~~62~~	~~78~~	~~84~~	~~91~~	~~108~~	~~119~~
info[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
		first, midPoint	last						

GREATER → first = midPoint + 1

~~15~~	~~26~~	~~38~~	57	~~62~~	~~78~~	~~84~~	~~91~~	~~108~~	~~119~~
info[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
			first, midPoint, last						

LESS → last = midPoint - 1

Trace concludes

item = 45

info[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
15	26	38	57	62	78	84	91	108	119
		last	first						

first > last → **found = false**

```
void  SortedType::RetrieveItem ( ItemType&  item,   bool&  found )
//  ASSUMES info ARRAY SORTED IN ASCENDING ORDER
{  int midPoint ;
   int  first  =  0;
   int  last  = length - 1 ;
   bool  moreToSearch  =  ( first  <=  last ) ;
   found = false ;

   while ( moreToSearch  &&  !found )
   {    midPoint  =  ( first + last ) / 2 ;
        switch ( item.ComparedTo( info [ midPoint ] ) )
        {  case   LESS        :    last = midPoint - 1 ;
                                   moreToSearch = ( first <= last ) ;
                                   break ;
              case  GREATER  :    first = midPoint + 1 ;
                                   moreToSearch = ( first <= last ) ;
                                   break ;
              case  EQUAL     :    found = true   ;
                                   item = info[ midPoint ] ;
                                   break ;
        }
   }
}
```

Comparison of Algorithms

- **When there are more than one algorithm that perform the same task, how do we choose the best algorithm?**
  - ***The amount of work that the computer does***

    c.f. amount of work that the programmer does
- **Objective measures for efficiency**
  - **Execution time**
  - **Number of statements**
  - ***Number of fundamental operations***
    - **Expensive and/or frequently occurring operations**

Order of Magnitude of a Function

The order of magnitude, or Big-O notation, of a function expresses the computing time of a problem as the term in a function that increases most rapidly relative to the size of a problem.

$$f(N) = N^4 + 100N^2 + 10N + 50$$

Names of Orders of Magnitude

$O(1)$	bounded (by a constant) time
$O(\log_2 N)$	logarithmic time
$O(N)$	linear time
$O(N*\log_2 N)$	$N*\log_2 N$ time
$O(N^2)$	quadratic time
$O(2^N)$	exponential time

N	$\log_2 N$	$N * \log_2 N$	N^2	2^N
1	0	0	1	2
2	1	2	4	4
4	2	8	16	16
8	3	24	64	256
16	4	64	256	65,536
32	5	160	1024	4,294,967,296
64	6	384	4096	
128	7	896	16,384	

Big-O Comparison of List Operations

OPERATION	UnsortedList	SortedList
RetrieveItem	O(N)	O(N) linear search $O(\log_2 N)$ binary search
InsertItem		
Find	O(1)	O(N) search
Put	O(1)	O(N) moving down
Combined	O(1)	O(N)
DeleteItem		
Find	O(N)	O(N) search
Put	O(1) swap	O(N) moving up
Combined	O(N)	O(N)