CH2. Mathematical Building Blocks of Deep-learning

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Mathematical Building Blocs of Deep-learning

A first look on neural network basic example & construction Tensorflow keras

Data representations for neural network

The gear of neural networks: tensor operations

The engine of neural networks: gradient-based optimization

Looking back at our first example

A first look on neural network

Figure 2.1 MNIST sample digits

Mnist data analysis example

hand written digit data set

Hand written digit (28,28 pixcels) for 10 categories (0-9)

60,000 training set and 10,000 test set

* There is keras module in Tensorflow. (TF)
- 케라스는 텐서플로라 해도 무방 오픈소스, 텐서플로에서도 케라스 사용
- In keras module, there is some widely popular used dataset. (MNIST is one of that)

Mnist data

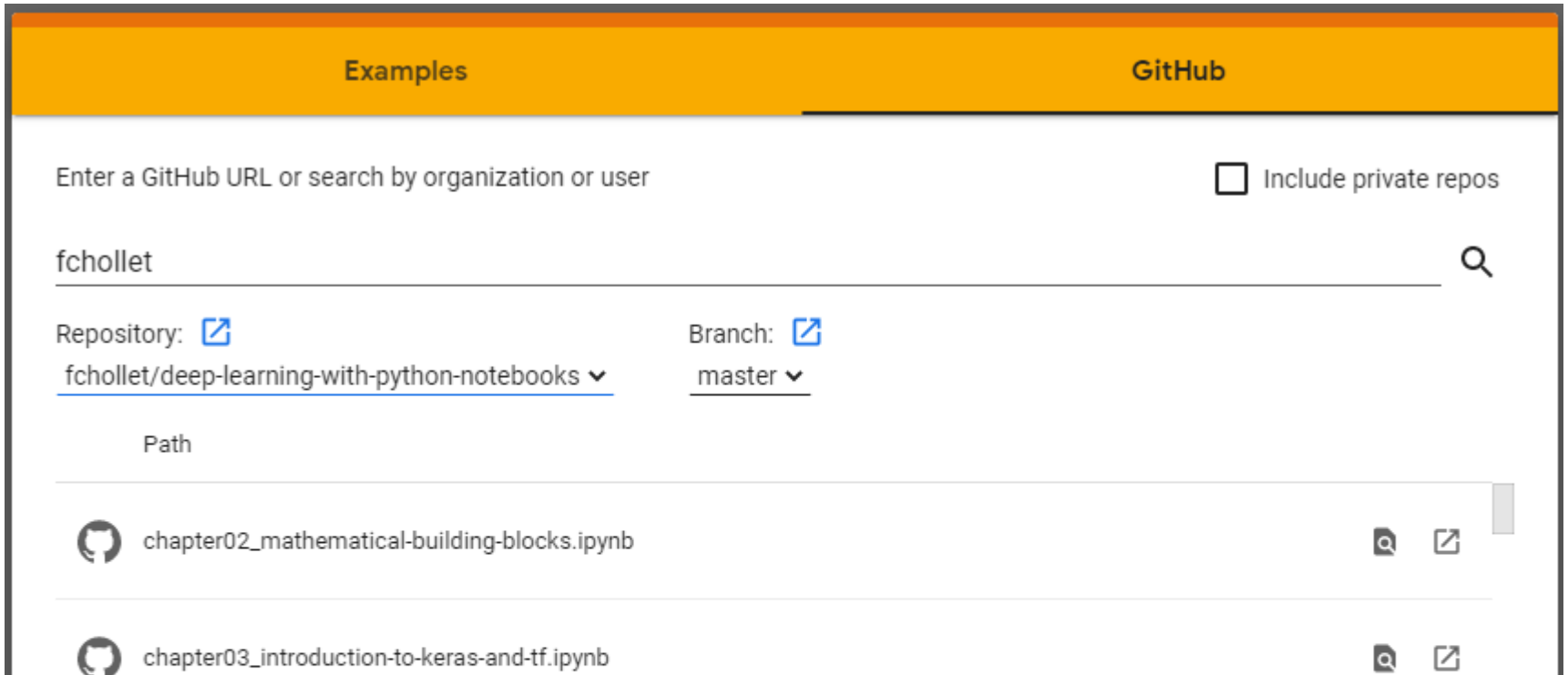
already in tensorflow module

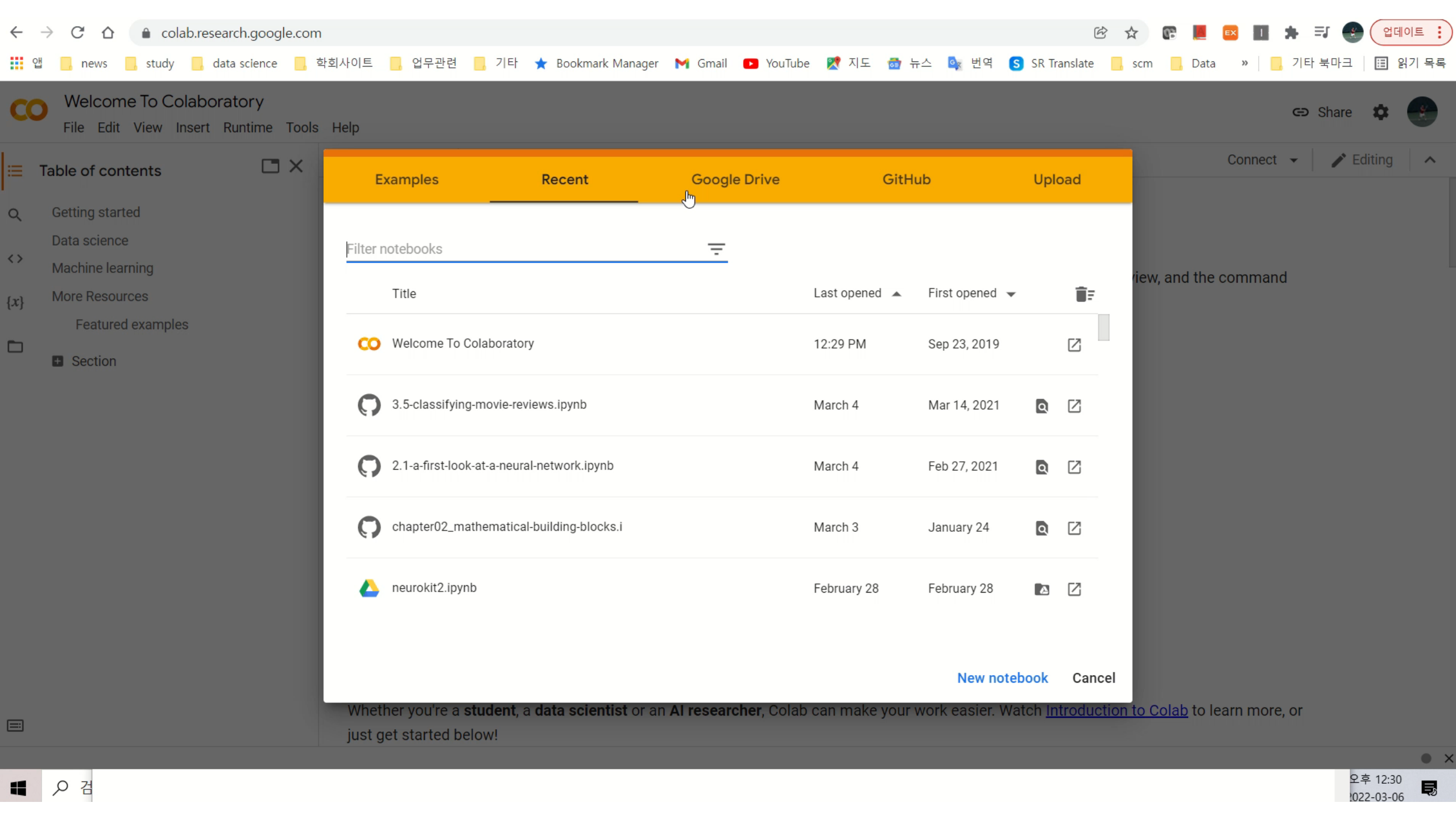
Listing 2.1 Loading the MNIST dataset in Keras

```
from tensorflow.keras.datasets import mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
```

{ calling importing } library, we load mnist data.

Try these codes from https://colab.research.google.com/github/

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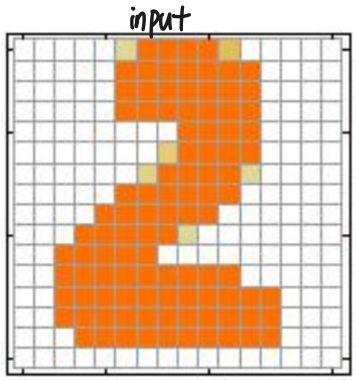


we define this model

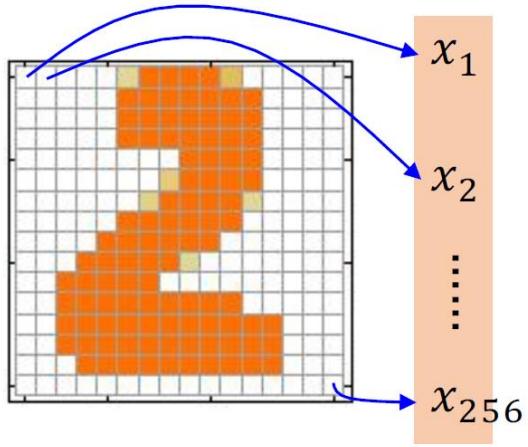
Machine

classifies the result.

"2"

output vector should be length '10'. (0~9)

input



machine (model)

output

	scores	
$\hat{y}_1$	0.09	is "1"
$\hat{y}_2$	0.71	is "2"
$\hat{y}_{10}$	0.03	is "0"

largest → classifies input as "2"

the image is "2"

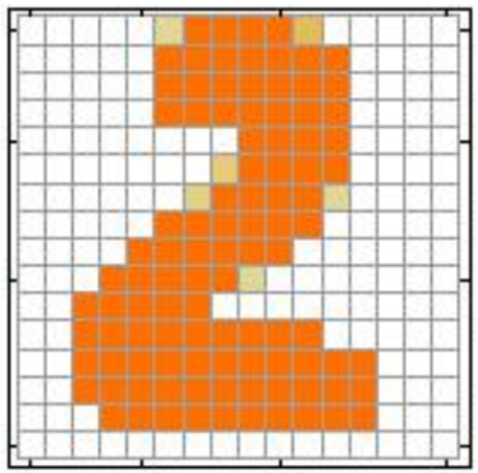
$16 \times 16 = 256$

ink → 1

no ink → 0

into input vector

Each dimension represents the confidence of a digit.



input

x_1

x_2

$\vdots$

x_{256}

Machine

output

$\hat{y}_1$

$\hat{y}_2$

$\vdots$

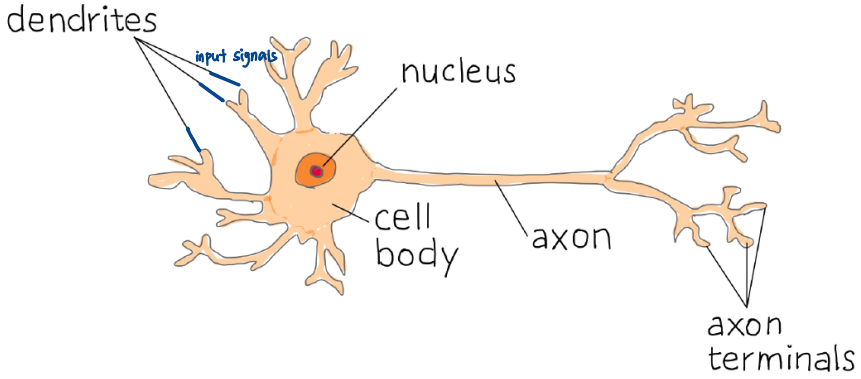
$\hat{y}_{10}$

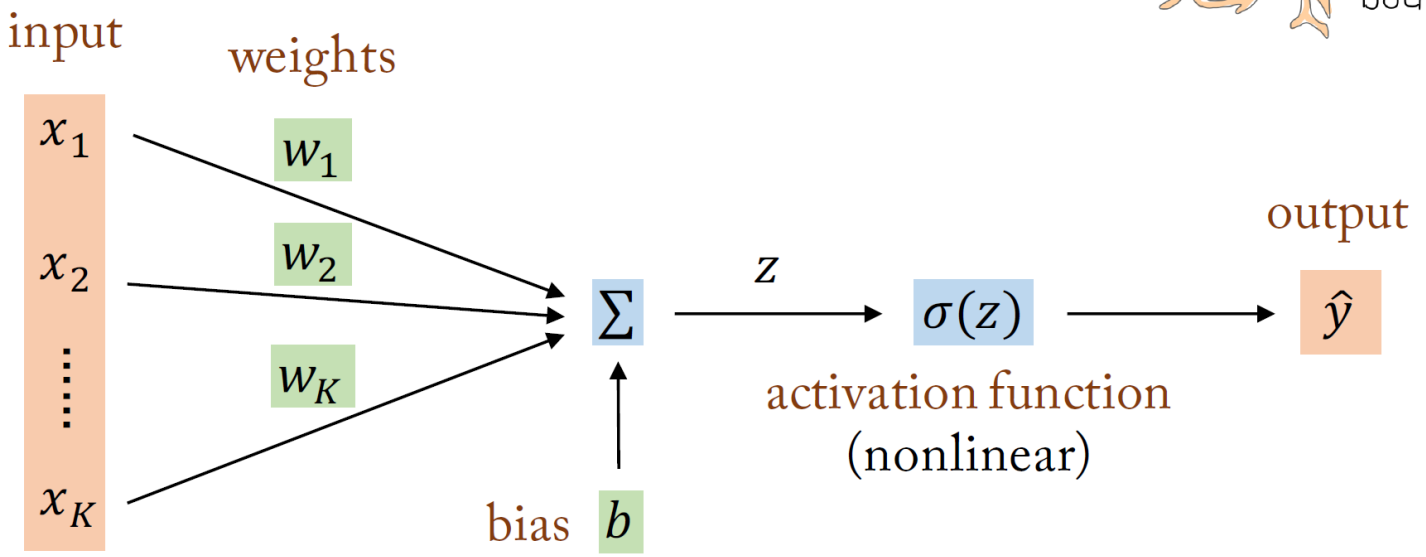
$f : R^{256} \rightarrow R^{10}$

In Deep Learning, the function f is represented by a Neural Network.

Perceptron (neuron) [Frank Rosenblatt, 1957]

- The simplest structure of DL is one-layer model.





$$f : \mathbb{R}^K \to \mathbb{R}$$

$= \sigma(Wx+b)$ (P×K, K×1, P×1; P×1 vector)

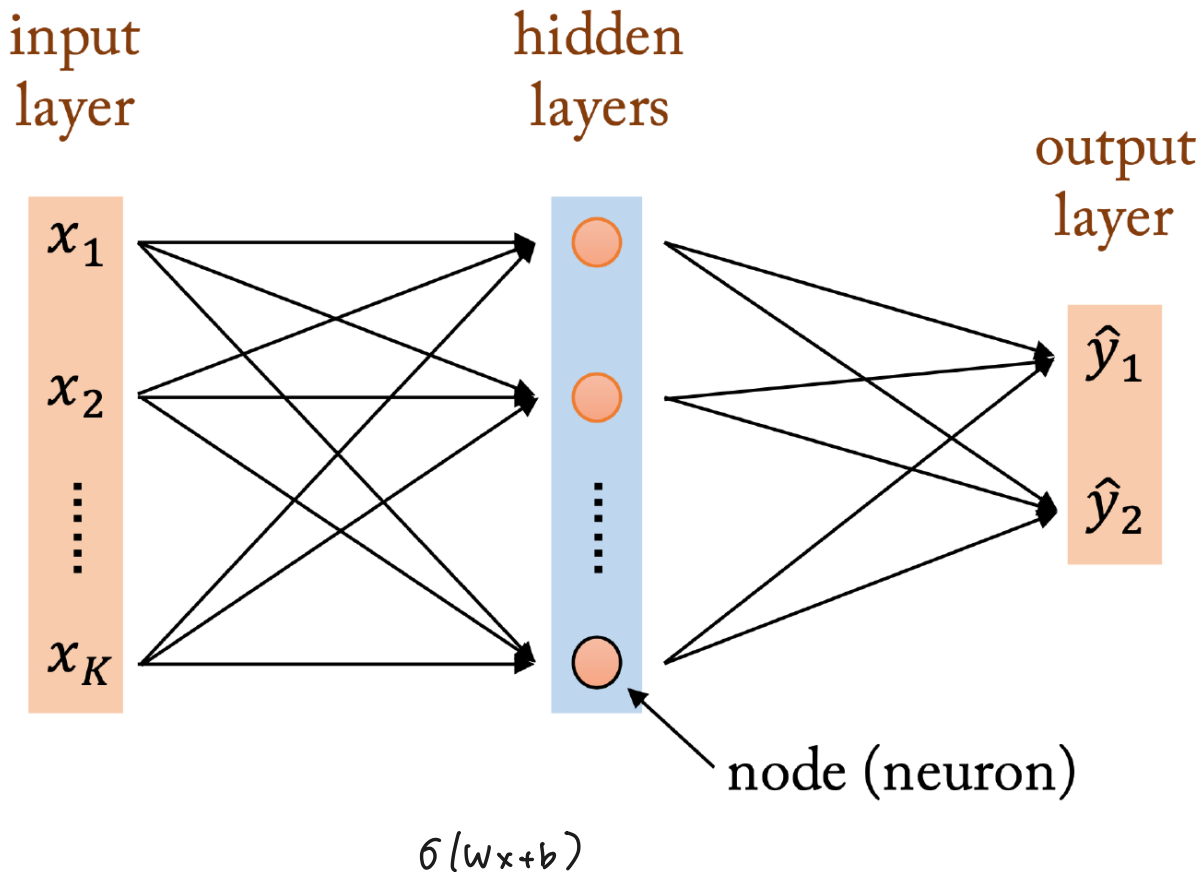
similar to dense layer.

$$\hat{y} = \sigma(z) = \sigma(w_1x_1 + w_2x_2 + \cdots + w_Kx_K + b)$$

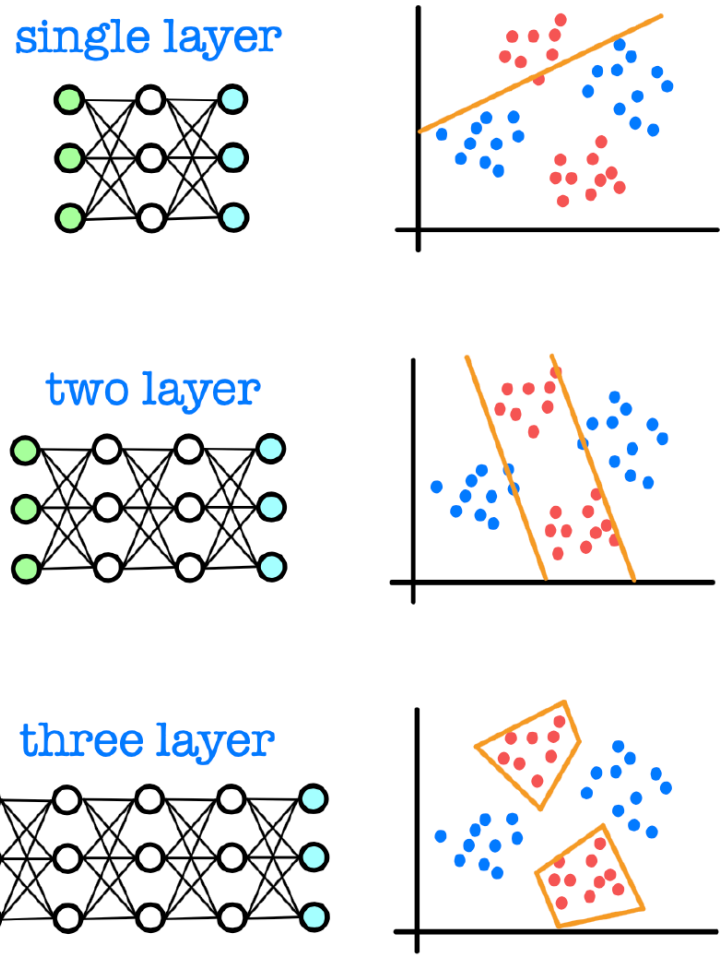
nonlinear — weighted sum (linear)

Multilayer Perceptron (MLP)

- Applying perceptron model layer by layer (= multi layer)



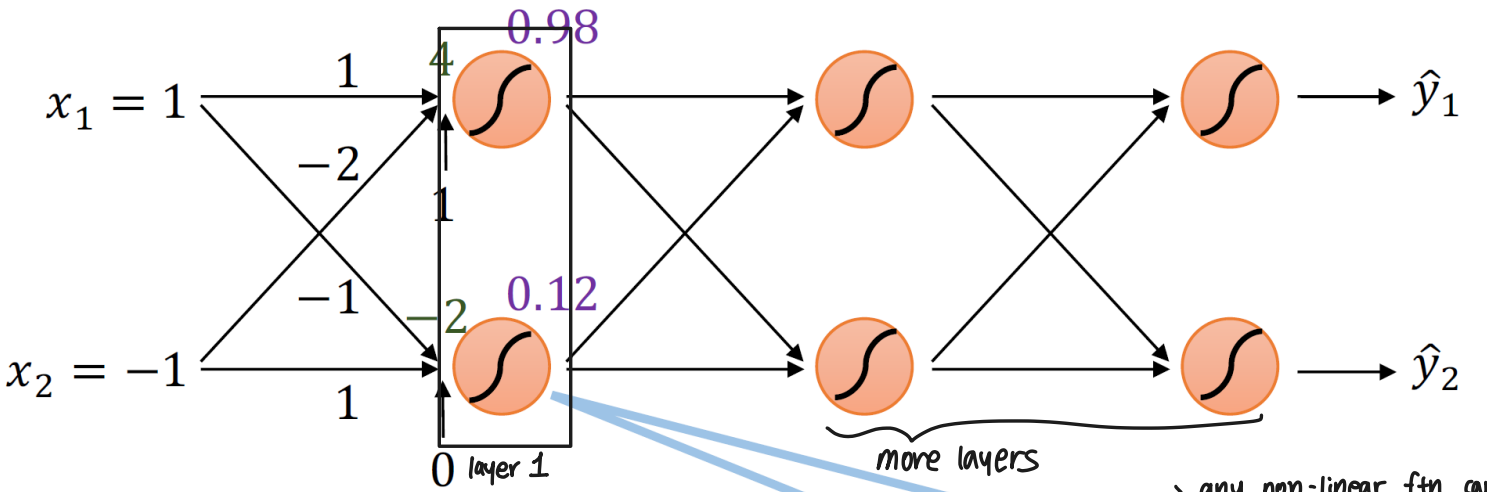
$\sigma(Wx+b)$



Forward pass computation 순전파

$$\sigma(w_1 x_1 + w_2 x_2 + \cdots + w_K x_K + b)$$

$$0.98 = \sigma(1 \cdot 1 + (-1) \cdot (-2) + 1)$$



activation ftn

$$\sigma\left(\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

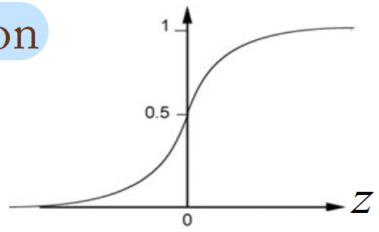
$$= \sigma\left(\begin{bmatrix} 4 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

W, b trainable parameters.
After setting up DL model,
we should optimize weights (w) & bias (b)
to make model better.

any non-linear ftn can be used in activation ftn.
eg) relu, sigmoid, etc.

eg)
sigmoid function

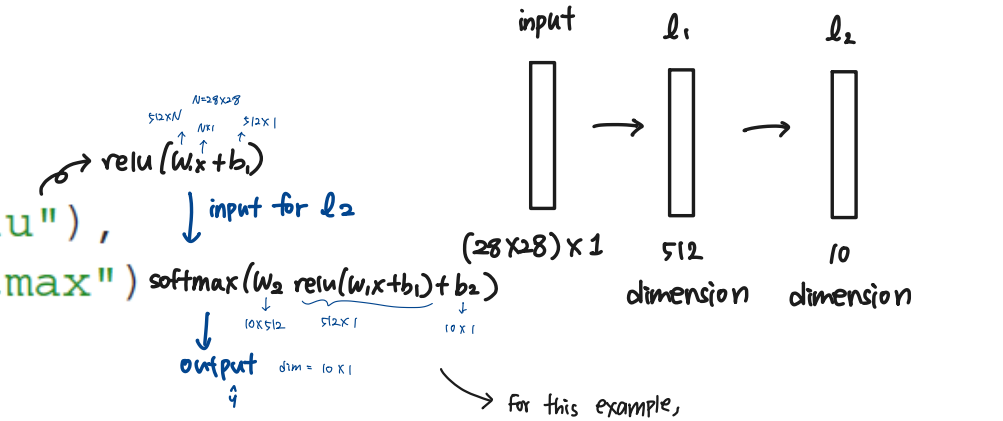
$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



activation function

Listing 2.2 The network architecture

```
from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
    layers.Dense(512, activation="relu"),
    layers.Dense(10, activation="softmax")
])
```



Core building block of NN is layers

Layers extract *representations*

This step define our model f(x; weights)

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```
[8] train_images.shape
```

```
(60000, 28, 28)
```

```
[9] len(train_labels)
```

```
60000
```

```
train_labels
```

```
array([5, 0, 4, ..., 5, 6, 8], dtype=uint8)
```

```
[11] test_images.shape
```

```
(10000, 28, 28)
```

```
[12] len(test_labels)
```

```
10000
```

```
[13] test_labels
```

```
array([7, 2, 1, ..., 4, 5, 6], dtype=uint8)
```

The network architecture

step 1) define our model $f_\theta(x)=y$

step 2) define criteria → loss function
How the network measure its performance.

What's next?

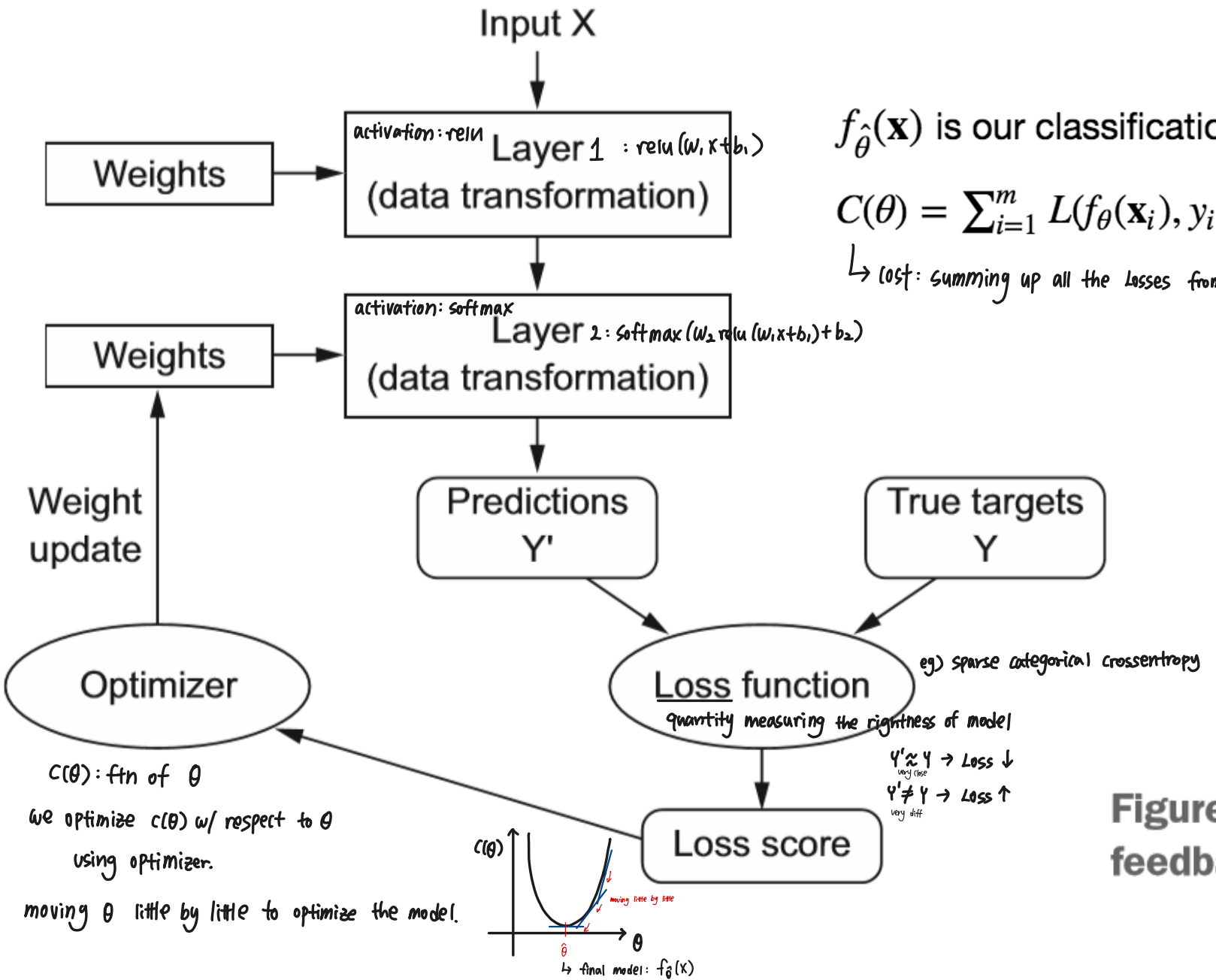
Loss function: How the network measure its performance

Optimizer: Mechanism through which the network will update itself based on the data it sees and its loss function

Metrics to monitor during training and testing: Accuracy

Listing 2.3 The compilation step

```
model.compile(optimizer="rmsprop",
              loss="sparse_categorical_crossentropy",
              metrics=["accuracy"])
```



$f_{\hat{\theta}}(\mathbf{x})$ is our classification model with estimated parameters $\hat{\theta}$

$$C(\theta) = \sum_{i=1}^{m} L(f_\theta(\mathbf{x}_i), y_i)$$

↳ cost: summing up all the losses from i to m.
(all indicies. / # training data set.)

Figure 1.9 The loss score is used as a feedback signal to adjust the weights.

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[14] 3s
```
from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
    layers.Dense(512, activation="relu"),
    layers.Dense(10, activation="softmax")
])
```

The compilation step

+ Code + Text

0s
```
model.compile(optimizer="rmsprop",
              loss="sparse_categorical_crossentropy",
              metrics=["accuracy"])
```

[ ]

Preparing the image data

[ ]
```
train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype("float32") / 255
test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype("float32") / 255
```

"Fitting" the model

[ ]
```
model.fit(train_images, train_labels, epochs=5, batch_size=128)
```

0s completed at 1:03 PM

Rescale values from [0,255] to [0,1]

Listing 2.4 Preparing the image data

```
train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype("float32") / 255
test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype("float32") / 255
```

Fitting the model

Listing 2.5 "Fitting" the model

```
>>> model.fit(train_images, train_labels, epochs=5, batch_size=128)
Epoch 1/5
60000/60000 [==============================] - 5s - loss: 0.2524 - acc: 0.9273
Epoch 2/5
51328/60000 [========================>.....] - ETA: 1s - loss: 0.1035 - acc: 0.9692
```

using training data 5 times

every iteration of the training how many mini batch sample that we are using?

Make Predictions

Listing 2.6 Using the model to make predictions

```
>>> test_digits = test_images[0:10]
>>> predictions = model.predict(test_digits)
>>> predictions[0]
array([1.0726176e-10, 1.6918376e-10, 6.1314843e-08, 8.4106023e-06,
       2.9967067e-11, 3.0331331e-09, 8.3651971e-14, 9.9999106e-01,
       2.6657624e-08, 3.8127661e-07], dtype=float32)
```

0 1 2 3 4 5 6 7 8 9

the result would be "7".

Evaluating the model

Listing 2.7 Evaluating the model on new data

```
>>> test_loss, test_acc = model.evaluate(test_images, test_labels)
>>> print(f"test_acc: {test_acc}")
test_acc: 0.9785
```

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3s

```
from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
    layers.Dense(512, activation="relu"),
    layers.Dense(10, activation="softmax")
])
```

The compilation step

[22] 0s

```
model.compile(optimizer="rmsprop",
              loss="sparse_categorical_crossentropy",
              metrics=["accuracy"])
```

Preparing the image data

```
train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype("float32") / 255
test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype("float32") / 255
```

"Fitting" the model

```
model.fit(train_images, train_labels, epochs=5, batch_size=128)
```

CH2. Mathematical Building Blocks of Deep-learning (2)

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Data representations for neural networks

Tensor is a container for data

0D tensors (Scalars)

```
>>> import numpy as np
>>> x = np.array(12)
>>> x
array(12)
>>> x.ndim
0
```

1D tensors (Vectors)

```
>>> x = np.array([12, 3, 6, 14])
>>> x
array([12, 3, 6, 14])
>>> x.ndim
1
```

2D tensors (Matrices)

```
>>> x = np.array([[5, 78, 2, 34, 0],
[6, 79, 3, 35, 1],
[7, 80, 4, 36, 2]])
>>> x.ndim
2
```

3D tensors and higher-dimensional tensors

```
>>> x = np.array([[[5, 78, 2, 34, 0],
[6, 79, 3, 35, 1],
[7, 80, 4, 36, 2]],
[[5, 78, 2, 34, 0],
[6, 79, 3, 35, 1],
[7, 80, 4, 36, 2]],
[[5, 78, 2, 34, 0],
[6, 79, 3, 35, 1],
[7, 80, 4, 36, 2]]])
>>> x.ndim
3
```

Key attributes for *tensors*

Number of axes (rank): can be checked by .ndim **for numpy objects**

0D, 1D, 2D

rank

Shape: tuple of integers that describes how many dimensions the tensor has along each axis. Can be checked by .shape **for numpy objects**

Data type (dtype**): type of the data in tensor. Ex:** float32, uint8, float64, **and so on.**

Key attributes for *tensors*

```
from keras.datasets import mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()

>>> print(train_images.ndim)
3

>>> print(train_images.shape)
(60000, 28, 28)

>>> print(train_images.dtype)
uint8
```

Displaying Digits

```
digit = train_images[4]

import matplotlib.pyplot as plt
plt.imshow(digit, cmap=plt.cm.binary)
plt.show()
```

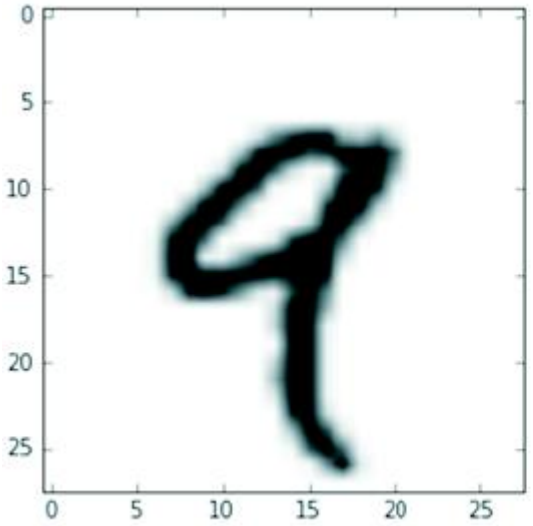

Figure 2.2 The fourth sample in our dataset

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```
x.ndim
```

Key attributes

```
from tensorflow.keras.datasets import mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
```

```
[ ] train_images.ndim
```

```
[ ] train_images.shape
```

```
[ ] train_images.dtype
```

Displaying the fourth digit

```
[ ] import matplotlib.pyplot as plt
digit = train_images[4]
plt.imshow(digit, cmap=plt.cm.binary)
plt.show()
```

```
[ ] train_labels[4]
```

Manipulating tensors in NumPy

1s completed at 1:16 PM

Manipulating tensors in Numpy

Tensor slicing

```
>>> my_slice = train_images[10:100]
>>> print(my_slice.shape)
(90, 28, 28)
>>> my_slice = train_images[10:100, :, :]
>>> my_slice.shape
(90, 28, 28)
>>> my_slice = train_images[10:100, 0:28, 0:28]
>>> my_slice.shape
(90, 28, 28)
```

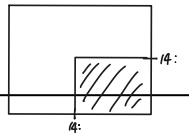
||

|| same

Manipulating tensors in Numpy

Select 14×14 pixels in bottom-right corner of all images

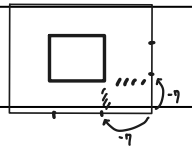
```
my_slice = train_images[:, 14:, 14:]
```



Crop 14 × 14 pixels centered in the middle

```
my_slice = train_images[:, 7:-7, 7:-7]
```

from backward




```
plt.imshow(my_slice[4], cmap=plt.cm.binary)
plt.show()
```

```
[47] train_labels[4]
```

```
9
```

Manipulating tensors in NumPy

```
my_sli Loading... images[10:100]
my_slice.shape
```

```
[ ] my_slice = train_images[10:100, :, :]
my_slice.shape
```

```
[ ] my_slice = train_images[10:100, 0:28, 0:28]
my_slice.shape
```

```
[ ] my_slice = train_images[:, 14:, 14:]
```

```
[ ] my_slice = train_images[:, 7:-7, 7:-7]
```

The notion of data batches

```
[ ] batch = train_images[:128]
```

```
[ ] batch = train_images[128:256]
```

The notion of data batches

The 1st axis of data tensors in deep-learning will be sample axis 60,000

Deep-learning models don't process an entire dataset at once Process little by little (using batches) Like mini batch gradient descent algorithm

```
# 1st batch
batch = train_images[:128]
# 2nd batch
batch = train_images[128:256]
# nth batch
batch = train_images[128 * n:128 * (n + 1)]
```

Real-world examples of data tensors

Vector data – 2D tensor of shape (samples, features)

Timeseries data or sequence data – 3D tensors of shape (samples, timesteps, features)

(cf. MNIST : 3D ∵ black-white image ⇒ it means 1 channel)

Images – 4D tensors of shape (samples, height, width, channels) **or** (samples, channels, height, width)

represents colors

sequence of images

it means 'frame'

Video – 5D tensors of shape (samples, frames, height, width, channels) **or** (samples, frames, channels, height, width)

Vector Data

Vector data – 2D tensor of shape (samples, features)

Example:

Dataset of people, where we consider each person's *age*, *ZIP code*, **and** *income*.
Each person can be characterized as a vector of 3 values
Entire dataset of 100,000 people can be stored in a 2D tensor of shape (100000, 3).

Timeseries data or sequence data

Timeseries data or sequence data – 3D tensors of shape (samples, timesteps, features)

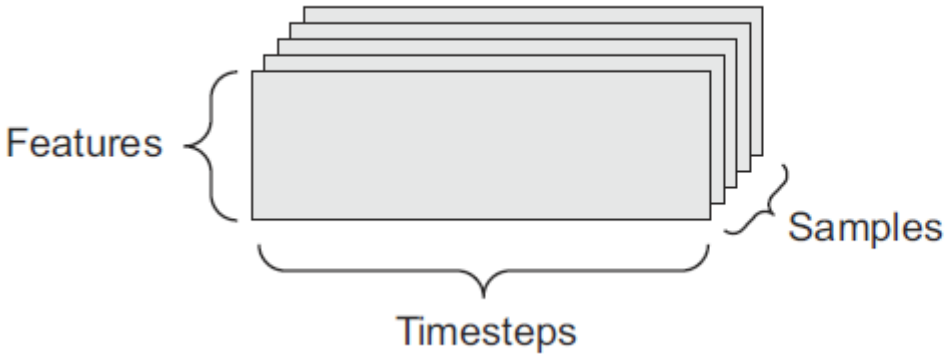


Figure 2.3 A 3D timeseries data tensor

Dataset of stock prices. Every minute, we store the *current price*, *the highest* **and** *the lowest price* **in the past minute. Encoded as a 2D tensor of shape (390, 3) for 390 minutes in a trading day**
250 days' data stored in 3D tensor of shape (250, 390, 3).

Image data

Images – 4D tensors of shape (samples, height, width, channels) **or** (samples, channels, height, width)

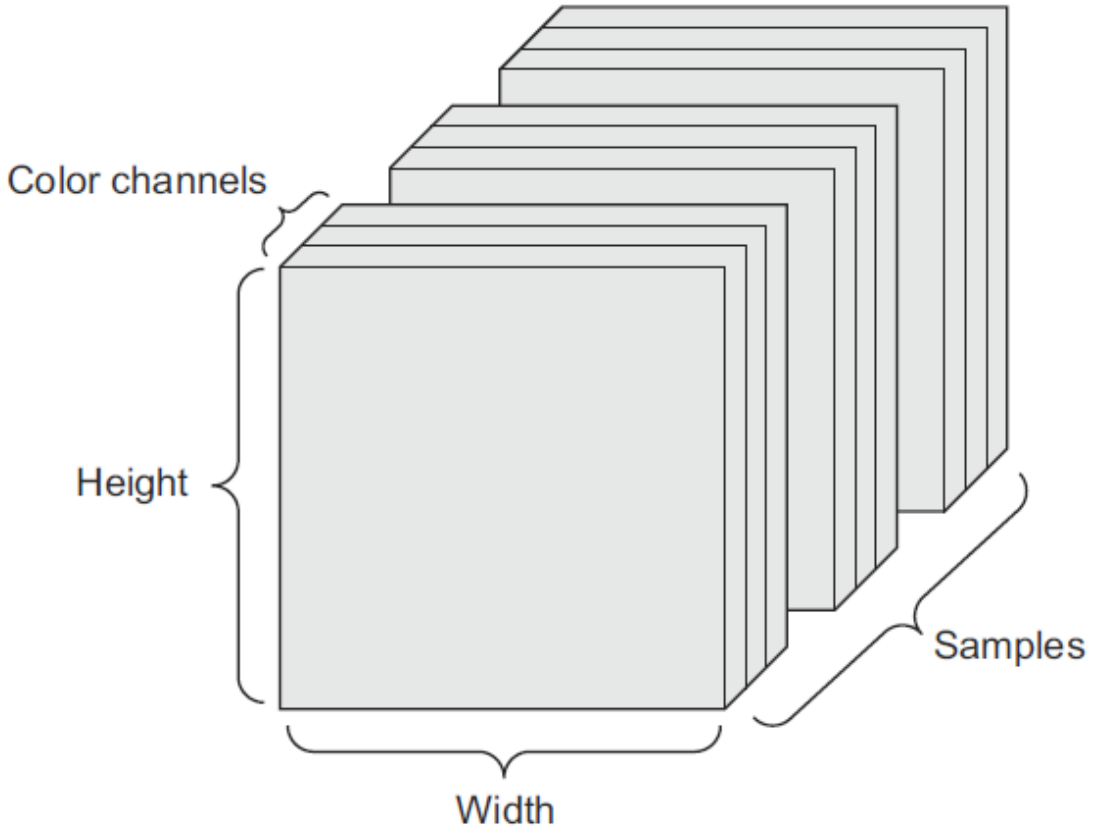


Figure 2.4 A 4D image data tensor (channels-first convention)

Video data

Video – 5D tensors of shape (samples, frames, height, width, channels) **or** (samples, frames, channels, height, width)

Example:

A 60-second, 144 × 256 YouTube video clip sampled at 4 frames per second would have 240 frames. A batch of four such video clips would be stored in a tensor of shape (4, 240, 144, 256, 3)

The gears of neural networks: tensor operations

Tensor operations **applied to tensors of numeric data.**

Example:

keras.layers.Dense(512, activation='relu')

Unpack this

output = relu(dot(W, input) + b)

relu: max(x, 0) — dot: dot product — +: addition

Three tensor operations:

dot product **(dot) between input tensor and a tensor W**

addition **(+) between the resulting 2D tensor and vector b**

relu operation: **relu(x) is max(x, 0).**

Element-wise operations

Ex) relu and addition

eg) (1,2,3) + (0,0,1) = (1,2,4)
Calculate elementwisely!

Naïve implementation of an element-wise relu operation:

x should be matrix form.

```
def naive_relu(x):
    assert len(x.shape) == 2, 'x.ndim() must 2'

    x = x.copy()
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            x[i, j] = max(x[i, j], 0)

    return x
```

chapter02_mathematical-building-blocks.i

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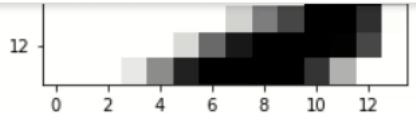


The notion of data batches

```
[ ] batch = train_images[:128]
```

```
[ ] batch = train_images[128:256]
```

```
[ ] n = 3
    batch = train_images[128 * n:128 * (n + 1)]
```

Real-world examples of data tensors

Vector data

Timeseries data or sequence data

Image data

0s completed at 1:36 PM

Element-wise operations

Ex) relu and addition

Naïve implementation of addition:

```
def naive_add(x, y):
    assert len(x.shape) == 2
    assert x.shape == y.shape

    x = x.copy()  ## avoid overwriting
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            x[i, j] += y[i, j]
    return x
```

input으로 넣은 x가 그대로 살아남게

and original values changing

Element-wise operations

Element-wise relu:

```
import numpy as np
z = np.maximum(z, 0.)
```

Element-wise add:

```
z = x + y
```

Broadcasting

What happens with addition when the shapes of the two tensors being added differ?

Broadcasting consists of two steps:

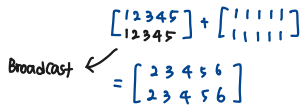
eg) X & Y
dims are diff
X + Y
X.ndim > Y.ndim ⇒ Y Broadcasting

1. **Axes (called broadcast axes) are added to the smaller tensor to match the ndim of the larger tensor.**
2. **The smaller tensor is repeated alongside these new axes to match the full shape of the larger tensor.**

Broadcasting

Example:

```
import numpy as np
x = np.array([1,2,3,4,5])
y = np.array([[1,1,1,1,1], [1,1,1,1,1]])
```




```
def naive_add_matrix_and_vector(x, y):
    assert len(x.shape) == 2
    assert len(y.shape) == 1
    assert x.shape[1] == y.shape[0]

    x = x.copy()
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            x[i, j] += y[j]
    return x
```

x is a 2D Numpy tensor.

y is a Numpy vector.

Avoid overwriting the input tensor.

Broadcasting

if one tensor has shape *(a, b, … n, n + 1, … m)* and the other has shape *(n, n + 1, … m)*. The broadcasting will then automatically happen for axes *a* through *n - 1*.

```
import numpy as np

x = np.random.random((64, 3, 32, 10))
y = np.random.random((32, 10))

z = np.maximum(x, y)
```

x is a random tensor with shape (64, 3, 32, 10).

y is a random tensor with shape (32, 10).

The output z has shape (64, 3, 32, 10) like x.

[73] x1

0s

```
array([[1, 0, 3],
       [2, 3, 0]])
```

```
def naive_add(x, y):
    assert len(x.shape) == 2
    assert x.shape == y.shape
    x = x.copy()
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            x[i, j] += y[i, j]
    return x
```

```
import time

x = np.random.random((20, 100))
y = np.random.random((20, 100))

t0 = time.time()
for _ in range(1000):
    z = x + y
    z = np.maximum(z, 0.)
print("Took: {0:.2f} s".format(time.time() - t0))
```

```
t0 = time.time()
for _ in range(1000):
```

Tensor dot

Dot product in math

$(a,b,c) \cdot (d,e,f) = ad + be + cf$

```
import numpy as np
z = np.dot(x, y)
```

```
def naive_vector_dot(x, y):
    assert len(x.shape) == 1
    assert len(y.shape) == 1
    assert x.shape[0] == y.shape[0]
    z = 0.
    for i in range(x.shape[0]):
        z += x[i] * y[i]
    return z
```

x and y are NumPy vectors.

$x_1y_1 + x_2y_2 + x_3y_3$

Matrix vector dot

```
def naive_matrix_vector_dot(x, y):
    assert len(x.shape) == 2
    assert len(y.shape) == 1
    assert x.shape[1] == y.shape[0]
    z = np.zeros(x.shape[0])
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            z[i] += x[i, j] * y[j]
    return z
```

(W x
axb bx1

x is a NumPy matrix. (W)

y is a NumPy vector. (x)

The first dimension of x must be the same as the 0th dimension of y!

This operation returns a vector of 0s with the same shape as y.

Matrix vector dot

```
def naive_matrix_vector_dot(x, y):
    assert len(x.shape) == 2
    assert len(y.shape) == 1
    assert x.shape[1] == y.shape[0]
    z = np.zeros(x.shape[0])
    for i in range(x.shape[0]):
        for j in range(x.shape[1]):
            z[i] += x[i, j] * y[j]
    return z
```

x is a NumPy matrix.

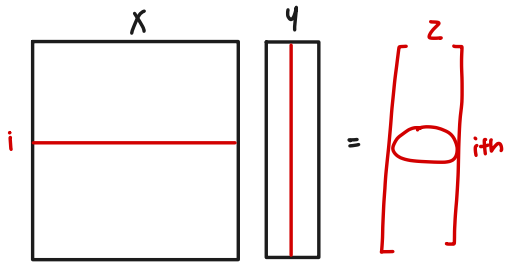
y is a NumPy vector.

The first dimension of x must be the same as the 0th dimension of y!

This operation returns a vector of 0s with the same shape as y.

Could also use the previous codes

```
def naive_matrix_vector_dot(x, y):
    z = np.zeros(x.shape[0])
    for i in range(x.shape[0]):
        z[i] = naive_vector_dot(x[i, :], y)
    return z
```



Dot product for 2D tensors

```
def naive_matrix_dot(x, y):
    assert len(x.shape) == 2
    assert len(y.shape) == 2
    assert x.shape[1] == y.shape[0]
    z = np.zeros((x.shape[0], y.shape[1]))
    for i in range(x.shape[0]):
        for j in range(y.shape[1]):
            row_x = x[i, :]
            column_y = y[:, j]
            z[i, j] = naive_vector_dot(row_x, column_y)
    return z
```

x and y are NumPy matrices.

The first dimension of x must be the same as the 0th dimension of y!

Iterates over the rows of x . . .

. . . and over the columns of y.

This operation returns a matrix of 0s with a specific shape.

The gears of neural networks: Tensor operations

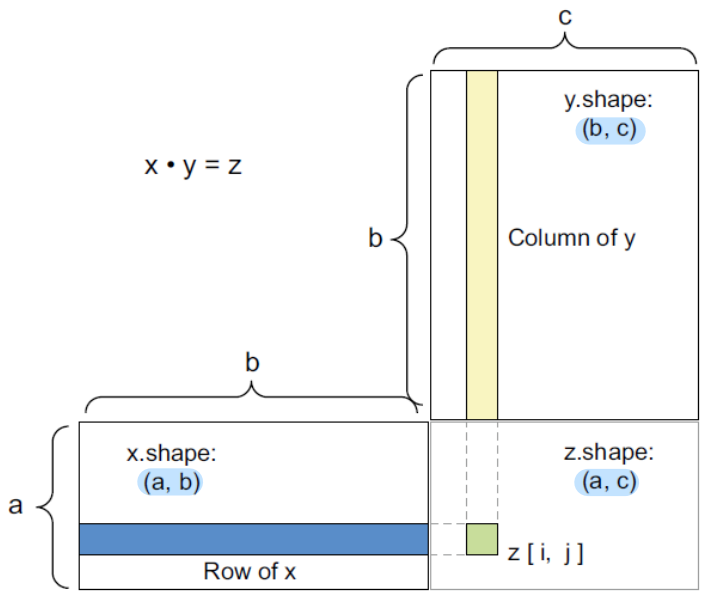


Figure 2.5 Matrix dot-product box diagram

Tensor dot

More generally, you can take the dot product between higher-dimensional tensors, following the same rules for shape compatibility as outlined earlier for the 2D case:

$a \times b \cdot b \times c \Rightarrow a \times c$

4D 1D

(a, b, c, d) . (d,) -> (a, b, c)

(a, b, c, d) . (d, e) -> (a, b, c, e)

Tensor reshaping

we preprocessed the digits data before feeding it into our network:

```
train_images = train_images.reshape((60000, 28 * 28))
```

Try it:

```
x = np.array([[0., 1.], [2., 3.], [4., 5.]])

x = x.reshape((6, 1))

x = x.reshape((2, 3))
```

```
 1.87539025, 1.41756671, 1.11338379, 0.96418242, 1.21331544],
[1.31749392, 0.94878296, 1.91253185, 0.38539614, 0.40926378,
 1.57273443, 1.31628261, 1.66346262, 1.51518779, 0.81068861]])
```

```
import numpy as np
x = np.random.random((64, 3, 32, 10))
y = np.random.random((32, 10))
z = np.maximum(x, y)
```

Tensor product

```
[ ]  x = np.random.random((32,))
     y = np.random.random((32,))
     z = np.dot(x, y)
```

```
[ ]  def naive_vector_dot(x, y):
         assert len(x.shape) == 1
         assert len(y.shape) == 1
         assert x.shape[0] == y.shape[0]
         z = 0.
         for i in range(x.shape[0]):
             z += x[i] * y[i]
         return z
```

```
[ ]  def naive_matrix_vector_dot(x, y):
         assert len(x.shape) == 2
         assert len(y.shape) == 1
         assert x.shape[1] == y.shape[0]
```

Geometric interpretation of tensor operations

Vector as an arrow

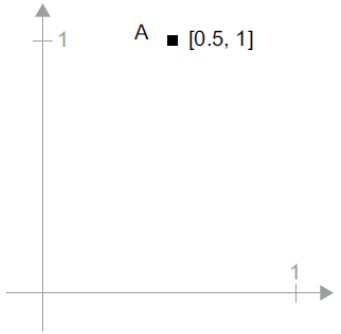


Figure 2.6 A point in a 2D space

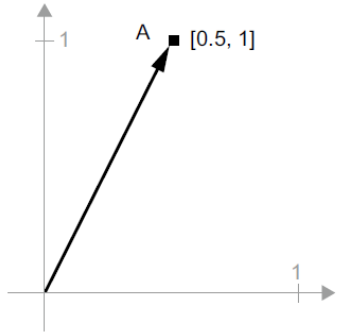


Figure 2.7 A point in a 2D space pictured as an arrow

Sum of two vectors

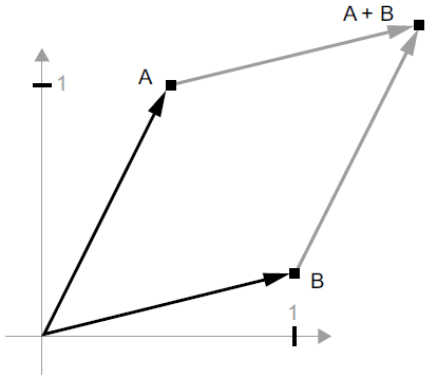


Figure 2.8 Geometric interpretation of the sum of two vectors

Translation as vector addition

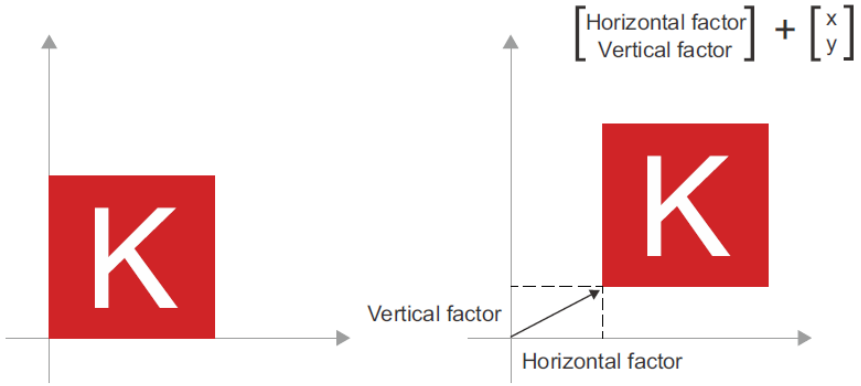


Figure 2.9 2D translation as a vector addition

2D rotation

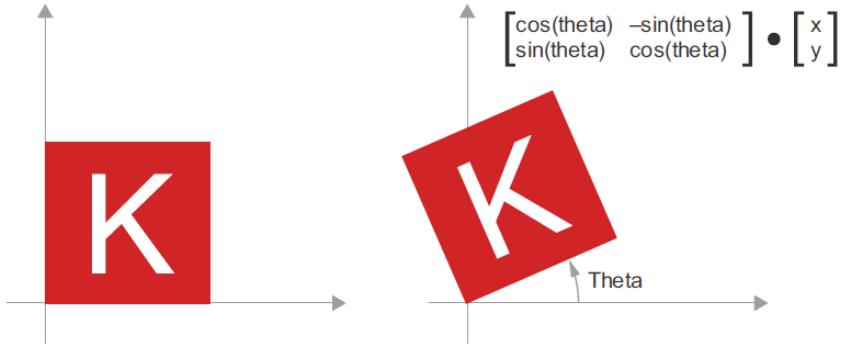


Figure 2.10 2D rotation (counterclockwise) as a dot product

Geometric interpretation of tensor operations

2D scaling

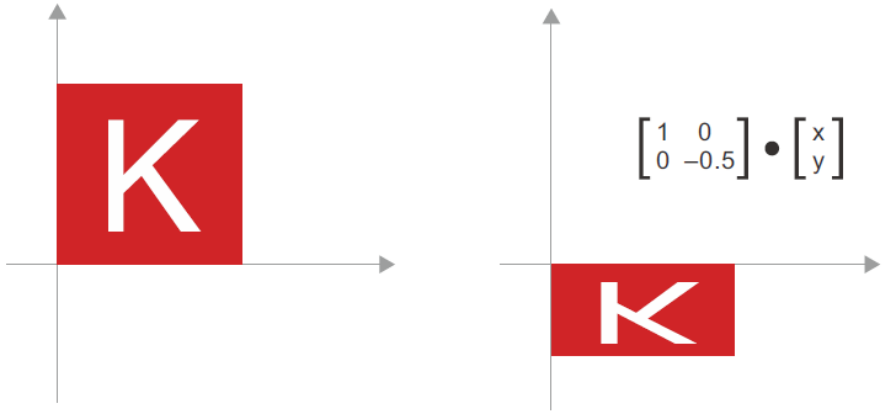


Figure 2.11 2D scaling as a dot product

Affine transform

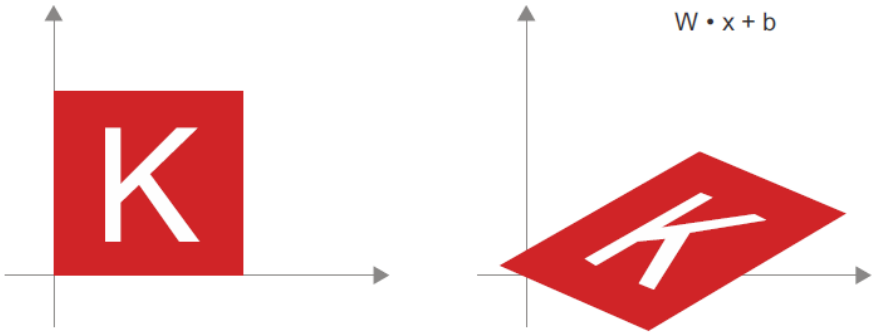


Figure 2.12 Affine transform in the plane

Affine transform followed by relu

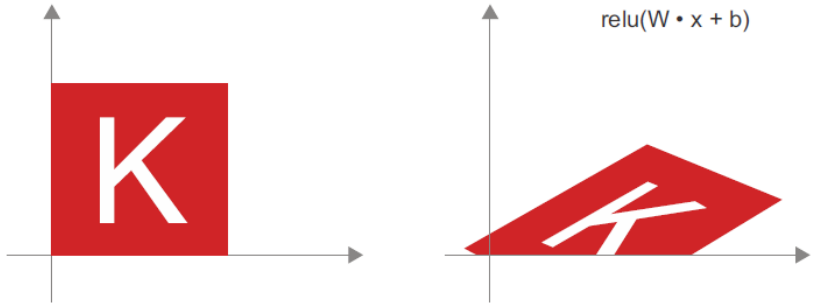


Figure 2.13 Affine transform followed by `relu` activation

Geometric interpretation of deep learning

Neural networks consist entirely of chains of tensor operations

These tensor operations are just simple geometric transformations of the input data

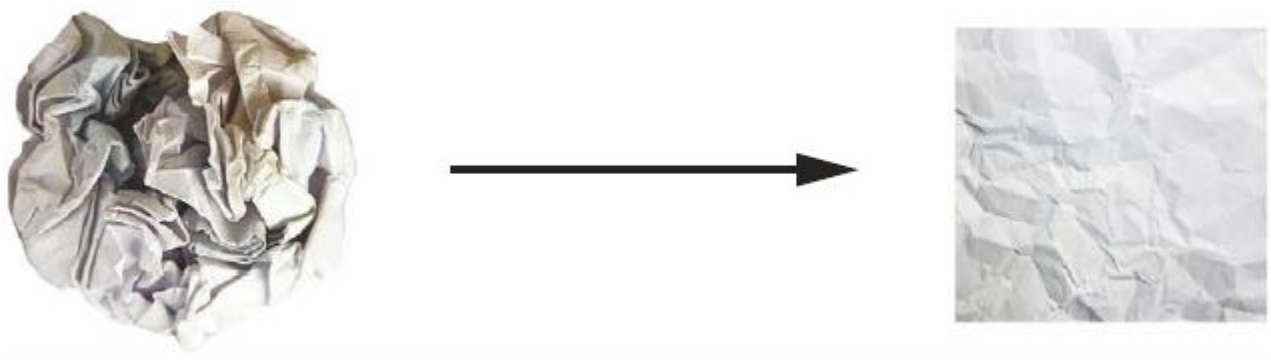


Figure 2.14 Uncrumpling a complicated manifold of data

Thank you! ☺