Advanced Machine Design

Junesh Gautam
Student ID: 101150362
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Problem 6.1: Plane Stress on a Free Surface

Given:

$$\sigma_x = 50\,\text{MPa}, \quad \sigma_y = 10\,\text{MPa}, \quad \tau_{xy} = -15\,\text{MPa}.$$

(A negative sign indicates that the shear stress acts in a sense opposite the chosen positive convention.)
Required:

(a) The two principal normal stresses σ_1, σ_2 in the xy-plane and the principal shear stress there, plus the coordinate system rotation angles.

(b) The maximum normal stress $\sigma_{\max}$ and maximum shear stress $\tau_{\max}$ (considering the free surface, implying $\sigma_z = 0$).

1) In-Plane Principal Normal Stresses

For a 2D plane stress state $(\sigma_x, \sigma_y, \tau_{xy})$, the principal normal stresses in the xy-plane are given by:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

Plugging in $\sigma_x = 50$, $\sigma_y = 10$, $\tau_{xy} = -15$:

$$\begin{aligned}
\frac{\sigma_x + \sigma_y}{2} &= \frac{50 + 10}{2} = 30, \\
\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 &= \left(\frac{50 - 10}{2}\right)^2 = 20^2 = 400, \\
\tau_{xy}^2 &= (-15)^2 = 225, \\
\sqrt{400 + 225} &= \sqrt{625} = 25.
\end{aligned}$$

Hence,

$$\sigma_1 = 30 + 25 = 55\,\text{MPa}, \quad \sigma_2 = 30 - 25 = 5\,\text{MPa}.$$

Principal Shear Stress in the xy-plane. In-plane, the maximum shear is

$$\tau_{(\text{plane})} = \frac{\sigma_1 - \sigma_2}{2} = \frac{55 - 5}{2} = 25\,\text{MPa}.$$

So the in-plane principal normal stresses are 55MPa and 5MPa, and the *principal* shear in that same plane is 25MPa.

2) Coordinate System Rotation Angles

The orientation θ_n of σ_1 relative to the original x-axis satisfies:

$$\tan(2\,\theta_n) = \frac{2\,\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times (-15)}{50 - 10} = \frac{-30}{40} = -0.75.$$

Hence

$$2\,\theta_n = \arctan(-0.75) \approx -36.87^\circ \quad \Longrightarrow \quad \theta_n \approx -18.435^\circ.$$

A negative angle indicates σ_1 is rotated about 18° clockwise from x. (Alternatively, you could add 180° to $2\,\theta_n$ to get a positive rotation measure.)

3) Maximum Normal Stress and Maximum Shear Stress (Including $\sigma_z = 0$)

Because this is a free surface, we also have $\sigma_z = 0$ as a third principal direction out-of-plane. Therefore, the three principal normal stresses at this point are $\{\sigma_1 = 55,\ \sigma_2 = 5,\ \sigma_3 = 0\}$.

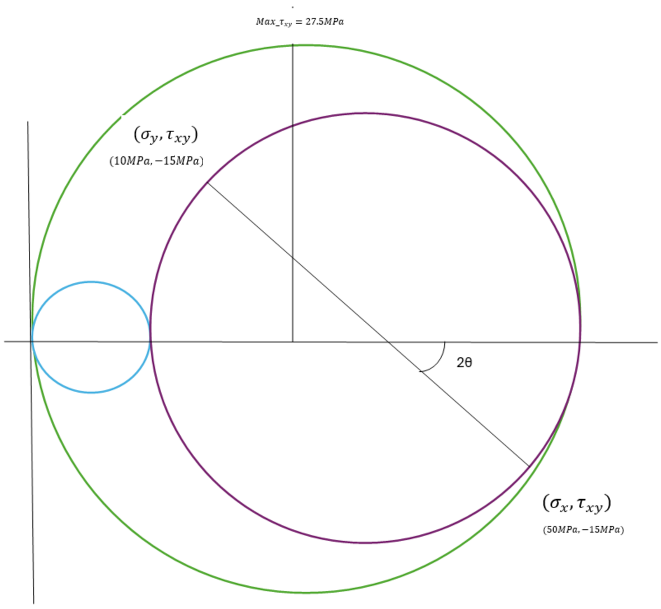
(a) Maximum Normal Stress.

$$\sigma_{\max} = \max\{\,55,\ 5,\ 0\} = 55\,\text{MPa}.$$

(b) Maximum Shear Stress in 3D. The *maximum* shear stress is half the difference between the largest and smallest principal normal stress. The smallest principal is 0, so:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{55 - 0}{2} = 27.5\,\text{MPa}.$$

(Note that this 27.5MPa is larger than the 25MPa in the xy-plane alone.)



Final Results

$$\boxed{\begin{array}{l} \text{(a) In-plane: } \sigma_1 = 55\,\text{MPa},\ \sigma_2 = 5\,\text{MPa},\ \tau_{(\text{plane})} = 25\,\text{MPa}, \\ \qquad \text{Rotation: } 2\theta_n = -36.87^\circ\ (\theta_n \approx -18.4^\circ). \\ \text{(b) Considering } \sigma_z = 0:\ \sigma_{\max} = 55\,\text{MPa},\ \tau_{\max} = 27.5\,\text{MPa}. \end{array}}$$

Interpretation: The negative τ_{xy} leads to a negative rotation angle for the principal axes. Within the xy plane, the largest principal normal is 55MPa, but if we include the free-surface direction ($\sigma_z = 0$), the maximum shear (27.5MPa) is slightly higher than the in-plane shear (25MPa).

Problem 6.10: Three-Dimensional State of Stress

Given State of Stress:

$$\sigma_x = 100\,\text{MPa}, \quad \sigma_y = 40\,\text{MPa}, \quad \sigma_z = -60\,\text{MPa}, \quad \tau_{xy} = 30\,\text{MPa}, \quad \tau_{yz} = 0, \quad \tau_{zx} = 0.$$

Objective:

(a) Determine the three principal normal stresses $\sigma_1, \sigma_2, \sigma_3$ and the associated principal shear stresses τ_1, τ_2, τ_3.

(b) Find the maximum normal stress and the maximum shear stress in 3D.

(c) Identify the directions of the principal normal axes.

Solution

1) Observing the Stress Matrix

Because $\tau_{yz} = 0$ and $\tau_{zx} = 0$, the stress matrix (in x, y, z coordinates) is

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} = \begin{bmatrix} 100 & 30 & 0 \\ 30 & 40 & 0 \\ 0 & 0 & -60 \end{bmatrix} \text{MPa}.$$

Note that the z-direction is *already* decoupled from x and y. This implies that $\sigma_z = -60\,\text{MPa}$ itself is a principal stress (the third axis has no shear).

2) In-Plane Principal Stresses (xy-plane)

We focus on the 2×2 submatrix in the xy-plane:

$$\begin{bmatrix} 100 & 30 \\ 30 & 40 \end{bmatrix}.$$

For a plane stress problem with σ_x, σ_y, τ_{xy}, the two in-plane principal normal stresses $\sigma_{(xy)1}$ and $\sigma_{(xy)2}$ follow the standard formulas:

$$\sigma_{(xy)1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

Plugging in $\sigma_x = 100$, $\sigma_y = 40$, and $\tau_{xy} = 30$:

$$\begin{aligned} \sigma_{(xy)1,2} &= \frac{100 + 40}{2} \pm \sqrt{\left(\frac{100 - 40}{2}\right)^2 + (30)^2} \\ &= 70 \pm \sqrt{(30)^2 + (30)^2} = 70 \pm \sqrt{900 + 900} \\ &= 70 \pm \sqrt{1800} \approx 70 \pm 42.426. \end{aligned}$$

Hence,

$$\sigma_{(xy)1} \approx 112.43\,\text{MPa}, \quad \sigma_{(xy)2} \approx 27.57\,\text{MPa}.$$

3) Full 3D Principal Normal Stresses

Including $\sigma_z = -60\,\text{MPa}$ as the third direction, we have three principal normal stresses:

$$\sigma_1 = 112.43\,\text{MPa}, \quad \sigma_2 = 27.57\,\text{MPa}, \quad \sigma_3 = -60.0\,\text{MPa},$$

where we order them $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

4) Principal Shear Stresses in 3D

For 3D states, the *principal shear stresses* are found by pairing each pair of principal normals:

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_2 = \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_3 = \frac{\sigma_1 - \sigma_3}{2}.$$

Numerically:

$$\begin{aligned} \tau_1 &= \frac{112.43 - 27.57}{2} = 42.43\,\text{MPa}, \\ \tau_2 &= \frac{27.57 - (-60.0)}{2} = 43.79\,\text{MPa}, \\ \tau_3 &= \frac{112.43 - (-60.0)}{2} = 86.21\,\text{MPa}. \end{aligned}$$

5) Part (b): Maximum Normal and Maximum Shear

- **Maximum normal stress** among $\{\sigma_1, \sigma_2, \sigma_3\}$ is $\sigma_{\max} = \sigma_1 \approx 112.4\,\text{MPa}$.
- **Maximum shear stress** is $\max\{\tau_1, \tau_2, \tau_3\} = \tau_3 \approx 86.2\,\text{MPa}$.

6) Part (c): Directions of the Principal Axes

Since $\tau_{yz} = 0$ and $\tau_{zx} = 0$, the z-axis is *already* a principal direction corresponding to $\sigma_3 = -60\,\text{MPa}$. The other two principal axes lie in the xy-plane, found by rotating x by the angle θ_n:

$$\tan(2\theta_n) = \frac{2\,\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \cdot 30}{100 - 40} = 1 \implies 2\theta_n = 45^\circ, \quad \theta_n = 22.5^\circ.$$

Hence:

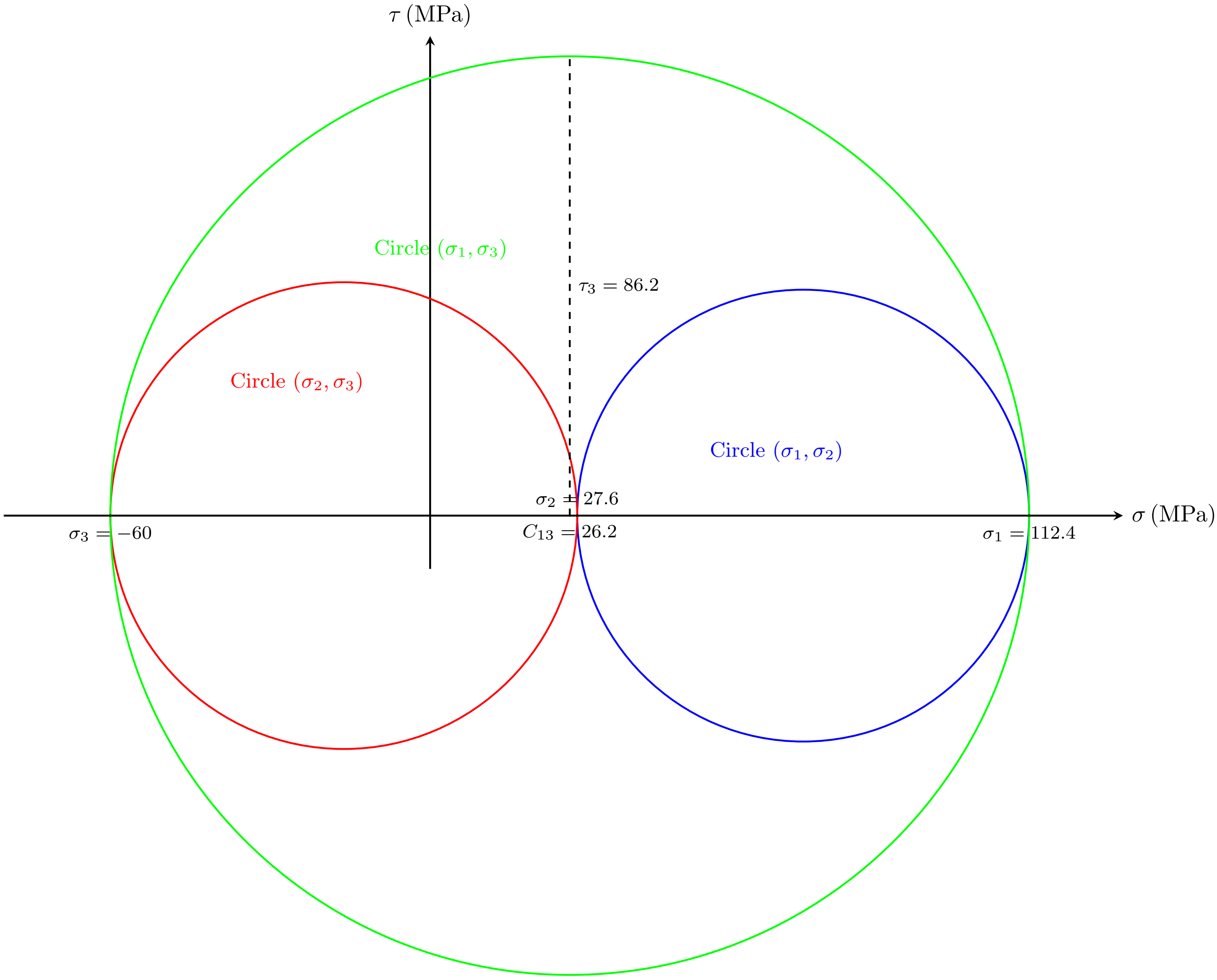
- $\sigma_1 = 112.43\,\text{MPa}$ acts along the direction 22.5° CCW from the x-axis,
- $\sigma_2 = 27.57\,\text{MPa}$ is 90° from that (in-plane),
- $\sigma_3 = -60\,\text{MPa}$ is along the z-axis.

Summary of Results

Principal normal stresses: $\sigma_1 = 112.4\,\mathrm{MPa}$, $\sigma_2 = 27.6\,\mathrm{MPa}$, $\sigma_3 = -60\,\mathrm{MPa}$.
Principal shear stresses: $\tau_1 = 42.4\,\mathrm{MPa}$, $\tau_2 = 43.8\,\mathrm{MPa}$, $\tau_3 = 86.2\,\mathrm{MPa}$.
Max normal stress: $\sigma_{\max} = \sigma_1 \approx 112.4\,\mathrm{MPa}$.
Max shear stress: $\tau_{\max} = \tau_3 \approx 86.2\,\mathrm{MPa}$.
Directions:
z-axis for σ_3, $\quad \theta_n = 22.5^\circ$ in xy-plane for σ_1.

Mohr's Circle Diagram (Three Circles)

Below is a rough illustration of the three-circle Mohr diagram for these stresses. In **3D**, each circle is defined by pairing two principal normals σ_i and σ_j. The largest circle spans from σ_3 to σ_1 and indicates the maximum shear τ_3.



Interpretation:

- The largest circle (green) spans from $\sigma_3 = -60$ MPa to $\sigma_1 = 112.4$ MPa, with radius $\tau_3 = 86.2$ MPa. This circle directly shows the **maximum shear stress** in 3D.
- The other circles (blue and red) each pass through $\sigma_2 = 27.6$ MPa and are "inside" the largest circle. They reveal smaller principal shears ($\tau_1 = 42.4$ MPa and $\tau_2 = 43.8$ MPa).
- The three principal normal stresses appear along the horizontal axis at -60, 27.6, and 112.4 MPa.

Problem 6.15: Special Case of Plane Stress with $\tau_{xy} = 0$

Given: A plane stress state with

$$\sigma_x = 60\,\text{MPa}, \quad \sigma_y = -80\,\text{MPa}, \quad \tau_{xy} = 0.$$

Find:

(a) The principal normal stresses σ_1, σ_2 and the maximum in-plane shear stress $\tau_{\max}$.

(b) Show that for $\tau_{xy} = 0$, the principal axes coincide with the original x and y axes.

(a) Principal Normal Stresses and Maximum Shear

Since $\tau_{xy} = 0$, the general formula for in-plane principal stresses,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

simplifies significantly. Plugging in $\sigma_x = 60\,\text{MPa}$, $\sigma_y = -80\,\text{MPa}$, and $\tau_{xy} = 0$:

$$\begin{aligned}\sigma_{1,2} &= \frac{60 + (-80)}{2} \pm \sqrt{\left(\tfrac{60-(-80)}{2}\right)^2 + 0} \\ &= \frac{-20}{2} \pm \frac{|60 - (-80)|}{2} \\ &= -10 \pm \frac{140}{2} = -10 \pm 70.\end{aligned}$$

Thus,

$$\sigma_1 = -10 + 70 = 60\,\text{MPa}, \quad \sigma_2 = -10 - 70 = -80\,\text{MPa}.$$

Interpretation: The principal normal stresses are exactly σ_x and σ_y themselves.

In-Plane Maximum Shear Stress, $\tau_{\max}$. For plane stress, $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$. Numerically:

$$\tau_{\max} = \frac{60 - (-80)}{2} = \frac{140}{2} = 70\,\text{MPa}.$$

Hence:

$$\sigma_1 = 60\,\text{MPa}, \quad \sigma_2 = -80\,\text{MPa}, \quad \tau_{\max} = 70\,\text{MPa}.$$

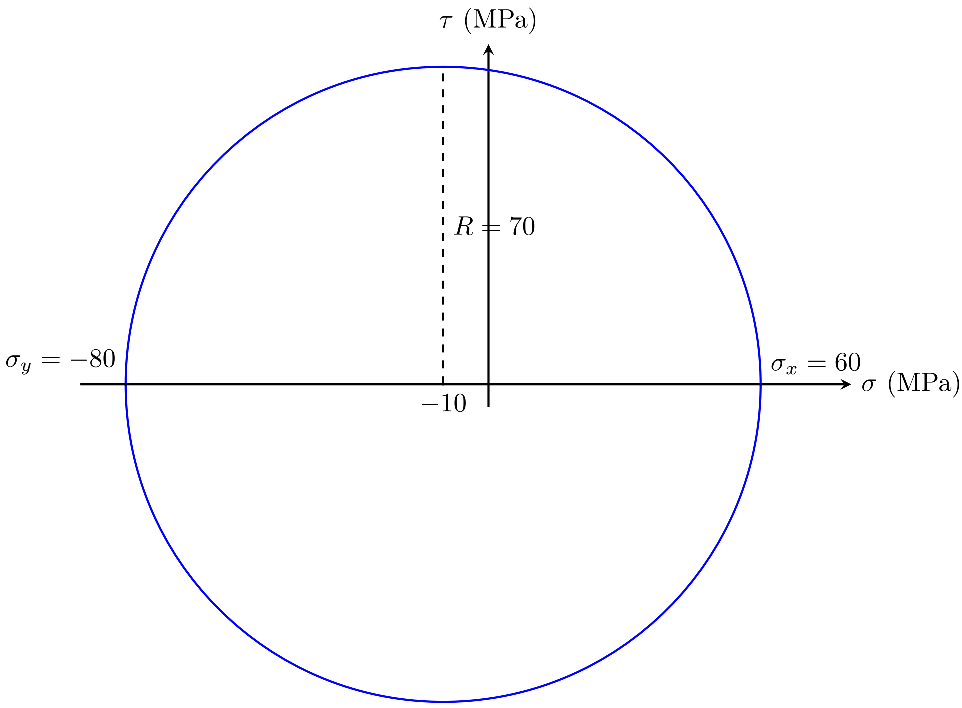
(b) Same Directions as the Original x, y Axes

Observe that $\tau_{xy} = 0$ on the original coordinate system. This means no shear stress acts on the x or y faces. When *both* σ_x and σ_y have zero shear coupling, those directions are already *principal axes* for the in-plane state of stress.

$$\boxed{\text{Hence the } x \text{ direction carries } \sigma_1 = 60\,\text{MPa and the } y \text{ direction carries } \sigma_2 = -80\,\text{MPa.}}$$

Optional Mohr's Circle Sketch

For completeness, we can draw a small Mohr's circle for plane stress:



Here, the "circle" intersects the σ-axis at $\sigma_x = 60$ and $\sigma_y = -80$, with the center at $\sigma = -10$ and radius 70. The top point of the circle is the maximum shear stress, $\tau_{\max} = 70\,\text{MPa}$, and it lies at $\sigma = -10$ (the center's σ-value).

Final Answers:

$$\sigma_1 = 60\,\text{MPa}, \quad \sigma_2 = -80\,\text{MPa}, \quad \tau_{\max} = 70\,\text{MPa}.$$

Moreover, the axes remain the same (i.e. x-axis is principal axis 1, y-axis is principal axis 2) since $\tau_{xy} = 0$ from the start.

Problem 6.19: Maximum Normal and Shear Stresses in a Pressurized, End-Loaded Pipe under Bending

Given:

- A thin-walled pipe (closed ends) with:
  - Outer diameter $D_{\text{out}} = 80\,\text{mm}$.
  - Wall thickness $t = 2.0\,\text{mm}$.
- Internal pressure $p = 10\,\text{MPa}$.
- Bending moment $M = 2.0\,\text{kN}\cdot\text{m}$.
- Neglect localized effects at the ends.

Required:

- Maximum normal stress, σ_{max}, in the pipe wall.
- Maximum shear stress, τ_{max}, in the pipe wall.

1. Determine the Internal-Pressure Stresses

Assuming a thin-walled cylinder, let $r \approx \frac{D_{\text{in}}+D_{\text{out}}}{2}$ be the mean radius. Since $D_{\text{out}} = 80\,\text{mm}$ and $t = 2\,\text{mm}$, the inner diameter $D_{\text{in}} \approx 76\,\text{mm}$. Hence,

$$r \approx \frac{80+76}{2} = 78\,\text{mm}/2 = 39\,\text{mm}.$$

For a closed-end cylinder under internal pressure p:

$$\text{Hoop (circumferential) stress:} \quad \sigma_{\text{hoop}} = \frac{p\,r}{t}, \qquad \text{Axial stress:} \quad \sigma_{\text{axial}} = \frac{p\,r}{2\,t}.$$

Plug in $p = 10\,\text{MPa}$, $r = 39\,\text{mm}$, $t = 2\,\text{mm}$:

$$\sigma_{\text{hoop}} = \frac{10\times 39}{2} = 195\,\text{MPa},$$
$$\sigma_{\text{axial}} = \frac{10\times 39}{2\times 2} = 97.5\,\text{MPa}.$$

2. Determine the Bending Stress

The pipe is also subjected to $M = 2.0\,\text{kN}\cdot\text{m} = 2000\,\text{N}\cdot\text{m}$. Converting to $\text{N}\cdot\text{mm}$:

$$M = 2000\,(\text{N}\cdot\text{m}) \times 1000\,\frac{\text{mm}}{\text{m}} = 2.0\times 10^{6}\,\text{N}\cdot\text{mm}.$$

We assume bending about a diameter axis, so the neutral axis passes through the center of the pipe's cross section. For a thin annular cross section of outer radius $R_{\text{out}} = 40\,\text{mm}$ and inner radius $R_{\text{in}} = 38\,\text{mm}$, the second moment of area about that neutral axis is:

$$I = \frac{\pi}{4}\big(R_{\text{out}}^4 - R_{\text{in}}^4\big).$$

Numerically:

$$R_{\text{out}}^4 = 40^4 = 2.56\times 10^6, \quad R_{\text{in}}^4 = 38^4 \approx 2.07\times 10^6, \quad \text{difference} = 0.49\times 10^6.$$

Thus

$$I = \frac{\pi}{4}\times 0.49\times 10^6 \approx 0.49\times 10^6 \times 0.785 = 3.85\times 10^5\,\text{mm}^4.$$

The distance from the neutral axis to the outermost fiber is $c = 40\,\text{mm}$, so the bending stress at the outer fiber is

$$\sigma_{\text{bend}} = \frac{M\,c}{I} = \frac{(2.0\times 10^6)\times 40}{3.85\times 10^5} \approx 208\,\text{MPa}.$$

3. Combine Axial and Bending Stresses

The *axial* direction stress due to internal pressure is $\sigma_{\text{axial}} = 97.5\,\text{MPa}$. Bending adds or subtracts from that, depending on whether we look at the tensile side or the compressive side of the pipe's cross section.

$$\text{Top fiber (tension side):} \quad \sigma_{\text{ax, top}} = 97.5 + 208 = 305.5\,\text{MPa},$$
$$\text{Bottom fiber (compression side):} \quad \sigma_{\text{ax, bottom}} = 97.5 - 208 = -110.5\,\text{MPa}.$$

4. State of Stress at the Outer Fiber

At a given point on the pipe's outer surface, the *hoop* stress is $\sigma_{\text{hoop}} = 195\,\text{MPa}$ and the *axial* stress is either 305.5 MPa (tension side) or -110.5 MPa (compression side). We assume no shear in these axes (thin-walled, principal directions).

Maximum Normal Stress.

- On the tension side, the axial stress is 305.5 MPa, while the hoop stress is 195 MPa. The larger of those is 305.5 MPa.
- On the compression side, the axial stress is -110.5, which is negative and smaller in magnitude than $+195$ or $+305.5$.

Hence, the **maximum normal stress** anywhere in the pipe wall is

$$\sigma_{\max} \;\approx\; 305.5\,\text{MPa}\,(\text{tension side}).$$

Maximum Shear Stress. In plane stress with zero shear, the maximum in-plane shear is half the difference between the two normal stresses. We check each side:

- *Tension side:* $\{\sigma_{\text{ax}} = 305.5,\ \sigma_{\text{hoop}} = 195\}$; difference = 110.5; half = 55.25 MPa.
- *Compression side:* $\{\sigma_{\text{ax}} = -110.5,\ \sigma_{\text{hoop}} = 195\}$; difference = 305.5; half = 152.75 MPa.

Thus the **maximum shear stress** is on the compression side and equals

$$\tau_{\max} = \frac{(195) - (-110.5)}{2} = 152.75\,\text{MPa}.$$

Summary of Results

$$\boxed{\begin{array}{l} \text{Maximum normal stress: } \sigma_{\max} \approx 305.5\,\text{MPa} \quad (\text{top fiber, tension side}). \\ \text{Maximum shear stress: } \tau_{\max} \approx 152.8\,\text{MPa} \quad (\text{bottom fiber, compression side}). \end{array}}$$

Note: The hoop stress is 195 MPa in every circumferential direction; the axial stress from pressure alone is 97.5 MPa, but the bending adds ± 208 MPa at the outer fibers, leading to these combined results.

Problem 6.26: Solid Shaft Under Axial Tension and Torsion

Given:

- Solid circular shaft with diameter $d = 75\,\text{mm}$.
- Axial tensile load $P = 400\,\text{kN}$.
- Applied torque $T = 3.5\,\text{kN} \cdot \text{m}$.

Required:

- Maximum normal stress, $\sigma_{\max}$.
- Maximum shear stress, $\tau_{\max}$.

1. Compute the Axial (Normal) Stress

The cross-sectional area of a solid circular shaft of diameter $d = 75\,\text{mm}$ is

$$A \;=\; \frac{\pi\, d^2}{4} = \frac{\pi \times 75^2}{4} \;\approx\; 4418\,\text{mm}^2.$$

Hence the axial (tensile) normal stress is

$$\sigma_{\text{ax}} = \frac{P}{A} = \frac{400\,000\,\text{N}}{4418\,\text{mm}^2} \;\approx\; 90.5\,\text{MPa}.$$

2. Compute the Torsional Shear Stress (at Outer Radius)

Polar Moment of Inertia. For a solid circular cross section,

$$J \;=\; \frac{\pi\, d^4}{32}.$$

With $d = 75\,\text{mm}$,

$$d^4 = 75^4 = 3.1640625 \times 10^7, \quad J \approx \frac{\pi \times 3.164 \times 10^7}{32} \;\approx\; 3.10 \times 10^6\,\text{mm}^4.$$

Shear Stress at Radius c. The outer radius is $c = d/2 = 37.5\,\text{mm}$. For a circular shaft under torque T, the maximum shear stress at the outer fiber is

$$\tau \;=\; \frac{T\,c}{J}.$$

Given $T = 3.5\,\text{kN}\cdot\text{m} = 3.5\times10^6\,\text{N}\cdot\text{mm}$,

$$\tau = \frac{(3.5\times10^6)\times 37.5}{3.10\times10^6} \;\approx\; 42.3\,\text{MPa}.$$

3. Combine Normal Stress and Shear Stress (2D Plane Stress)

At any point on the shaft's outer surface, we have:

$$\begin{cases} \text{Normal stress along the shaft axis:} & \sigma_z \;\approx\; 90.5\,\text{MPa}, \\ \text{Shear stress on planes containing the axis:} & \tau \;\approx\; 42.3\,\text{MPa}, \\ \text{No other normal or shear components.} & \end{cases}$$

This is effectively a plane stress situation in the (z, r) plane:

$$\sigma_r = 0, \quad \sigma_z = 90.5, \quad \tau_{zr} = 42.3.$$

Mohr's Circle Analysis. Let $\sigma_x = 0$ and $\sigma_y = 90.5$, with $\tau_{xy} = 42.3$ to represent that same 2D state. The center of the circle is

$$\sigma_{\text{avg}} = \frac{0 + 90.5}{2} = 45.25,$$

and the radius is

$$R = \sqrt{\left(\frac{90.5 - 0}{2}\right)^2 + (42.3)^2} = \sqrt{(45.25)^2 + (42.3)^2} \;\approx\; 61.9.$$

Hence the principal normal stresses in that plane are:

$$\sigma_1 = 45.25 + 61.9 = 107.15\,\text{MPa}, \quad \sigma_2 = 45.25 - 61.9 = -16.65\,\text{MPa}.$$

Because there is a third out-of-plane direction with zero stress, the largest principal normal in 3D is $\sigma_1 \approx 107.2\,\text{MPa}$.

4. Maximum Normal Stress $\sigma_{\max}$ and Maximum Shear Stress $\tau_{\max}$

- $\sigma_{\max} \approx 107.2\,\text{MPa}$ (the largest principal normal).
- The minimum principal normal is about $-16.7\,\text{MPa}$, and the out-of-plane direction is 0. So in 3D, we order them as $\sigma_1 \approx 107.2 > \sigma_2 = 0 > \sigma_3 \approx -16.7$.
- The **maximum shear stress** is

  $$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \;\approx\; \frac{107.2 - (-16.7)}{2} = 61.95\,\text{MPa}.$$

  (This also equals the Mohr circle radius for the in-plane problem.)

Final Answers

$$\boxed{\sigma_{\max} \;\approx\; 107\,\text{MPa}, \qquad \tau_{\max} \;\approx\; 62\,\text{MPa}.}$$

Interpretation: The axial tension alone gives $\sigma_z = 90.5\,\text{MPa}$, but because of the torsional shear ($42.3\,\text{MPa}$), the plane containing (z, r) yields a principal normal stress of about $107\,\text{MPa}$ and a maximum shear of about $62\,\text{MPa}$ in the shaft.