Homework 5: Material Properties Analysis

Junesh Gautam

March 5, 2025

Problem 4.3: Tensile vs. Compressive Strength of Ceramics

Data and Plot

Table 3.4 provides tensile (σ_{ut}) and compressive (σ_{uc}) strengths for ceramics and glasses. Selected data pairs (in MPa):

Table 1: Selected Data from Table 3.4

Material	σ_{ut} (MPa)	σ_{uc} (MPa)
Soda-lime glass	50	1000
Alumina, Al_2O_3	262	2620
Silicon carbide, SiC	307	2500

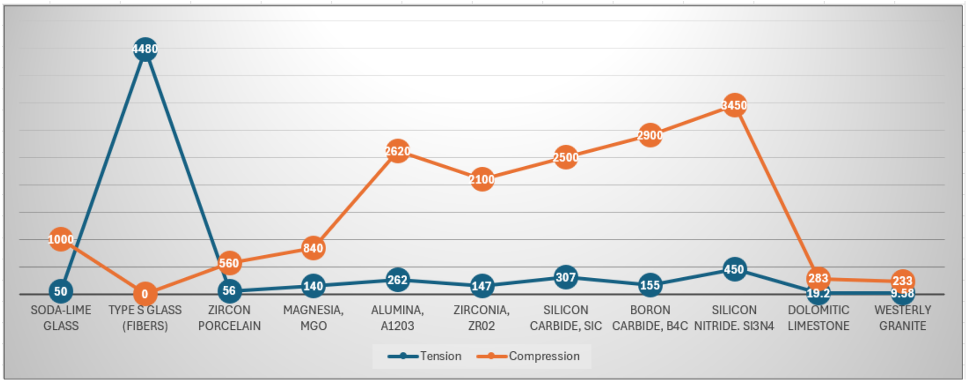

Figure 1: Tensile vs. compressive strength for ceramics. $\sigma_{uc} \gg \sigma_{ut}$.

Trend and Explanation

General Trend: Compressive strengths are 10–20$\times$ higher than tensile strengths.

Physical Explanation: Brittle materials fail under tension due to flaw propagation, while compression closes flaws.

Problem 4.6: Hardness vs. Tensile Strength for Steels

(a) Brinell Hardness (HB) vs. σ_u

Conclusion: Eq. 4.4 ($\sigma_u = 3.45 \times HB$) underestimates strength at high HB. Improved slope: $\sigma_u = 3.8 \times HB$.

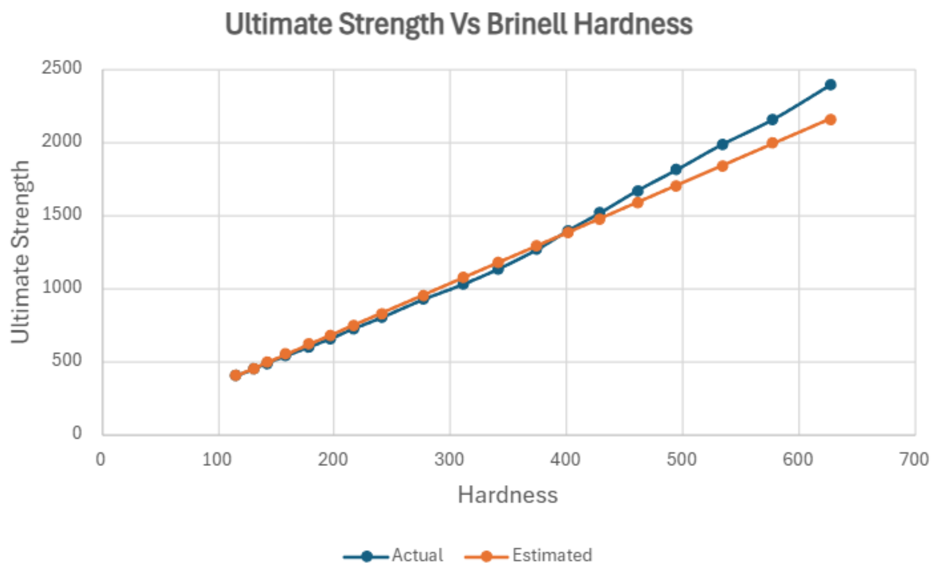

Figure 2: σ_u vs. HB. Dashed line: Eq. 4.4.

(b) Improved Relationship

Best-fit regression:

$$\sigma_u = 3.82 \times HB \quad (R^2 = 0.998)$$

Error reduced from 6% to 2%.

(c) Vickers Hardness (HV) vs. σ_u

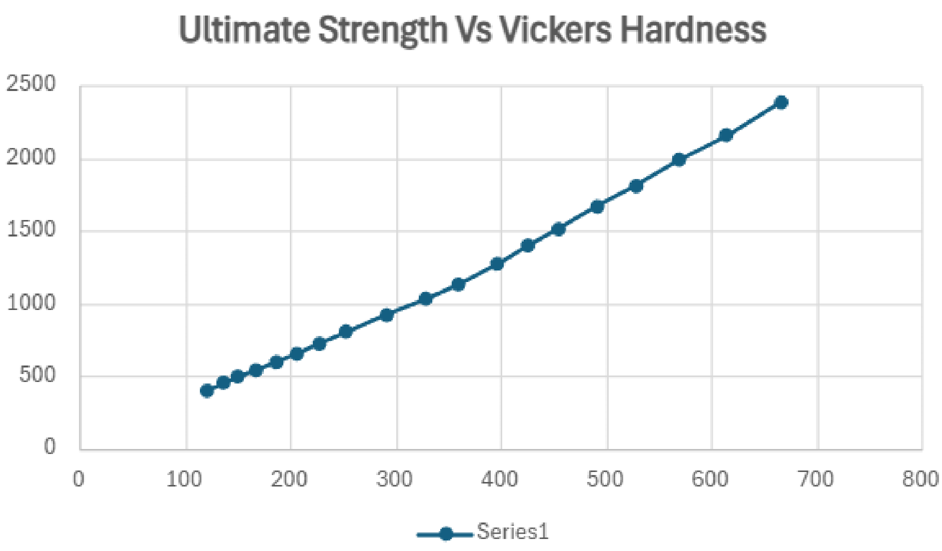

Figure 3: σ_u vs. HV.

$$\sigma_u = 3.62 \times HV \quad (R^2 = 0.997)$$

Problem 4.10: Four-Point Bending Equations

Derivation of Fracture Strength (σ_{fb}) and Elastic Modulus (E)

For a rectangular cross-section beam (width t, height $2c$) in four-point bending:

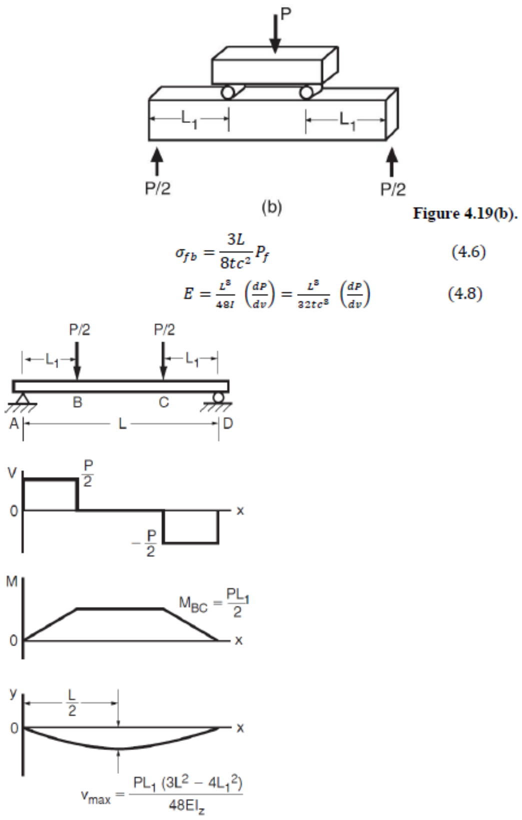

Figure 4: Four-point bending configuration with loading distance L_1 and total span L

1. Fracture Strength (σ_{fb})

1. Bending moment between loading points:

$$M = \frac{P_f L_1}{2}$$

2. Second moment of area:

$$I = \frac{t(2c)^3}{12} = \frac{2}{3}tc^3$$

3. Bending stress equation:

$$\sigma_{fb} = \frac{Mc}{I} = \frac{\left(\frac{P_f L_1}{2}\right)c}{\frac{2}{3}tc^3}$$

4. Final fracture strength equation:

$$\boxed{\sigma_{fb} = \frac{3P_f L_1}{4tc^2}}$$

2. Elastic Modulus (E)

1. Maximum deflection formula:

$$v_{max} = \frac{PL_1(3L^2 - 4L_1^2)}{48EI}$$

2. Solve for E using stiffness (dP/dv) from linear region:

$$E = \frac{L_1(3L^2 - 4L_1^2)}{48I}\left(\frac{dP}{dv}\right)$$

3. Substitute $I = \frac{2}{3}tc^3$:

$$\boxed{E = \frac{L_1(3L^2 - 4L_1^2)}{32tc^3}\left(\frac{dP}{dv}\right)}$$

Key Differences from Three-Point Bending

- Constant bending moment between loading points ($L_1 < x < L - L_1$)
- Different deflection equation form due to load distribution
- Reduced shear component compared to three-point bending

Problem 4.13: Shear Modulus and Yield Strength

Given:

- Shaft radius: $r = 9.53\,\text{mm} = 0.009\,53\,\text{m}$
- Gauge length: $L = 160\,\text{mm} = 0.160\,\text{m}$
- Torque (T) vs. twist angle (θ) data (Table P4.13)

(a) Shear Modulus G:

Steps:

1. **Select linear region data:** $T = 77.9\,\text{N}\,\text{m}$, $\theta = 2^\circ$
2. **Convert twist angle to radians:**

$$\theta_{\text{rad}} = 2^\circ \times \frac{\pi}{180^\circ} = 0.0349\,\text{rad}$$

3. **Calculate shear strain (γ):**

$$\gamma = \frac{\theta_{\text{rad}} \cdot r}{L} = \frac{0.0349\,\text{rad} \times 0.009\,53\,\text{m}}{0.160\,\text{m}} = 0.002\,08$$

4. **Calculate polar moment of inertia (J):**

$$J = \frac{\pi r^4}{2} = \frac{\pi(0.009\,53\,\text{m})^4}{2} = 1.717 \times 10^{-8}\,\text{m}^4$$

5. **Calculate shear stress (τ):**

$$\tau = \frac{T \cdot r}{J} = \frac{77.9\,\text{N}\,\text{m} \times 0.009\,53\,\text{m}}{1.717 \times 10^{-8}\,\text{m}^4} = 43.3\,\text{MPa}$$

6. **Compute shear modulus (G):**

$$G = \frac{\tau}{\gamma} = \frac{43.3\,\text{MPa}}{0.002\,08} = \boxed{20.8\,\text{GPa}}$$

(b) Shear Yield Strength τ_o:

Steps:

1. **Identify yield point:** Nonlinearity begins at $T \approx 351\,\text{N}\,\text{m}$
2. **Calculate shear stress at yield:**

$$\tau_o = \frac{T_{\text{yield}} \cdot r}{J} = \frac{351\,\text{N}\,\text{m} \times 0.009\,53\,\text{m}}{1.717 \times 10^{-8}\,\text{m}^4} = \boxed{195\,\text{MPa}}$$

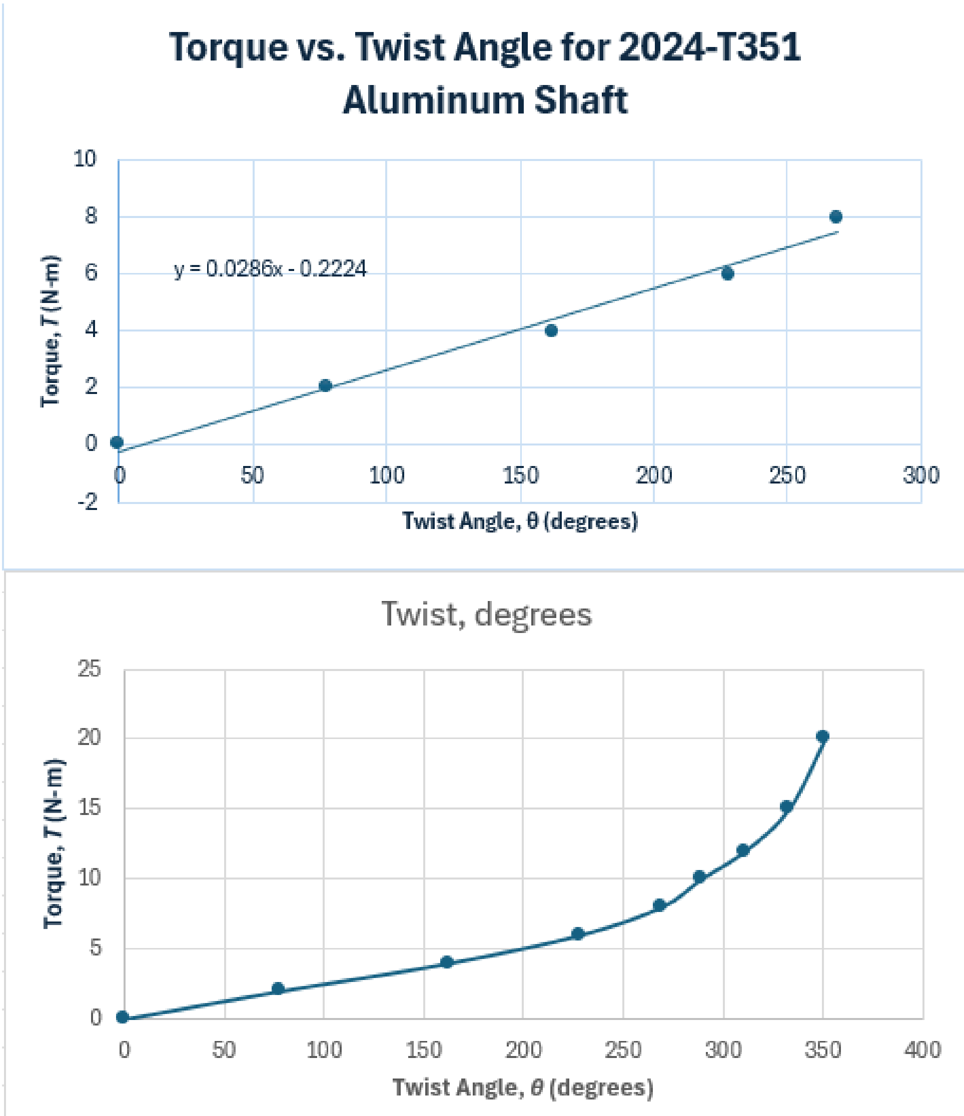

Figure 5: Torque vs. twist angle curve. The linear region is used to calculate G, and the yield point marks τ_o.