Homework 9

Advanced Machine Design

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6.27 (15 points) A solid shaft of diameter d is subjected to an axial load P and a torque T.

(a) Derive an expression for the maximum shear stress as a function of d, P, and T.

(b) If P = 200 kN and T = 1.5 kN·m, what is the smallest diameter such that the maximum shear stress does not exceed 100MPa?

Derivation of the maximum shear stress

A solid circular shaft of diameter d carries

$$\text{normal load } P \quad \text{and} \quad \text{torque } T.$$

1. Normal (axial) stress on the x–faces $\sigma = \dfrac{4P}{\pi d^2}$.
2. Shear stress at the outer fibre due to torsion $\tau = \dfrac{16T}{\pi d^3}$.
3. Treating the state as *plane stress* ($\sigma_y = \tau_{yz} = \tau_{zx} = 0$) the principal shear in the x–y plane is, from Mohr's-circle result

$$\tau_{\max} = \tfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\tfrac{4P}{\pi d^2}\right)^2 + 4\left(\tfrac{16T}{\pi d^3}\right)^2}.$$

   Simplifying,

$$\boxed{\tau_{\max}(d) = \frac{2}{\pi d^3}\sqrt{P^2 d^2 + 64\,T^2}}.$$

Required diameter for $\tau_{\max} \le 100$ MPa

Let $\tau_{\text{allow}} = 100\,\text{MPa}$. Set $\tau_{\max} = \tau_{\text{allow}}$ and solve for d:

$$\frac{2}{\pi d^3}\sqrt{P^2 d^2 + 64T^2} = \tau_{\text{allow}} \implies \left(\frac{\tau_{\text{allow}}\pi}{2}\right)^2 d^6 - P^2 d^2 - 64T^2 = 0.$$

Substituting $P = 200\,\text{kN}$, $T = 1.5\,\text{kN}\,\text{m}$ and solving the cubic in $x = d^2$

$$d_{\min} = \boxed{46\,\text{mm}}.$$

After isolating $\sqrt{\cdots}$ we squared both sides, giving a d^6 term and hence a cubic in d^2. Only one real, positive root is physical.

6.31 (20 points) A thick-walled tube has closed ends and is loaded with an internal pressure of 80 MPa and a torque of 15 kN·m. The inner and outer diameters are 60 and 80 mm, respectively.

(a) Determine the maximum shear stress in the tube. Neglect the localized effects of the end closure. (Suggestion: Using Fig. A.6(a), calculate $\tau_{\max}$ at the inner and outer walls and at several intermediate radial positions.)

(b) Plot the variations of σ_r, σ_t, and σ_x due to the pressure, τ_{tx} due to the torsion, and $\tau_{\max}$, all versus R.

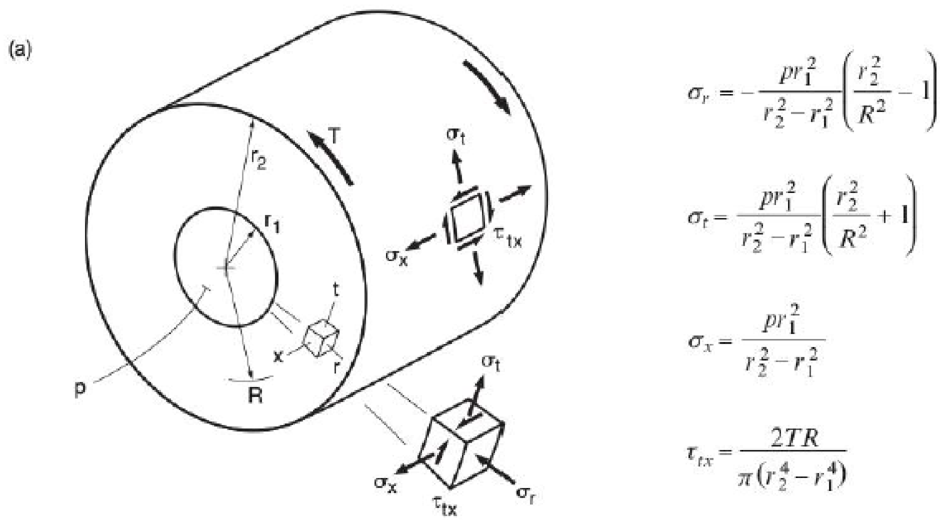


Figure A.6 Stresses in thick-walled pressure vessels, (a) tubular

Given data

$$p_i = 80\,\text{MPa},\ T = 15\,\text{kN m},\ r_i = 30\,\text{mm},\ r_o = 40\,\text{mm}.$$

Normal stresses due to pressure

Using Lame's formulas (tube with closed ends, Fig. A.6 a):

$$\sigma_r(r) = -\frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right), \quad \sigma_t(r) = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right), \quad \sigma_x = \frac{p_i r_i^2}{r_o^2 - r_i^2} = \text{constant}.$$

Shear stress due to torsion

$$\tau_{tx}(r) = \frac{T\,r}{J}, \qquad J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right).$$

(a) Maximum shear at any r

At each radius the stress tensor is $\sigma_{ij} = \mathrm{diag}(\sigma_r, \sigma_t, \sigma_x)$ with τ_{tx} the only off-diagonal term. The 3-D principal shear is

$$\tau_{\max}(r) = \tfrac{1}{2}\max\left\{|\sigma_t - \sigma_r|,\ |\sigma_r - \sigma_x|,\ |\sigma_x - \sigma_t|\right\}.$$

Evaluating numerically (outer surface governs):

$$\boxed{\tau_{\max}(r_o) = 224\,\text{MPa}}.$$

(b) Stress variation plot

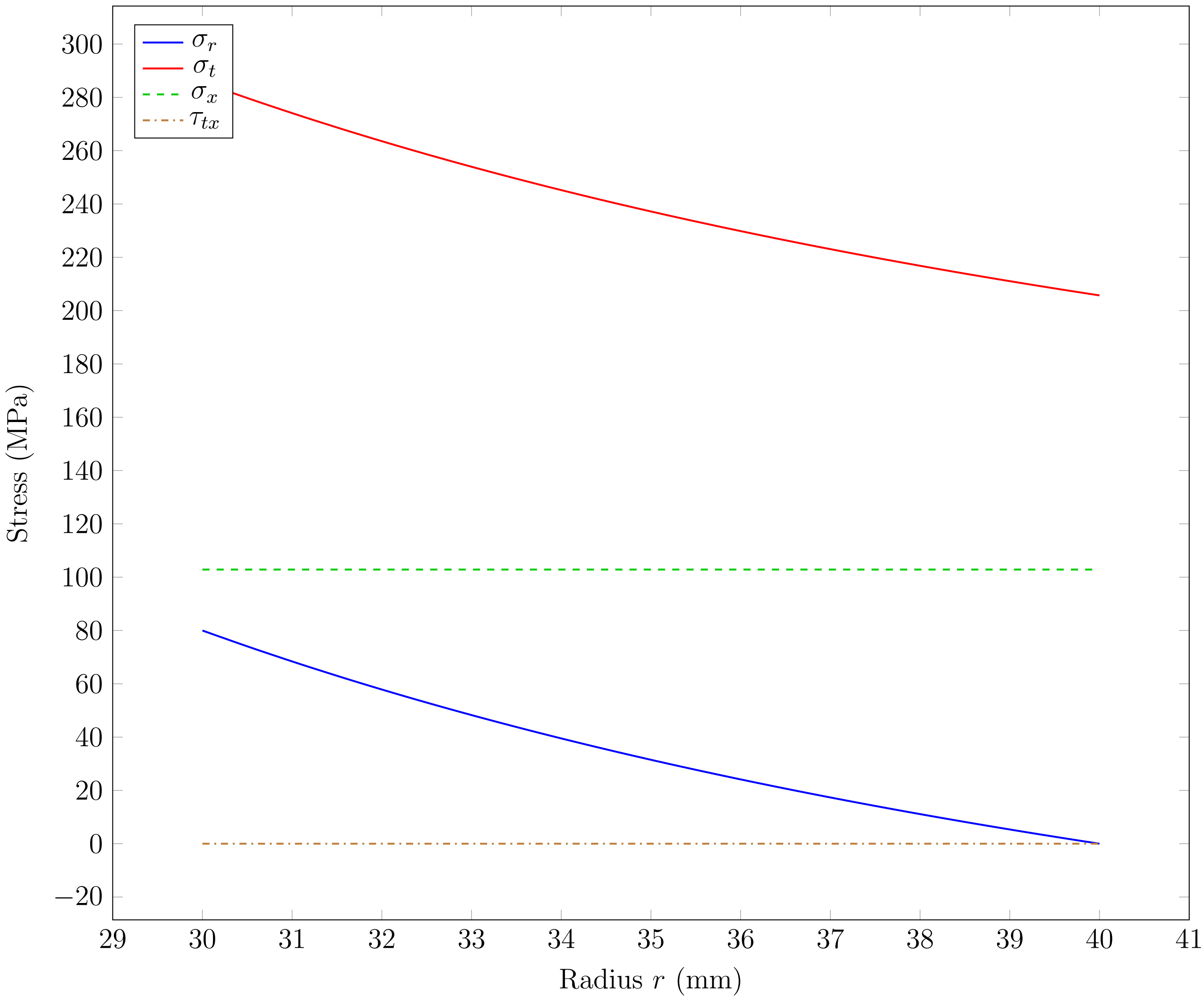


Figure 1: Variation of σ_r, σ_t, σ_x and τ_{tx} across the tube wall.

6.37 (15 points) An element of material is subjected to the following state of stress: $\sigma_x = -40$, $\sigma_y = 100$, $\sigma_z = 30$, $\tau_{xy} = -50$, $\tau_{yz} = 12$, and $\tau_{zx} = 0$ MPa. Determine the following:

(a) Principal normal stresses and principal shear stresses.

(b) Maximum normal stress and maximum shear stress.

(c) Direction cosines for each principal normal stress axis.

Stress tensor

$$[\sigma] = \begin{bmatrix} -40 & -50 & 0 \\ -50 & 100 & 12 \\ 0 & 12 & 30 \end{bmatrix} \text{ MPa.}$$

(a) Principal normal and shear stresses

The characteristic equationgives the eigenvalues

$$\sigma_1 = 118\,\text{MPa},\ \sigma_2 = 29\,\text{MPa},\ \sigma_3 = -56\,\text{MPa}.$$

Principal shears (Eq. 6.18) are

$$\tau_1 = \tfrac{1}{2}(\sigma_2 - \sigma_3) = 42\,\text{MPa},\ \tau_2 = \tfrac{1}{2}(\sigma_3 - \sigma_1) = 87\,\text{MPa},\ \tau_3 = \tfrac{1}{2}(\sigma_1 - \sigma_2) = 44\,\text{MPa}.$$

(b) Maximum normal and shear

$$\boxed{\sigma_{\max} = \sigma_1 = 118\,\text{MPa}}, \qquad \boxed{\tau_{\max} = |\tau_2| = 87\,\text{MPa}}.$$

(c) Principal directions

Direction cosines $\{\ell, m, n\}$ are the normalised eigenvectors:

Axis	ℓ	m	n
σ_1	0.300	−0.945	−0.130
σ_2	0.080	−0.110	0.991
σ_3	−0.951	−0.308	0.043

6.48 (10 points) Consider a case of plane stress where the only nonzero components for the x-y-z coordinate system chosen are σ_x and τ_{xy}. (For example, this situation occurs at the surface of a shaft under combined bending and torsion.) Develop equations in terms of σ_x and τ_{xy} for the following: maximum normal stress, maximum shear stress, and octahedral shear stress.

Treat $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$. From Eq. (6.7) for principal normals and Eq. (6.10) for shear:

$$\boxed{\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}}, \qquad \boxed{\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}}.$$

The octahedral shear (Eq. 6.36) simplifies to

$$\boxed{\tau_{\text{oct}} = \frac{1}{3}\sqrt{\sigma_x^2 + 6\,\tau_{xy}^2}}.$$

6.54 (10 points) For the strains measured on the free surface of a mild steel part in Prob. 6.16, i.e., $\varepsilon_x = 190 \times 10^{-6}$, $\varepsilon_y = -760 \times 10^{-6}$, and $\gamma_{xy} = 300 \times 10^{-6}$. Poisson's ratio from Table 5.2 is 0.293. Determine the principal normal strains and the principal shear strains. Assume that no yielding has occurred.

Given (free surface of a mild-steel part)

$$\varepsilon_x = 190 \times 10^{-6}, \qquad \varepsilon_y = -760 \times 10^{-6}, \qquad \gamma_{xy} = 300 \times 10^{-6}.$$

Step 1 – determine out-of-plane strain ε_z

At a free surface the normal stress $\sigma_z = 0$. Hooke's law for plane stress gives

$$\varepsilon_z = -\nu(\varepsilon_x + \varepsilon_y) = -0.293\big(190 - 760\big) \times 10^{-6} = 167 \times 10^{-6}.$$

Step 2 – build the strain Mohr's-circle data

Average normal strain in the x–y plane:

$$\varepsilon_{\text{avg}} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = \tfrac{1}{2}(190 - 760) \times 10^{-6} = -285 \times 10^{-6}.$$

Radius of the circle (Dow Eq. 6.37 with strains)

$$R = \sqrt{\left(\tfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\tfrac{\gamma_{xy}}{2}\right)^2} = \sqrt{(475)^2 + (150)^2} \times 10^{-6} = 498 \times 10^{-6}.$$

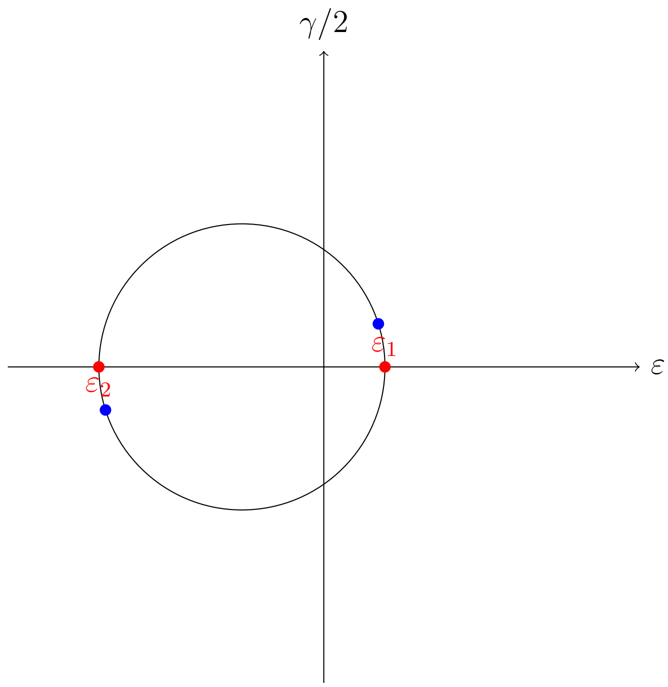
Step 3 – principal normal and shear strains

$$\boxed{\begin{aligned} \varepsilon_1 &= \varepsilon_{\text{avg}} + R = -285 + 498 = \ 213\ \mu\varepsilon, \\ \varepsilon_2 &= \varepsilon_{\text{avg}} - R = -285 - 498 = -783\ \mu\varepsilon. \end{aligned}}$$

Maximum in-plane shear strain ($\gamma_{\max}$ is the full peak-to-peak value on Mohr's circle):

$$\boxed{\gamma_{\max}^{(xy)} = 2R = 996\ \mu\varepsilon,} \qquad \boxed{\gamma_{1,2}^{(\text{principal})} = \pm\tfrac{1}{2}\gamma_{\max} = \pm 498\ \mu\varepsilon.}$$

Mohr's-circle sketch



6.56 (10 points) A strain gauge rosette of the type shown in Fig. 6.16(a) is employed to measure strains on the free surface of a titanium alloy part, with the result being $\varepsilon_x = 2200 \times 10^{-6}$, $\varepsilon_y = 1800 \times 10^{-6}$, and $\varepsilon_{45} = 1500 \times 10^{-6}$. Poisson's ratio from Table 5.2 is 0.361. Determine the principal normal strains and the principal shear strains. Assume that no yielding has occurred.

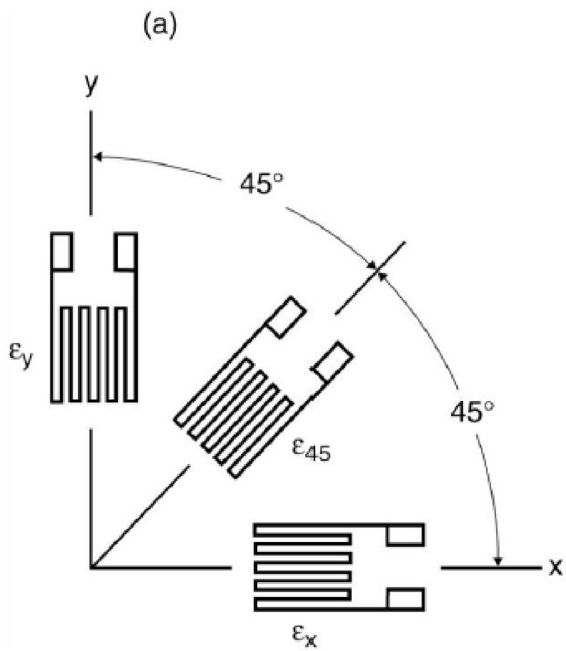


Figure 6.16 Two strain gauge rosette configurations for measurements in three directions.

Given (45° rectangular rosette, Fig. 6.16 a)

$$\varepsilon_x = 2200\ \mu\varepsilon, \quad \varepsilon_y = 1800\ \mu\varepsilon, \quad \varepsilon_{45^\circ} = 1500\ \mu\varepsilon, \quad \nu = 0.361.$$

Step 1 – back-out γ_{xy} from the 45° gauge

For a rectangular rosette

$$\varepsilon_{45^\circ} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}\gamma_{xy},$$

so

$$\gamma_{xy} = 2\Big(\varepsilon_{45^\circ} - \tfrac{1}{2}(\varepsilon_x + \varepsilon_y)\Big) = 2\big(1500 - 2000\big) = -1000\ \mu\varepsilon.$$

Step 2 – principal strains in the x–y plane

$$\varepsilon_{\text{avg}} = \tfrac{1}{2}(2200 + 1800) = 2000\ \mu\varepsilon,$$

$$R = \sqrt{\left(\tfrac{2200-1800}{2}\right)^2 + \left(\tfrac{-1000}{2}\right)^2} = \sqrt{200^2 + 500^2} = 539\ \mu\varepsilon.$$

$$\boxed{\varepsilon_1 = 2000 + 539 = 2540\ \mu\varepsilon,} \qquad \boxed{\varepsilon_2 = 2000 - 539 = 1460\ \mu\varepsilon.}$$

Step 3 – maximum shear strain

$$\gamma_{\max}^{(xy)} = \varepsilon_1 - \varepsilon_2 = 1080\ \mu\varepsilon, \qquad \gamma_{1,2}^{(\text{principal})} = \pm\tfrac{1}{2}\gamma_{\max} = \pm 540\ \mu\varepsilon.$$

Step 4 – out-of-plane strain for completeness

$$\varepsilon_z = -\nu(\varepsilon_x + \varepsilon_y) = -0.361(4000)\ \mu\varepsilon = -1444\ \mu\varepsilon.$$

Hence the full set of principal normal strains is

$$\boxed{\varepsilon_1 = 2540\ \mu\varepsilon,\ \varepsilon_2 = 1460\ \mu\varepsilon,\ \varepsilon_3 = -1444\ \mu\varepsilon.}$$