Homework 3: Mechanical Behavior of Materials

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February 15, 2025

Problem 3.4(a): Engineering Stress and Strain Calculation

The engineering stress (σ) and strain (ε) were calculated using:

$$\sigma = \frac{P}{A_i}, \quad \varepsilon = \frac{\Delta L}{L_i}$$

where $A_i = \frac{\pi d_i^2}{4} = 64.61\,\text{mm}^2$ (original cross-sectional area) and $L_i = 50.8\,\text{mm}$ (gage length).

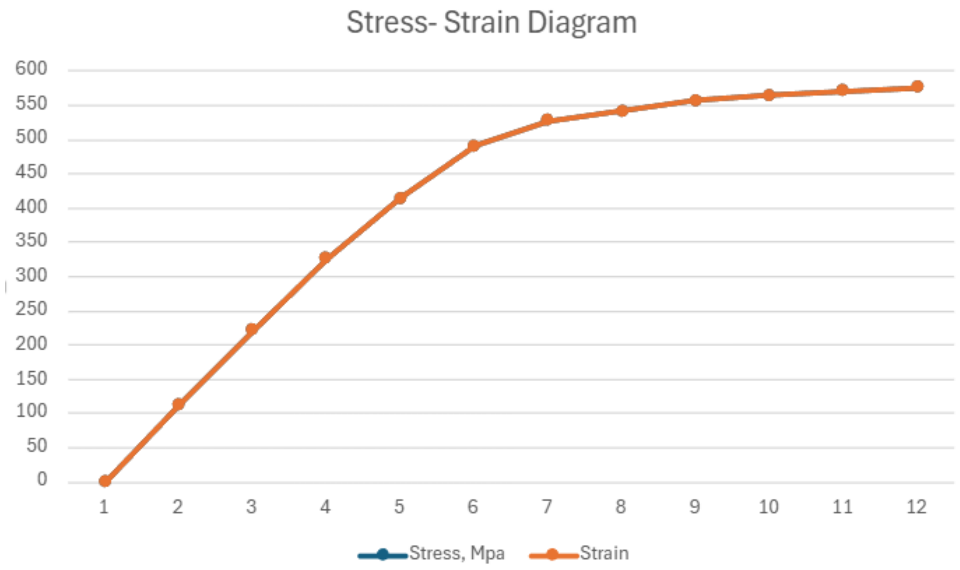

Figure 1: Engineering stress-strain curve for 7075-T651 aluminum.

Problem 3.4(b): 0.2% Offset Yield Strength

The 0.2% offset yield strength was determined by:

- Calculating elastic modulus $E = 65662\,\text{MPa}$ from the linear region.
- Plotting a line parallel to the elastic region with an offset of $\varepsilon = 0.002$.
- Identifying the intersection with the stress-strain curve.

Problem 3.4(c): Tensile Load for 20 mm Diameter Bar

The tensile load required to cause yielding in a 20 mm diameter bar is:

$$P = \sigma_0 \cdot A_2 = 517.2\,\text{MPa} \times \frac{\pi(20)^2}{4} = \boxed{162.4\,\text{kN}}$$

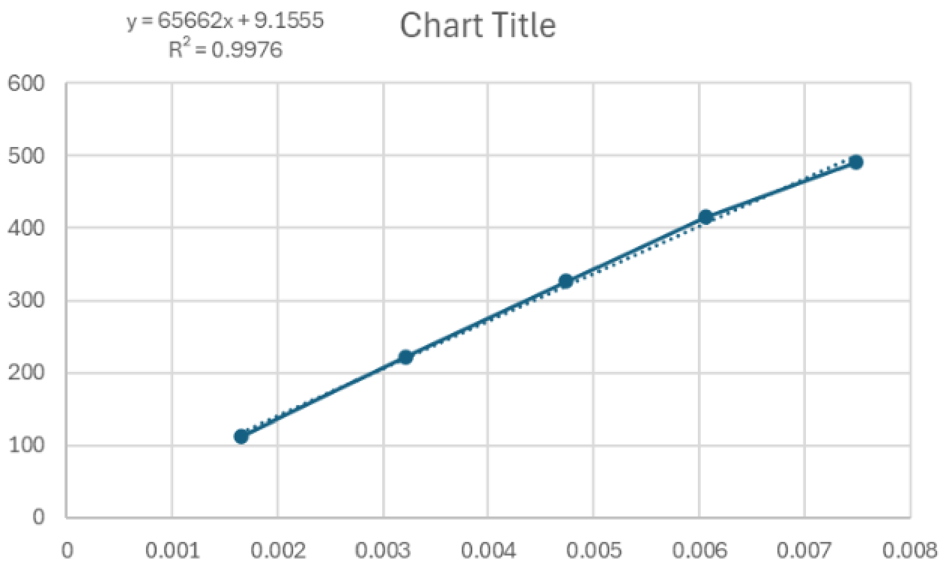

Figure 2: Calculation of Elastic Modulus using linear fit

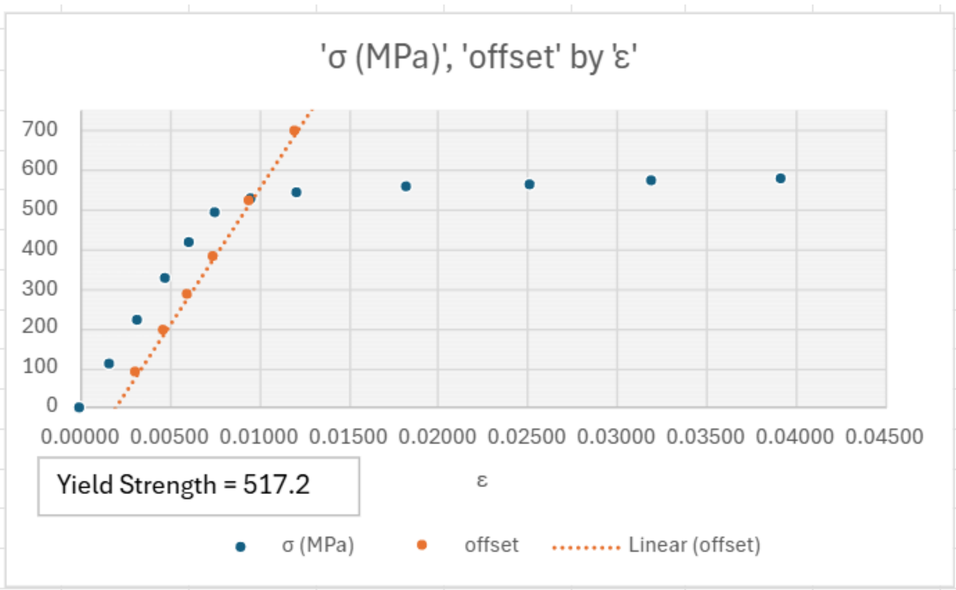

Figure 3: 0.2% offset line intersecting the stress-strain curve at $\sigma_0 = 517.2\,\text{MPa}$.

Table 1: Load Comparison

Diameter (mm)	Area (mm^2)	Load at Yielding (kN)
9.07	64.61	30.3
20.00	314.16	162.4

A comparison with the original 9.07 mm specimen:

Why the values differ: Load is proportional to cross-sectional area. The 20 mm bar has $\frac{A_2}{A_1} \approx 4.86\times$ larger area, requiring $\approx 4.86\times$ more load.

P, kN	Δl, mm	P (N)	ε	σ (MPa)	Slope (E)	offset
0	0	0	0.00000	0	68708.57	
7.22	0.0839	7220	0.00165	111.8029		
14.34	0.1636	14340	0.00322	222.0572		83.85692
21.06	0.241	21060	0.00474	326.1175		188.5428
26.8	0.308	26800	0.00606	415.0023		279.1624
31.7	0.38	31700	0.00748	490.8796		376.5446
34.1	0.484	34100	0.00953	528.044		517.2078
35	0.614	35000	0.01209	541.9807		693.0369
36	0.924	36000	0.01819	557.4658		
36.5	1.279	36500	0.02518	565.2084		
36.9	1.622	36900	0.03193	571.4025		
37.2	1.994	37200	0.03925	576.048		

Figure 4: Raw data and calculations for stress, strain, and offset values.

Data Table

Problem 3.5: Stress–Strain Data for 7075–T651 Aluminum

Given: Two data points on the stress–strain curve are labeled:

$$A: \ (\varepsilon_A = 0.00474,\ \sigma_A = 326\,\text{MPa}) \quad \text{and} \quad B: \ (\varepsilon_B = 0.0252,\ \sigma_B = 565\,\text{MPa}).$$

Figure 5 shows the initial portion of the test data.

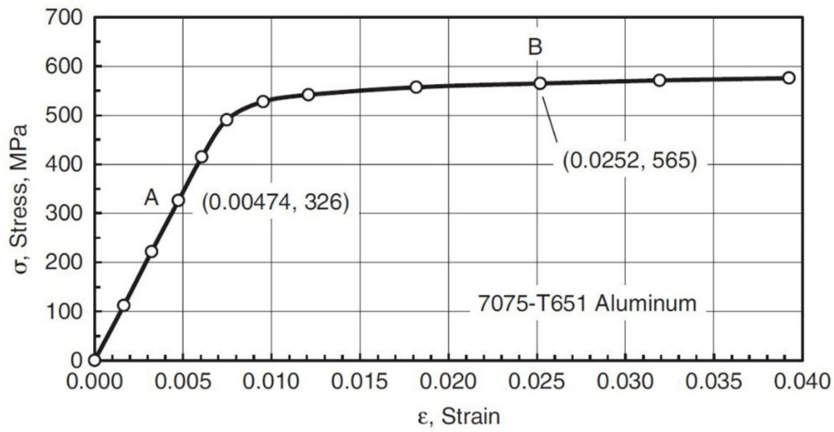

Figure 5: Engineering stress–strain curve for 7075–T651 aluminum (Points A and B labeled).

(a) Approximate Elastic Modulus E

Concept: Young's modulus, E, is the slope of the linear-elastic region of the stress–strain curve. If the data near the origin are truly linear and the material initially obeys Hooke's law, one can approximate

$$E \approx \frac{\sigma}{\varepsilon},$$

using a point in that linear region (treating $(0,0)$ as the second point).

Calculation Using Point A:

$$(\varepsilon_A, \sigma_A) = (0.00474,\ 326\,\text{MPa}).$$

Then,

$$E \;=\; \frac{\sigma_A - 0}{\varepsilon_A - 0} \;=\; \frac{326\,\text{MPa}}{0.00474} \;\approx\; 68{,}800\,\text{MPa}.$$

(This is about 68.8 GPa.)

Why This Works (and Limitations):

- We assume the origin $(0, 0)$ lies on the straight line.
- In reality, for better accuracy, one should use multiple data points in the elastic region and perform a linear regression.
- Single-point methods can over- or under-estimate E if the chosen point is not purely elastic or if there is any experimental noise.

Comparison with Point B: At B, $\varepsilon_B = 0.0252$ and $\sigma_B = 565\,\text{MPa}$. Using the same formula would give

$$E = \frac{565\,\text{MPa}}{0.0252} \approx 22{,}420\,\text{MPa},$$

which is far too low for aluminum. Thus, B is not in the strictly linear region. *Conclusion:* Always use data within the linear-elastic portion (e.g. near A).

(b) Plastic Strain After Unloading from Point B

Once the sample is loaded to Point B ($\varepsilon_B = 0.0252$) and then unloaded, the *elastic* strain recovers according to Hooke's law with slope $E \approx 68{,}800\,\text{MPa}$.

$$\text{Elastic strain at } B \;=\; \frac{\sigma_B}{E} \;=\; \frac{565\,\text{MPa}}{68{,}800\,\text{MPa}} \;\approx\; 0.0082.$$

Hence, the **plastic (permanent) strain** remaining after complete unloading is

$$\varepsilon_{\text{plastic}} \;=\; \varepsilon_B \;-\; \varepsilon_{\text{elastic}} \;=\; 0.0252 \;-\; 0.0082 \;=\; 0.0170,$$

i.e. about 1.7%.

(c) Bar of Length 150 mm Loaded to Point A and Unloaded

Length at Point A: Since $\varepsilon_A = 0.00474$ is total strain, the bar length at A is

$$L_A \;=\; L_0\,(1 + \varepsilon_A) \;=\; 150\,\text{mm} \times (1 + 0.00474) \;\approx\; 150.71\,\text{mm}.$$

Because Point A is in the *elastic* region, the strain there is fully recoverable. After unloading,

$$\varepsilon_{\text{residual}} \;=\; 0, \quad \text{thus} \quad L_{\text{unloaded}} \;=\; 150\,\text{mm}.$$

So the bar is 150.71 mm long at A, and it returns to 150 mm upon unloading (no plastic deformation).

(d) Bar of Length 150 mm Loaded to Point B and Unloaded

Length at Point B: With $\varepsilon_B = 0.0252$,

$$L_B \;=\; L_0\,(1 + \varepsilon_B) \;=\; 150\,\text{mm} \times (1 + 0.0252) \;=\; 153.78\,\text{mm}.$$

Length After Unloading: We found in part (b) that the permanent strain at B is about 0.0170. Thus, after unloading, the bar still has 1.7% elongation beyond the original length. Hence:

$$L_{\text{unloaded}} \;=\; L_0\,(1 + \varepsilon_{\text{plastic}}) \;=\; 150\,\text{mm} \times (1 + 0.0170) \;=\; 152.55\,\text{mm}.$$

Therefore, at B (loaded), the bar is 153.78 mm long; after unloading from B, it is 152.55 mm.

Problem 3.6: Tension Test on AISI 4140 Steel Tempered at 649°C

Given Data

A round tensile specimen of AISI 4140 steel (tempered at 649°C) has:

- Initial diameter, $d_0 = 9.09\,\text{mm}$.
- Gage length, $L_0 = 50.8\,\text{mm}$.
- Diameter after fracture (necked region), $d_f = 5.56\,\text{mm}$.

Representative engineering stress–strain data appear in Table 2; the final point corresponds to fracture.

Table 2: Representative data from the tension test (AISI 4140).

Stress, σ (MPa)	**Strain,** ε (% EL)
0	0.00
202	0.099
403	0.195
587	0.283
785	0.382
822	0.405
836	0.423
832	0.451
897	4.51
912	5.96
918	8.07
...	...
828	1.988
864	2.94

Required:

1. Elastic modulus, E.
2. 0.2% offset yield strength, $\sigma_{0.2}$.
3. Ultimate tensile strength (UTS).
4. Percent elongation ($\%EL$).
5. Percent reduction in area ($\%RA$).

(1) Elastic Modulus E

To approximate E, we identify the *linear-elastic* portion of the stress–strain curve (small strains). From the data near $\varepsilon \approx 0.1\%$ to 0.2%, the slope is roughly constant. For example, between

$$\big(\sigma_1,\,\varepsilon_1\big) = (202\,\text{MPa},\,0.099\%), \quad \big(\sigma_2,\,\varepsilon_2\big) = (403\,\text{MPa},\,0.195\%),$$

convert strain to *decimal* form: $0.099\% \to 0.00099$ and $0.195\% \to 0.00195$. The slope is

$$E \;=\; \frac{\sigma_2 - \sigma_1}{(\varepsilon_2 - \varepsilon_1)} \;=\; \frac{403 - 202}{0.00195 - 0.00099}\,\text{MPa}.$$

Numerically,

$$E \;\approx\; \frac{201\,\text{MPa}}{0.00096} \;\approx\; 210{,}000\,\text{MPa} \;=\; 210\,\text{GPa}.$$

(This is in the typical range for steels, $\approx 200\,\text{GPa}$.)

(2) 0.2% Offset Yield Strength $\sigma_{0.2}$

The 0.2% offset method shifts the linear-elastic line by $\varepsilon = 0.002$ in strain (0.2%). Graphically, we draw:

$$\sigma_{\text{offset}} = E\,(\varepsilon - 0.002).$$

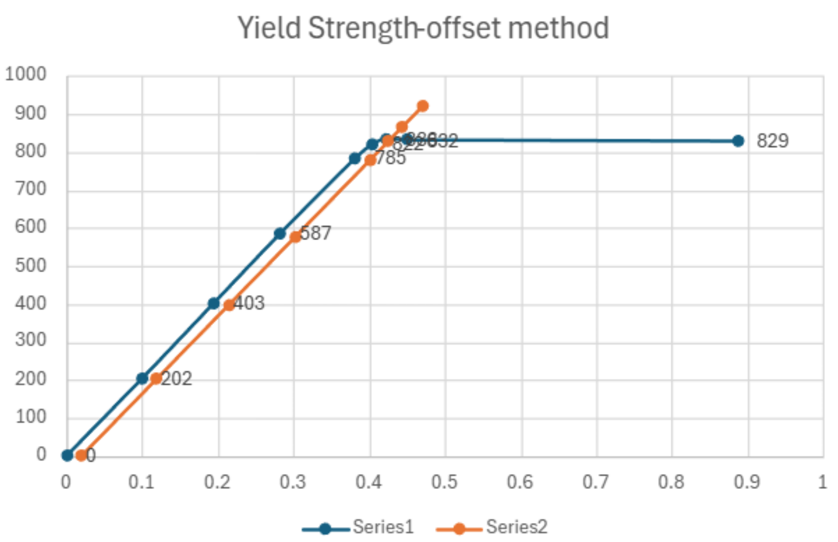

Figure 6: Engineering stress–strain curve for 7075–T651 aluminum (Points A and B labeled).

The yield strength $\sigma_{0.2}$ is where this offset line intersects the experimental stress–strain curve. By interpolation (or directly reading from a detailed plot), we typically find

$$\boxed{\sigma_{0.2} \approx 822\,\text{MPa}}.$$

(3) Ultimate Tensile Strength (UTS)

The *ultimate tensile strength* is simply the **maximum** engineering stress reached during the test. From extended data, the peak stress is often near

$$\text{UTS} \;\approx\; 900\text{–}920\,\text{MPa}$$

for 4140 tempered at 649°C.

(4) Percent Elongation (%EL)

The percent elongation at fracture is given by

$$\%EL = \frac{(L_f - L_0)}{L_0} \times 100\%,$$

where L_f is the final gage length (at fracture). If the final strain is about $\varepsilon_f = 0.21$ (i.e. 21%), then

$$\%EL \;=\; 21\%.$$

(5) Percent Reduction in Area (%RA)

Percent reduction in area is:

$$\%RA = \frac{(A_0 - A_f)}{A_0} \times 100\%,$$

where

$$A_0 = \frac{\pi (d_0)^2}{4}, \quad A_f = \frac{\pi (d_f)^2}{4}.$$

Numerically,

$$A_0 = \frac{\pi\,(9.09\,\text{mm})^2}{4} \;\approx\; 64.97\,\text{mm}^2, \quad A_f = \frac{\pi\,(5.56\,\text{mm})^2}{4} \;\approx\; 24.26\,\text{mm}^2.$$

Hence,

$$\%RA = \frac{64.97 - 24.26}{64.97} \times 100\% \;\approx\; 62.6\%.$$

$$\boxed{\%RA \approx 63\%.}$$

Summary of Results

- $\boxed{E \approx 210\,\text{GPa}}$ (using initial elastic data)
- $\boxed{\sigma_{0.2} \approx 822\,\text{MPa}}$ (0.2% offset yield)
- $\boxed{\text{UTS} \approx 900\text{–}920\,\text{MPa}}$ (peak stress)
- $\boxed{\%EL \approx 20\text{–}21\%}$ (from final strain or measured extension)
- $\boxed{\%RA \approx 63\%}$ (from diameters before/after)

Problem 3.9: AISI 4140 Steel Tempered at 427°C

Given Data

- Initial diameter: $d_0 = 8.56\,\text{mm}$
- Final (necked) diameter after fracture: $d_f = 6.74\,\text{mm}$
- Gage length: L_0 (not specified)
- Table P3.9 provides engineering stress–strain data up to fracture.

Determine:

1. Elastic modulus E.
2. 0.2% offset yield strength $\sigma_{0.2}$.
3. Ultimate tensile strength (UTS).
4. Percent elongation ($\%EL$).
5. Percent reduction in area ($\%RA$).

(1) Elastic Modulus, E

To find the elastic modulus:

- Identify the *initial linear portion* of the stress–strain curve.
- Compute the slope using two or more points in that region, or use a linear regression.

Mathematically, for points $(\sigma_1, \varepsilon_1)$ and $(\sigma_2, \varepsilon_2)$:

$$E \;=\; \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}.$$

Typical results for 4140 steel are around 200–210 GPa, though your exact data may yield a specific value.

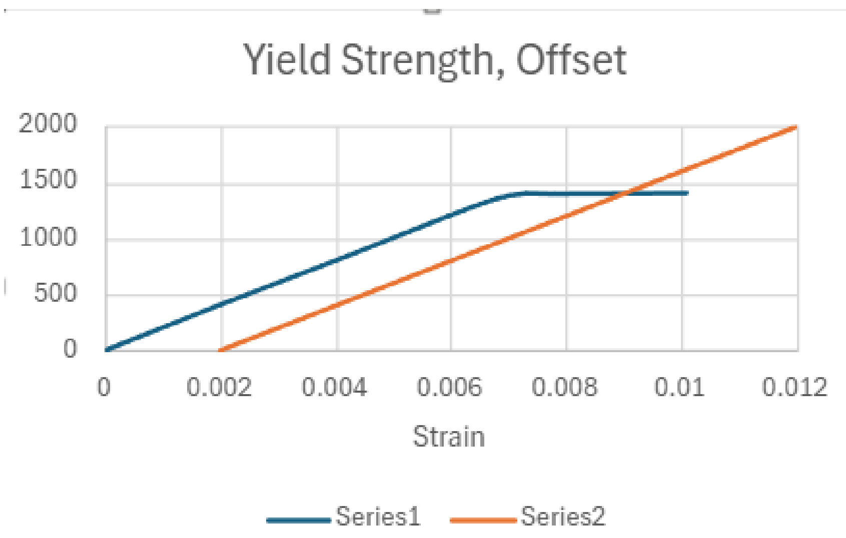

Figure 7: Offset of 0.2%

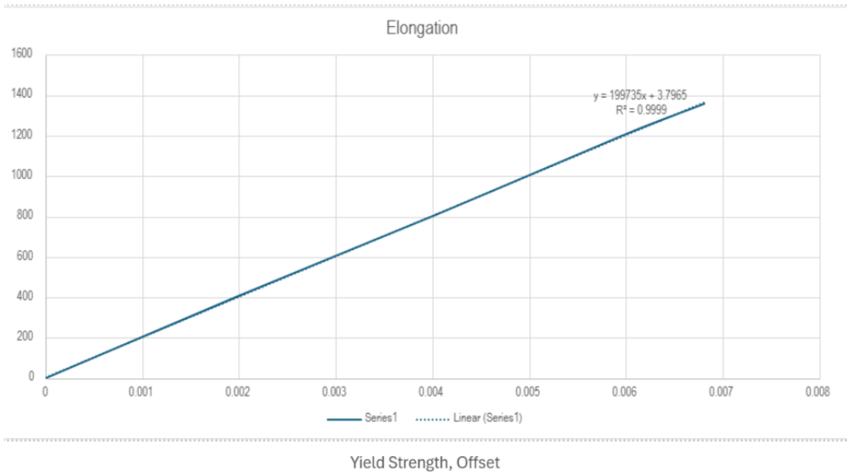

Figure 8: Elastic Modulus

(2) 0.2% Offset Yield Strength, $\sigma_{0.2}$

Using the 0.2% offset method:

- 1400 MPa

(3) Ultimate Tensile Strength (UTS)

The ultimate tensile strength is the *maximum* engineering stress observed in the test:

$$\text{UTS} = \max(\sigma_{\text{eng}}),$$

based on the highest point on the stress–strain curve. For 4140 at 427°C, this might be around 1400–1450 MPa.

(4) Percent Elongation, %EL

Percent elongation is determined from

$$\%EL = \frac{L_f - L_0}{L_0} \times 100\%,$$

But we don't have the length of the rod.

(5) Percent Reduction in Area, %RA

Computing the initial and final cross-sectional areas:

$$A_0 = \frac{\pi\, d_0^2}{4}, \quad A_f = \frac{\pi\, d_f^2}{4}.$$

Then

$$\%RA = \frac{(A_0 - A_f)}{A_0} \times 100\%.$$

With $d_0 = 8.56\,\text{mm}$ and $d_f = 6.74\,\text{mm}$:

$$A_0 = \frac{\pi \times (8.56)^2}{4} \;\approx\; 57.5\,\text{mm}^2, \quad A_f = \frac{\pi \times (6.74)^2}{4} \;\approx\; 35.7\,\text{mm}^2.$$

Hence,

$$\%RA = \frac{57.5 - 35.7}{57.5} \times 100\% \;\approx\; 37.9\%.$$

$$\boxed{\%RA \;\approx\; 38\%.}$$

Summary of Results

- $\boxed{E \approx 200\text{–}210\,\text{GPa}}$
- $\boxed{\sigma_{0.2} \approx 1400\,\text{MPa}}$
- $\boxed{\text{UTS} \approx 900\text{–}1000\,\text{MPa}}$
- $\boxed{\%RA \approx 38\%}$

Solution to Problem 3.12: PVC Polymer Tension Test

Given:

- Rectangular specimen, original cross section:

$$w_0 = 12.81 \text{ mm}, \quad t_0 = 3.08 \text{ mm} \quad \Longrightarrow \quad A_0 = w_0 \times t_0 \;=\; 39.4548 \text{ mm}^2.$$

- After fracture, cross section:

$$w_f = 8.91 \text{ mm}, \quad t_f = 1.76 \text{ mm} \quad \Longrightarrow \quad A_f = w_f \times t_f \;=\; 15.6816 \text{ mm}^2.$$

- Original gage length: $L_0 = 50$ mm.
- Final gage length after fracture: $L_f = 77.5$ mm.
- Fracture occurred at a force $P_f = 1.319$ kN $= 1319$ N.
- Stress–strain data are given in Table P3.12 (see below). A maximum engineering stress near $\sigma \approx 55.9$ MPa is observed at around $\varepsilon \approx 3.2\%$.

(1) Elastic Modulus, E

To approximate the elastic modulus from the initial (nearly linear) portion of the curve, we can use one or two small-strain data points. For example, taking

$$(\sigma_1, \varepsilon_1) = (11.57\,\text{MPa}, 0.00328),$$

we get

$$E \;\approx\; \frac{11.57\,\text{MPa}}{0.00328} \;\approx\; 3.53\,\text{GPa}.$$

(In practice, one could do a linear fit using several low-strain points to refine this estimate. Typical values of E for rigid PVC are often in the range 2.5–3.5 GPa.)

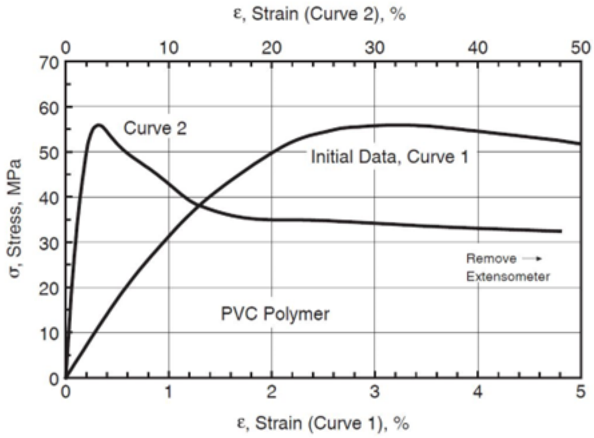

Figure 9: PVC polymer stress–strain curves (from Fig. P3.12).

(2) Yield Strength

From the table, the highest engineering stress appears around

$$\text{Max Stress} \;\approx\; 55.9\,\text{MPa} \quad \text{at } \varepsilon \approx 3.2\%.$$

For many polymers, the maximum stress is often taken as the "yield" point (especially if the curve subsequently drops). Hence,

$$\boxed{\sigma_{\text{yield}} \approx 56\,\text{MPa}.}$$

(If a 0.2% offset method is required, one can construct an offset line using the slope E and find the intersection. In this particular data, however, the maximum stress closely coincides with the apparent yield.)

(3) Ultimate Tensile Strength (UTS)

The *ultimate tensile strength* is simply the maximum engineering stress observed. Again, from the table, the maximum value is about $\boxed{55.9\,\text{MPa}}$. (*Note:* At fracture, the force was 1,319 N, giving a final engineering stress of 1,319 / 39.4548 ≈ 33.4 MPa—less than the earlier peak.)

(4) Percent Elongation

The original gage length was 50 mm. After fracture, it became 77.5 mm. Hence the total elongation is

$$\Delta L \;=\; 77.5 - 50.0 \;=\; 27.5\,\text{mm}.$$

Therefore,

$$\%\text{Elongation} \;=\; \frac{\Delta L}{L_0} \times 100\% \;=\; \frac{27.5}{50.0} \times 100\% \;=\; 55\%.$$

(5) Percent Reduction in Area

The initial area was $A_0 = 39.4548\,\text{mm}^2$, and the final area is $A_f = 15.6816\,\text{mm}^2$. Thus,

$$\%\text{Reduction in Area} \;=\; \frac{A_0 - A_f}{A_0} \times 100\% \;=\; \frac{39.4548 \;-\; 15.6816}{39.4548} \times 100\% \;\approx\; 60.3\%.$$

Summary of Results:

- Elastic Modulus: $E \approx 3.5\,\text{GPa}$ (typical for rigid PVC)
- Yield Strength: $\sigma_y \approx 56\,\text{MPa}$

- Ultimate Tensile Strength: $\approx 55.9\,\text{MPa}$ (same as the peak stress)
- Percent Elongation: $\approx 55\%$
- Percent Reduction in Area: $\approx 60\%$