Homework 4: Tension Test Analysis

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Problem 3.22(a): Engineering Stress-Strain Properties

Calculations

- **Elastic Modulus (E)**: Calculated using the slope of the linear elastic region (first three data points):
$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{257\text{ MPa} - 0}{0.0012 - 0} = \boxed{214\text{ GPa}}$$
- **Yield Strength (σ_y)**: Determined using the 0.2% offset method (Fig. 2):
$$\boxed{333\text{ MPa}}$$
- **Ultimate Tensile Strength (σ_u)**: Maximum engineering stress observed in the dataset:
$$\boxed{576\text{ MPa}}$$
- **Percent Reduction in Area**: Initial area $A_0 = \frac{\pi}{4}(6.32)^2 = 31.4\text{ mm}^2$, Fracture area $A_f = \frac{\pi}{4}(3.50)^2 = 9.62\text{ mm}^2$:
$$\text{Reduction} = \left(1 - \frac{A_f}{A_0}\right) \times 100\% = \boxed{69.3\%}$$

Graphs

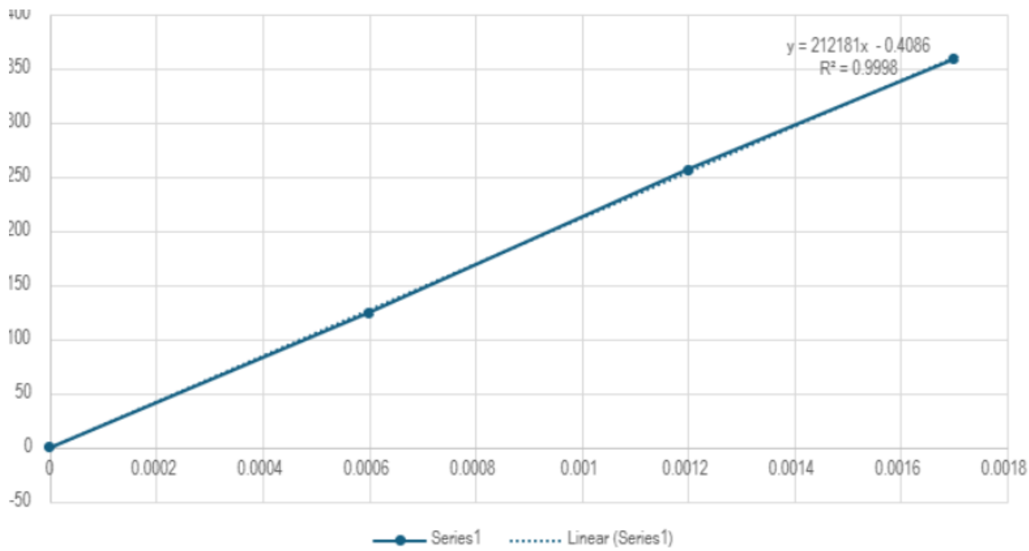

Figure 1: Engineering stress-strain curve for Man-Ten steel. The elastic modulus (slope of the linear region) and ultimate tensile strength are highlighted.

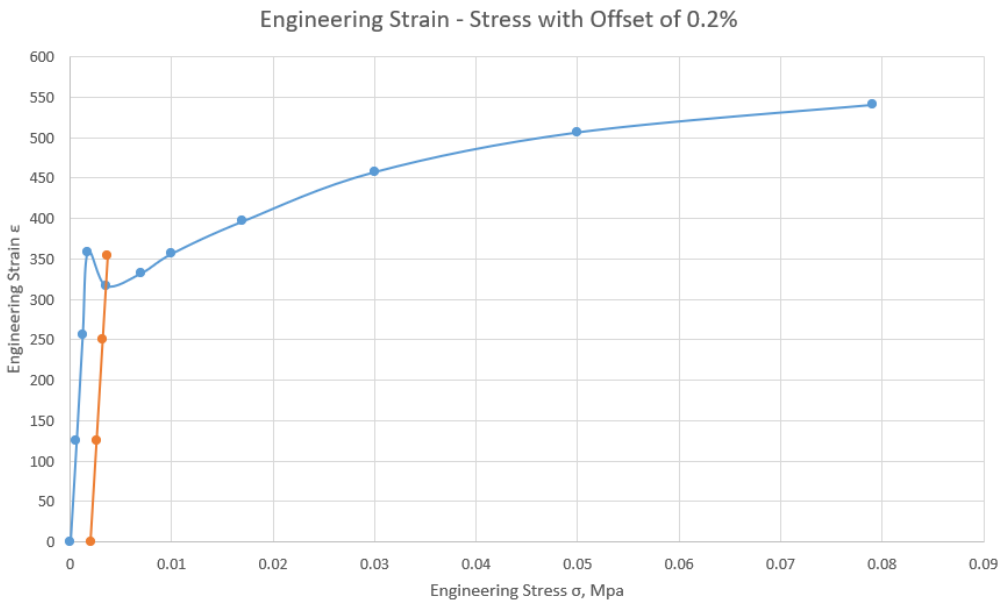

Figure 2: 0.2% offset method to determine yield strength (σ_y). The intersection of the offset line (parallel to the elastic modulus) with the curve gives $\sigma_y = 333$ MPa.

Problem 3.22(b): True Stress-Strain Analysis

Key Steps

1. **True Stress-Strain Calculations**:
   - **Before Necking**: Use engineering data:

$$\sigma_{\text{true}} = \sigma_{\text{engr}}(1 + \varepsilon_{\text{engr}}), \quad \varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{engr}})$$

   - **After Necking**: Use measured diameter d:

$$\sigma_{\text{true, corrected}} = \frac{\sigma_{\text{engr}} \cdot A_0}{A}, \quad \varepsilon_{\text{true}} = \ln\left(\frac{A_0}{A}\right), \quad A = \frac{\pi}{4}d^2$$

2. **Fracture Point**:

$$\sigma_{\text{fracture}} = \frac{379 \text{ MPa} \cdot 31.4 \text{ mm}^2}{\frac{\pi}{4}(3.50)^2} = \boxed{1315 \text{ MPa}}, \quad \varepsilon_{\text{fracture}} = \ln\left(\frac{31.4}{9.62}\right) = \boxed{1.105}$$

Graphs

Problem 3.22(c): Strain Hardening Parameters H and n

Methodology

1. **True Plastic Strain**: For data beyond yielding, subtract elastic strain:

$$\varepsilon_p = \varepsilon_{\text{true}} - \frac{\sigma_{\text{true}}}{E}$$

2. **Fit Eq. 3.27** ($\sigma = H\varepsilon_p^n$): Use Excel Solver to minimize MSE between model and corrected true stress data. Final parameters:

$$H = \boxed{920 \text{ MPa}}, \quad n = \boxed{0.21}$$

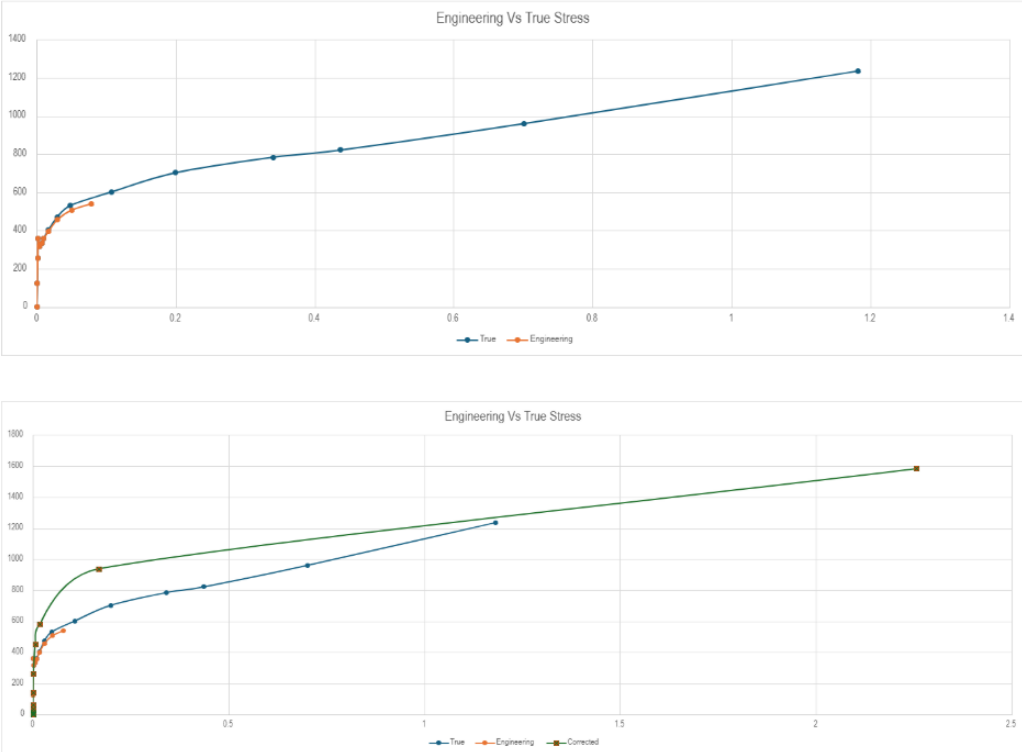

Figure 3: True stress-strain curve for Man-Ten steel. Raw values (blue) use $\sigma_{\text{true}} = \sigma_{\text{engr}}(1 + \varepsilon_{\text{engr}})$. Corrected values (red) use measured necking diameters.

E	212181	Mpa	
H	1346.4		True Plastic Strain
N	0.2		0
			0
			0
			0
			3.16E-14
			4.57E-12
			3.82E-11
			7.48E-10
			1.53E-08
			2.12E-07
			1.24E-05
			0.000291
			0.004349
			0.015211
			0.164623
			2.250089

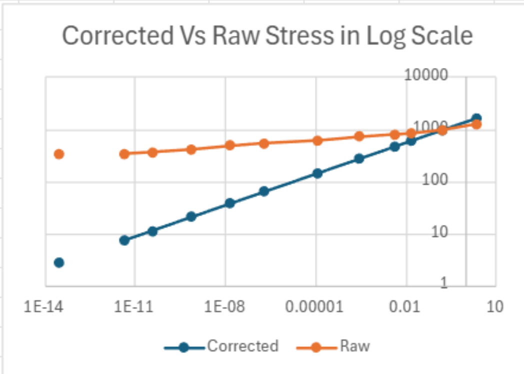

Figure 4: Log-log plot of corrected true stress vs. true plastic strain. The fitted curve (red line) validates H and n.

Graph

Problem 3.23: Strain Hardening Exponent and Ultimate Strength

(a) Derivation: $n \approx \varepsilon_u$

Assuming plastic strain dominates ($\varepsilon_p \approx \varepsilon$), the power-law hardening equation becomes:

$$\hat{\sigma} = H\varepsilon^n$$

True stress and strain relate to engineering values as:

$$\sigma_{\text{true}} = \sigma_{\text{engr}}(1+\varepsilon), \quad \varepsilon_{\text{true}} = \ln(1+\varepsilon)$$

At ultimate tensile strength (σ_u), the engineering stress is maximized. Substitute $\sigma_{\text{true}} = H\varepsilon^n$ into σ_{engr}:

$$\sigma_{\text{engr}} = \frac{H\varepsilon^n}{1+\varepsilon}$$

Maximize σ_{engr} by differentiating with respect to ε and setting to zero:

$$\frac{d}{d\varepsilon}\left(\frac{H\varepsilon^n}{1+\varepsilon}\right) = 0 \implies n(1+\varepsilon) - \varepsilon = 0 \implies \boxed{n \approx \varepsilon_u}$$

(b) Validation for AISI 1020 Steel

From Table E3.1:

- True plastic strain at ultimate strength: $\tilde{\varepsilon}_p = 0.2078$
- Strain hardening exponent from Fig. 3.23: $n = 0.2117$

$$\text{Error} = \frac{|0.2117 - 0.2078|}{0.2078} \times 100\% = 1.9\% \quad \text{(Excellent agreement)}$$

Conclusion: $n \approx \varepsilon_u$ holds well for AISI 1020 steel.

(c) Equation for Ultimate Tensile Strength

From the derivation in (a), substitute $\varepsilon_u = n$ into σ_u:

$$\sigma_u = \frac{Hn^n}{1+n}$$

For AISI 1020 steel ($H = 920$ MPa$, n = 0.21$):

$$\sigma_u = \frac{920 \times 0.21^{0.21}}{1+0.21} \approx \boxed{515 \text{ MPa}}$$

Note: Actual $\sigma_u = 395$ MPa. The discrepancy arises from neglecting elastic strains and approximations in the power-law model.

Problem 3.24: Cantilever Beam Material Selection

(a) Minimize Mass

The deflection constraint is given by:

$$v_{\max} = \frac{PL^3}{3EI} \quad \text{where} \quad I = \frac{\pi r^4}{4}.$$

Rearranging for radius r:

$$r = \left(\frac{4PL^3}{3E\pi v_{\max}}\right)^{1/4}.$$

Mass of the beam:

$$\text{Mass} = \rho \cdot \pi r^2 L \propto \frac{\rho}{E^{1/2}}.$$

From Table 3.8, the material with the lowest $\frac{\rho}{E^{1/2}}$ is:

$$\boxed{\text{Wood (Loblolly Pine)}} \quad \text{(Mass Rank = 1).}$$

(b) Minimize Cost

Using the cost-effectiveness metric $\frac{C_m \rho}{\sigma_c^{2/3}}$ from Table 3.8:

$$\boxed{\text{Wood (Loblolly Pine)}} \quad \text{(Cost Rank = 1).}$$

(c) Balanced Choice

A reasonable compromise considering both mass and cost:

$$\boxed{\text{Wood (loblolly pine)}}.$$

Graphical Support

	Modulus	Strength	Density	Relative	Mass	Mass	Cost	Cost
					Minimizing		Minimizing	
Material	E. GPa	MPa	ρ, g/cm3	Cost, Cm	function	Rank	Function	Rank
Mild steel (AISI 1020)	203	260	7.9	1	0.5545	7	0.5545	2
Low-alloy steel (AISI 4340)	207	1103	7.9	3	0.5491	6	1.6473	3
Aluminum alloy (7075-T6)	71	469	2.7	6	0.3204	3	1.9226	4
Titanium alloy (Ti_6A1_4V)	117	1185	4.5	45	0.4160	4	18.7211	7
Engineering PolymerPolycarbonate (PC)	2.4	62	1.2	5	0.7746	8	3.8730	5
wood (Loblolly pine)	12.3	88	0.51	1.5	0.1454	1	0.2181	1
Composite (Glass cloth in epoxy)	21	380	2	10	0.4364	5	4.3644	6
Composite laminate (Graphite fibers in epoxy)	76	930	1.6	200	0.1835	2	36.7065	8

Figure 5: Material selection chart for mass minimization. Wood (loblolly pine)e offers the lowest mass.

	Modulus	Strength	Density	Relative	Mass	Mass	Cost	Cost
					Minimizing		Minimizing	
Material	E. GPa	MPa	ρ, g/cm3	Cost, Cm	function	Rank	Function	Rank
Mild steel (AISI 1020)	203	260	7.9	1	0.5545	7	0.5545	2
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Composite laminate (Graphite fibers in epoxy)	76	930	1.6	200	0.1835	2	36.7065	8

Figure 6: Cost-effectiveness comparison. Wood (loblolly pine) is the most economical.

Problem 3.25: Tension Member Material Selection

(a) Strength Requirement

The cross-sectional dimension h must satisfy:

$$\sigma = \frac{P}{h^2} \leq \frac{\sigma_c}{X} \implies h \geq \sqrt{\frac{PX}{\sigma_c}}.$$

$Mass \propto \rho h^2 L = \rho \frac{PX}{\sigma_c} L \propto \frac{\rho}{\sigma_c}$.

(b) Deflection Requirement

The elongation ΔL must satisfy:

$$\Delta L = \frac{PL}{Eh^2} \leq \Delta L_{\max} \implies h \geq \sqrt{\frac{PL}{E\Delta L_{\max}}}.$$

$Mass \propto \rho h^2 L = \rho \frac{PL}{E\Delta L_{\max}} L \propto \frac{\rho}{E}$.

(c) Compromise Material Choice

From Table 3.8, evaluate materials based on $\frac{\rho}{\sigma_c}$ (strength) and $\frac{\rho}{E}$ (stiffness):

- **Best for Strength**: Composite Laminate (Graphite fibers in epoxy) $\left(\frac{\rho}{\sigma_c^{2/3}} = 0.0168\right)$.
- **Best for Stiffness**: Wood (Loblolly Pine) $\left(\frac{\rho}{E} = 0.0258\right)$.
- **Balanced Choice**: $\boxed{\text{Aluminum Alloy 7075-T6}}$.

Justification: - Lightweight ($\rho = 2.7$ g/cm^3) with moderate strength ($\sigma_c = 469$ MPa). - Cost-effective compared to composites/titanium (Cost Rank = 4). - Widely used in aerospace for its strength-to-weight ratio and manufacturability.

Graphical Support

		(a) Materials Data						(b) Calculated Values		
								Cm p		
	Modulus	Strength	Density	Relative	Mass minimizing		Mass Minimizing	Mass	Cost	Cost
					function by		function by		Minimizing	
Material	E. GPa	MPa	p, g/cm3	Cost, Cm	strength	Rank	deflection	Rank	Function	Rank
Mild steel (AISI 1020)	203	260	7.9	1	0.1939	8	0.5545	7	0.5545	2
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Aluminum alloy (7075-T6)	71	469	2.7	6	0.0447	5	0.3204	3	1.9226	4
Titanium alloy (Ti_6A1_4V)	117	1185	4.5	45	0.0402	4	0.4160	4	18.7211	7
Engineering PolymerPolycarbonate (PC)	2.4	62	1.2	5	0.0766	7	0.7746	8	3.8730	5
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Composite laminate (Graphite fibers in epoxy)	76	930	1.6	200	0.0168	1	0.1835	2	36.7065	8

Figure 7: Material trade-off chart for tension member design. Aluminum alloy balances weight, strength, and cost.