Mechanics of Materials Homework

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5.4 An aluminum alloy is represented by the elastic, linear hardening model of Fig. 5.3(c), with constants of E_1 = 70 GPa, E_2 = 2.5 GPa, and σ_o = 300MPa. Plot the stress–strain response for loading to a strain of ε = 0.016. Of this total strain, how much is elastic, and how much is plastic?

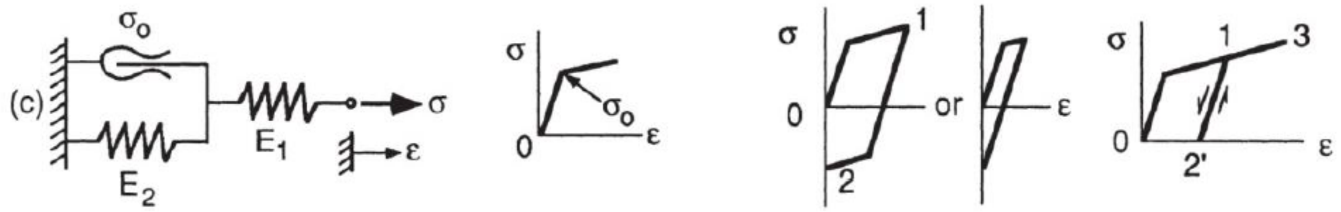

Figure 5.3 Rheological models for plastic deformation and their responses to three different strain inputs. Model (c) elastic, linear hardening.

Given:

- Elastic modulus: $E_1 = 70\,\mathrm{GPa}$
- Hardening modulus: $E_2 = 2.5\,\mathrm{GPa}$
- Yield strength: $\sigma_o = 300\,\mathrm{MPa}$
- Total strain: $\varepsilon = 0.016$

Solution Outline

The total strain ε can be broken into:

$$\varepsilon = \varepsilon_e + \varepsilon_p,$$

where ε_e is elastic strain and ε_p is plastic strain.

1) Elastic Phase ($\sigma \leq \sigma_o$):

$$\varepsilon_{\text{elastic}} = \frac{\sigma}{E_1}.$$

At $\sigma = \sigma_o = 300\,\mathrm{MPa}$, the elastic strain is

$$\varepsilon_{\text{elastic}} = \frac{300}{70{,}000} = 0.004286.$$

Since our total strain is 0.016 (which is greater than 0.004286), the material has yielded.

2) Linear-Hardening Phase ($\sigma > \sigma_o$): Beyond yield, the stress–strain curve follows:

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma - \sigma_o}{E_2}.$$

Rearranging to solve for σ,

$$\sigma = \frac{E_1 E_2}{E_1 + E_2}\,\varepsilon \; + \; \frac{\sigma_o\, E_2}{E_1 + E_2}.$$

Plug in the given values: $E_1 = 70\,\text{GPa}$, $E_2 = 2.5\,\text{GPa}$, $\sigma_o = 300\,\text{MPa}$, and $\varepsilon = 0.016$. Numerically:

$$\sigma = \frac{(70)(2.5)}{72.5}\,(0.016) + \frac{300 \times 2.5}{72.5} \approx 328.3\,\text{MPa}.$$

Strain Decomposition:

$$\varepsilon_e = \frac{\sigma}{E_1} = \frac{328.3}{70{,}000} \approx 0.00469, \quad \varepsilon_p = \varepsilon - \varepsilon_e = 0.016 - 0.00469 = 0.01131.$$

Numerical Results

$$\boxed{\begin{aligned} \sigma &\approx 328.3\,\text{MPa}, \\ \varepsilon_e &\approx 0.00469, \\ \varepsilon_p &\approx 0.01131. \end{aligned}}$$

The elastic portion of strain at $\varepsilon = 0.016$ is about 0.00469, so over 70% of the deformation is plastic at that point.

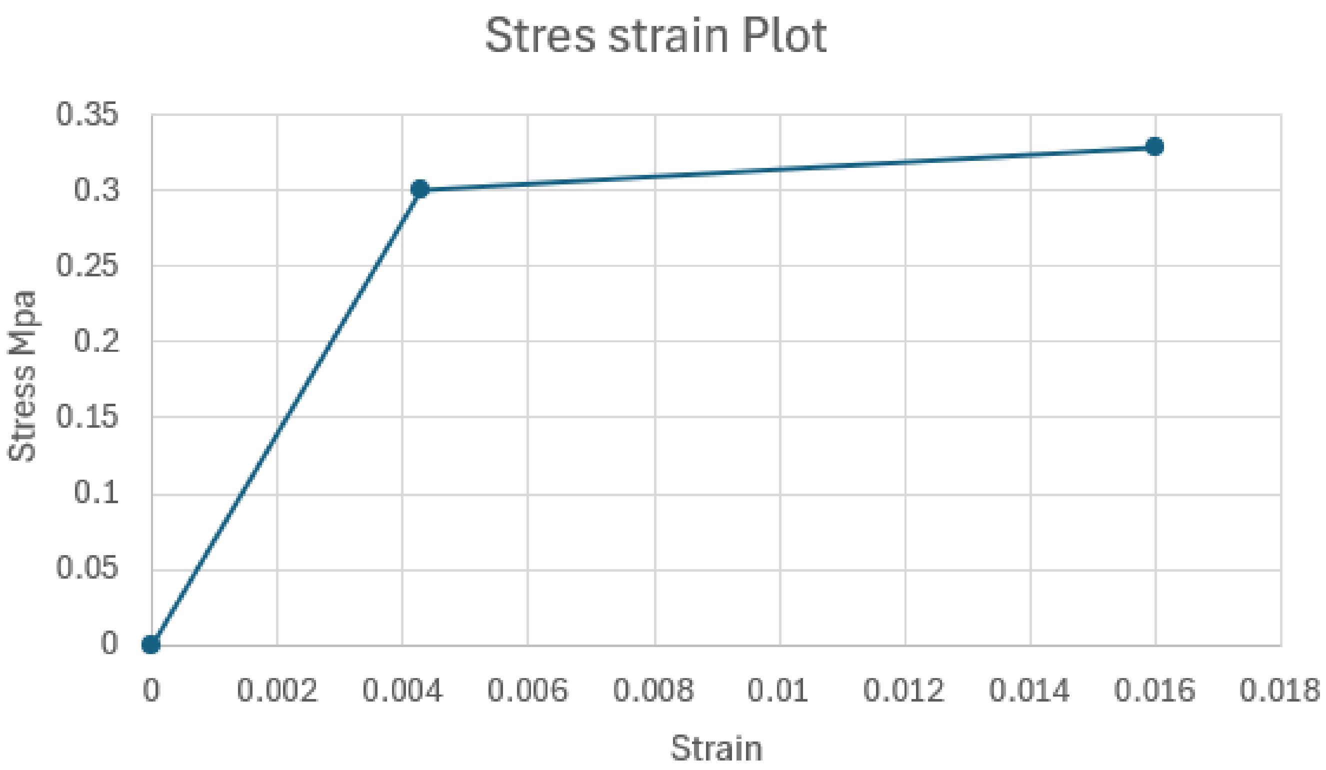

Figure 1: Stress–strain plot for the elastic–linear hardening model. The yield point is at 300 MPa, and beyond that the slope is $E_2 = 2.5\,\text{GPa}$.

5.7 A polymer has constants for the elastic, transient creep model of Fig. 5.5(b) of $E_1 = 4.0$ GPa, $E_2 = 5.0$ GPa, and $\eta_2 = 1.20\times10^5$ GPa·s. Determine and plot the strain versus time response for a stress of 55 MPa applied for one day.

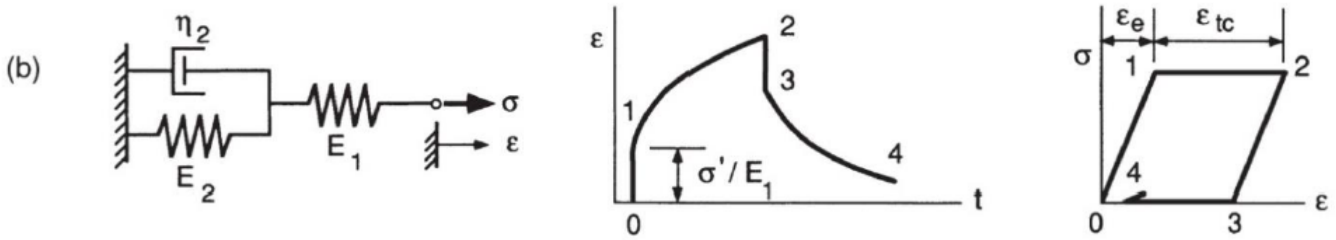

Figure 5.5 Rheological models having time-dependent behavior and their responses to a stress–time step. Both strain–time and stress–strain responses are shown. Model (b) transient creep with elastic strain added.

Given:

- $E_1 = 4\,\text{GPa},\ \ E_2 = 5\,\text{GPa}$
- $\eta_2 = 1.2 \times 10^5\,\text{GPa s}$
- Applied stress: $\sigma = 55\,\text{MPa}$
- Time: 1 day $= 86\,400\,\text{s}$

Solution:

Total strain equation (Eq. 5.17):

$$\varepsilon(t) = \underbrace{\frac{\sigma}{E_1}}_{\text{Elastic}} + \underbrace{\frac{\sigma}{E_2}\left(1 - e^{-E_2 t/\eta_2}\right)}_{\text{Transient Creep}}$$

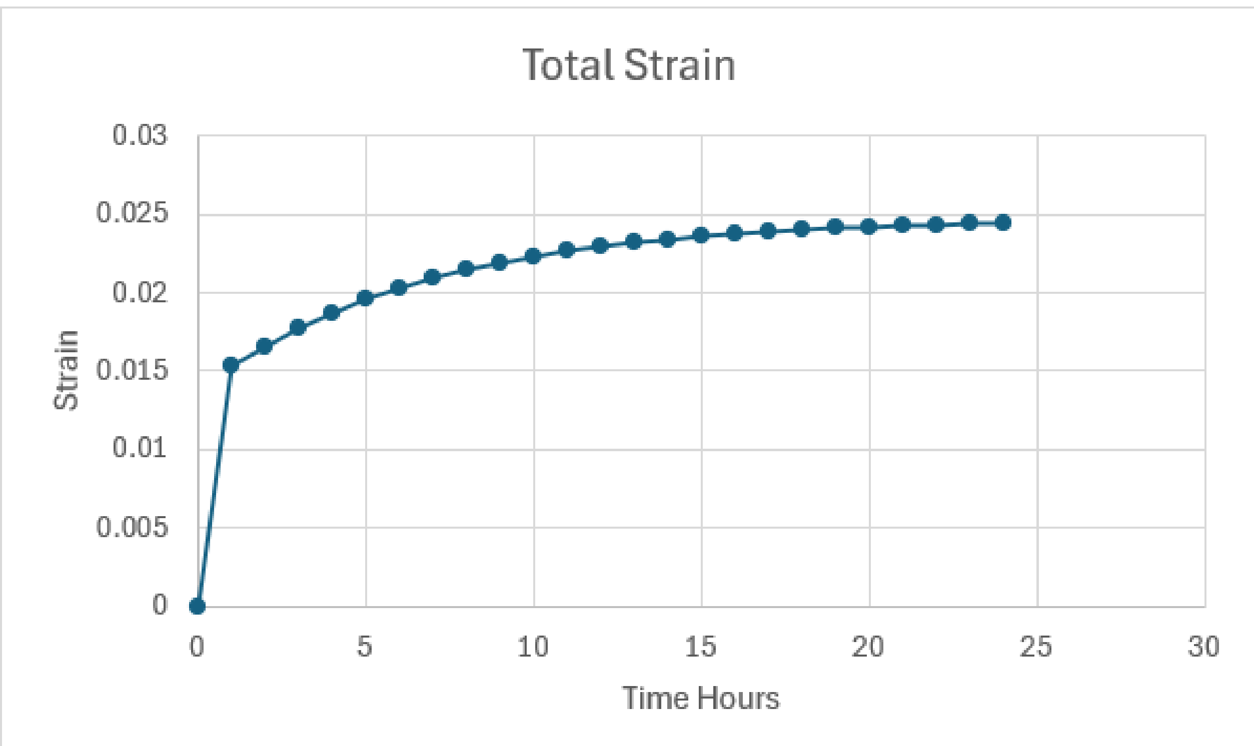

Figure 2: Total Strain Plot

Problem 5.11: Tensile Loading Analysis

5.11 A bar of a high-strength aluminum alloy is 200 mm long and has a circular cross section of diameter 50 mm. It is subjected to a tensile load of 250 kN, which gives a stress **well below the material's yield strength**. Determine the following: (a) stress in the length direction, (b) strain in the length direction, (c) strain in the transverse direction, (d) length while under load, and (e) diameter while under load. (**Young's modulus and Poisson's ratio refer to Table 5.2)**

Given:

- Aluminum bar: $L_0 = 200\,\text{mm},\ d_0 = 50\,\text{mm}$
- Load: $P = 250\,\text{kN}$
- Material: Aluminum alloy (Table 5.2)
  - $E = 70.3\,\text{GPa}$
  - $\nu = 0.345$

Solution:

1. **Cross-sectional Area:**

$$A_0 = \frac{\pi d_0^2}{4} = \frac{\pi(50)^2}{4} = 1963.5\,\text{mm}^2$$

2. **Axial Stress:**

$$\sigma_x = \frac{P}{A_0} = \frac{250000}{1963.5} = 127.3\,\text{MPa}$$

3. **Axial Strain:**

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{127.3}{70300} = 0.001811$$

4. **Transverse Strain:**

$$\varepsilon_y = -\nu\varepsilon_x = -0.345 \times 0.001811 = -0.000625$$

5. **New Dimensions:**

$$L = L_0(1 + \varepsilon_x) = 200 \times 1.001811 = 200.36\,\text{mm}$$
$$d = d_0(1 + \varepsilon_y) = 50 \times 0.999375 = 49.97\,\text{mm}$$

Problem 5.13: Plane Stress Analysis

5.13 For the special case of plane stress, $\sigma_z = \tau_{yz} = \tau_{zx} = 0$, proceed as follows:

(a) Write the resulting simplified version of Hooke's law, Eqs. 5.26 and 5.27.

(b) Then invert the simplified forms of Eq. 5.26(a) and (b) to obtain relationships that give the stresses σ_x and σ_y, each as a function of strains and materials constants only.

(c) Also derive the equation that gives ε_z as a function of the other two strains and materials constants.

Given: Plane stress condition ($\sigma_z = \tau_{yz} = \tau_{zx} = 0$)

Part (a): Simplified Hooke's Law

Objective: Derive simplified stress-strain relationships under plane stress.

Step 1: Start with general 3D Hooke's Law

$$\begin{cases} \varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \\ \gamma_{xy} = \frac{\tau_{xy}}{G} \\ \gamma_{yz} = \frac{\tau_{yz}}{G} \\ \gamma_{zx} = \frac{\tau_{zx}}{G} \end{cases}$$

Step 2: Apply plane stress conditions:

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

Substitute $\sigma_z = 0$ into equations:

$$\begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) && \text{(Normal strain in x)} \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) && \text{(Normal strain in y)} \\ \varepsilon_z &= -\frac{\nu}{E}(\sigma_x + \sigma_y) && \text{(Out-of-plane strain)} \\ \gamma_{xy} &= \frac{\tau_{xy}}{G} && \text{(Shear strain)} \\ \gamma_{yz} &= \gamma_{zx} = 0 && \text{(Zero shear strains)} \end{aligned}$$

Part (b): Stress-Strain Relationships

Objective: Express σ_x and σ_y in terms of strains.

Step 1: Equations (1) and (2) from Part (a) as a system:

$$\begin{cases} E\varepsilon_x = \sigma_x - \nu\sigma_y \\ E\varepsilon_y = \sigma_y - \nu\sigma_x \end{cases}$$

Step 2: Arranging in matrix form:

$$\begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = E \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}$$

Step 3: Finding matrix inverse:

$$\frac{1}{1-\nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix}$$

Why: Matrix inversion is required to solve for σ_x and σ_y

Step 4: Multiply both sides by inverse matrix:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}$$

Step 5: Perform matrix multiplication:

$$\begin{aligned} \sigma_x &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \\ \sigma_y &= \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \end{aligned}$$

Part (c): Out-of-Plane Strain ε_z

Objective: Find relationship for ε_z in terms of ε_x and ε_y

Step 1: Start with result from Part (a):

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Step 2: Substitute σ_x and σ_y from Part (b):

$$\sigma_x + \sigma_y = \frac{E}{1-\nu^2}[\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x]$$

Step 3: Factor common terms:

$$= \frac{E(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y)$$

Step 4: Simplify denominator using difference of squares:

$$1 - \nu^2 = (1-\nu)(1+\nu)$$

$$= \frac{E}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

Step 5: Substitute back into ε_z equation:

$$\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

Key Results

(a) Simplified Hooke's Law:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

(b) Stress Solutions:

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

(c) Out-of-Plane Strain:

$$\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

5.17 Strains are measured on the surface of a mild steel part as follows: $\varepsilon_x = 190 \times 10^{-6}$, $\varepsilon_y = -760 \times 10^{-6}$, and $\gamma_{xy} = 300 \times 10^{-6}$. Estimate in-plane stresses σ_x, σ_y, and τ_{xy}, and also the strain ε_z normal to the surface. (The same assumptions apply as for Prob. 5.14. i.e., Assume that the gages were bonded to the metal when there was no load on the part, that there has been no yielding, and that no loading is applied directly to the surface, so that $\sigma_z = \tau_{yz} = \tau_{zx} = 0$.)

(a) In-Plane Stresses

$$\begin{aligned}\sigma_x &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)\\ &= \frac{212\,\text{GPa}}{1-0.293^2}\,(190 + 0.293(-760)) \times 10^{-6}\\ &= -7.54\,\text{MPa}\,(\text{Compressive})\end{aligned}$$

$$\begin{aligned}\tau_{xy} &= G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}\\ &= \frac{212\,\text{GPa}}{2(1+0.293)} \times 300 \times 10^{-6}\\ &= 24.62\,\text{MPa}\end{aligned}$$

(b) Normal Strain ε_z

$$\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = 227$$

Summary of Results

$$\boxed{\begin{aligned} \sigma_x &= -7.54\,\mathrm{MPa}\,(\text{Compressive}) \\ \tau_{xy} &= 24.62\,\mathrm{MPa} \\ \varepsilon_z &= 227 \end{aligned}}$$