Advanced Machine Design

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5.20 A thin-walled *spherical* vessel contains a pressure p and has inner radius r and wall thickness t. It is made of an isotropic material that behaves in a linear-elastic manner. Determine the each of following as a function of the pressure, geometric dimensions, and material constants involved: (a) change in radius, Δr, and (b) change in wall thickness, Δt.

Step-by-Step Solution

Given: - Internal pressure p, inner radius r, wall thickness t. - Material constants: E (elastic modulus), ν (Poisson's ratio).

Objective: Find Δr and Δt.

Step 1: Circumferential Stress

For a thin-walled sphere, the hoop stress is:

$$\sigma = \frac{pr}{2t}$$

Step 2: Circumferential Strain (Change in Radius)

Using Hooke's Law for biaxial stress ($\sigma_x = \sigma_y = \sigma$, $\sigma_z = 0$):

$$\varepsilon_\theta = \frac{1}{E}[\sigma - \nu(\sigma + 0)] = \frac{\sigma(1-\nu)}{E}$$

Substitute $\sigma = \frac{pr}{2t}$:

$$\varepsilon_\theta = \frac{pr(1-\nu)}{2tE}$$

The change in radius is:

$$\Delta r = \varepsilon_\theta \cdot r = \boxed{\frac{pr^2(1-\nu)}{2tE}}$$

Step 3: Radial Strain (Change in Thickness)

Poisson's effect causes radial strain:

$$\varepsilon_r = -\frac{\nu}{E}(\sigma + \sigma) = -\frac{2\nu\sigma}{E}$$

Substitute $\sigma = \frac{pr}{2t}$:

$$\varepsilon_r = -\frac{\nu pr}{tE}$$

The change in thickness is:

$$\Delta t = \varepsilon_r \cdot t = \boxed{-\frac{\nu pr}{E}}$$

5.25 A block of isotropic material is stressed in the x- and y-directions, but rigid walls prevent deformation in the z-direction, as shown in Fig. P5.25. The ratio of the two applied stresses is a constant, so that $\sigma_y = \lambda\sigma_x$.

(a) Does a stress develop in the z-direction? If so, obtain an equation for σ_z as a function of σ_x, λ, and the elastic constant ν for the material.

(b) Determine the stiffness $E' = \sigma_x / \varepsilon_x$ for the x-direction as a function of only λ and the elastic constants E and ν for the material.

(c) Compare this apparent modulus E' with the elastic modulus E as obtained from a uniaxial test. (Suggestion: Assume that $\nu = 0.3$ and consider λ values of -1, 0, and 1.)

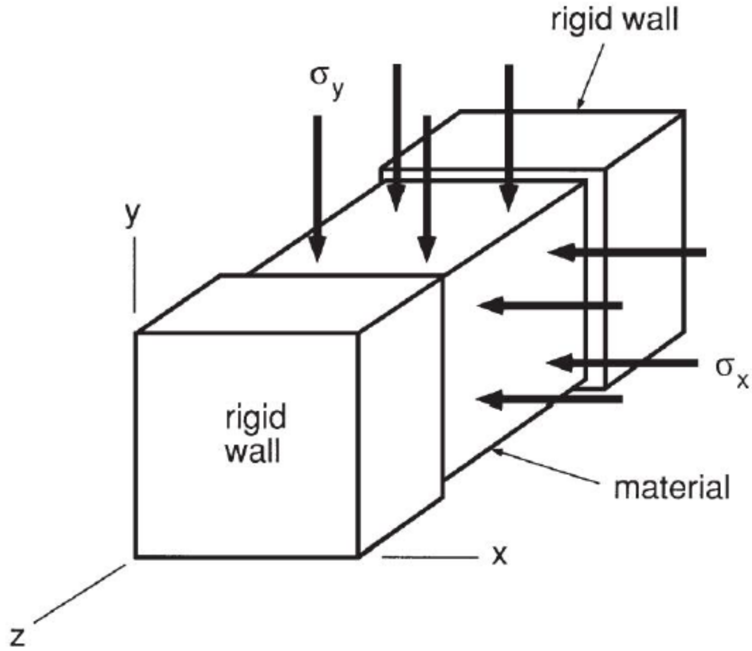


Figure P5.25

Step-by-Step Solution

Given: - Stresses: σ_x, $\sigma_y = \lambda\sigma_x$. - Constraint: $\varepsilon_z = 0$.

Part (a): Stress in z-Direction

From Hooke's Law for ε_z:

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = 0$$

Substitute $\sigma_y = \lambda\sigma_x$:

$$\sigma_z = \nu(\sigma_x + \lambda\sigma_x) = \boxed{\nu(1+\lambda)\sigma_x}$$

Part (b): Apparent Stiffness $E' = \sigma_x/\varepsilon_x$

From Hooke's Law for ε_x:

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

Substitute $\sigma_y = \lambda\sigma_x$ and $\sigma_z = \nu(1+\lambda)\sigma_x$:

$$\varepsilon_x = \frac{\sigma_x}{E}\left[1 - \nu\lambda - \nu^2(1+\lambda)\right]$$

Apparent stiffness:

$$E' = \frac{\sigma_x}{\varepsilon_x} = \boxed{\frac{E}{1 - \nu\lambda - \nu^2(1+\lambda)}}$$

Part (c): Comparison with $\nu = 0.3$

$$\begin{aligned}
\lambda = -1: \quad E' &= \frac{E}{1-(-0.3)-0.09(0)} = \frac{E}{1.3} \approx 0.77E \quad \text{(Softer)} \\
\lambda = 0: \quad E' &= \frac{E}{1-0-0.09} = \frac{E}{0.91} \approx 1.10E \quad \text{(Stiffer)} \\
\lambda = 1: \quad E' &= \frac{E}{1-0.3-0.18} = \frac{E}{0.52} \approx 1.92E \quad \text{(Much Stiffer)}
\end{aligned}$$

5.30 Equation 5.41 is sometimes used as a basis for making a preliminary comparison of the *thermal shock* resistance of ceramic materials by calculating the maximum ΔT that can occur without the material reaching its ultimate strength. The compressive ultimate σ_{uc} applies for a temperature increase (upward shock), and the tensile ultimate σ_{ut} applies for a temperature decrease (downward shock). Coefficients of thermal expansion, α, and Poisson's ratio, ν, for some of the ceramic materials of Table 3.4 are given in Table P5.30.

(a) Calculate $\Delta T_{\max}$ for each ceramic for both upward shock and downward shock.

(b) Briefly discuss the trends observed. Include your opinion and supporting logic as to which of these materials might be the best choice for high temperature engine parts, such as turbine

blades, where rapid temperature changes occur.

$$\sigma_x = \sigma_y = -\frac{E\alpha(\Delta T)}{1-\nu} \qquad (5.41)$$

Step-by-Step Solution

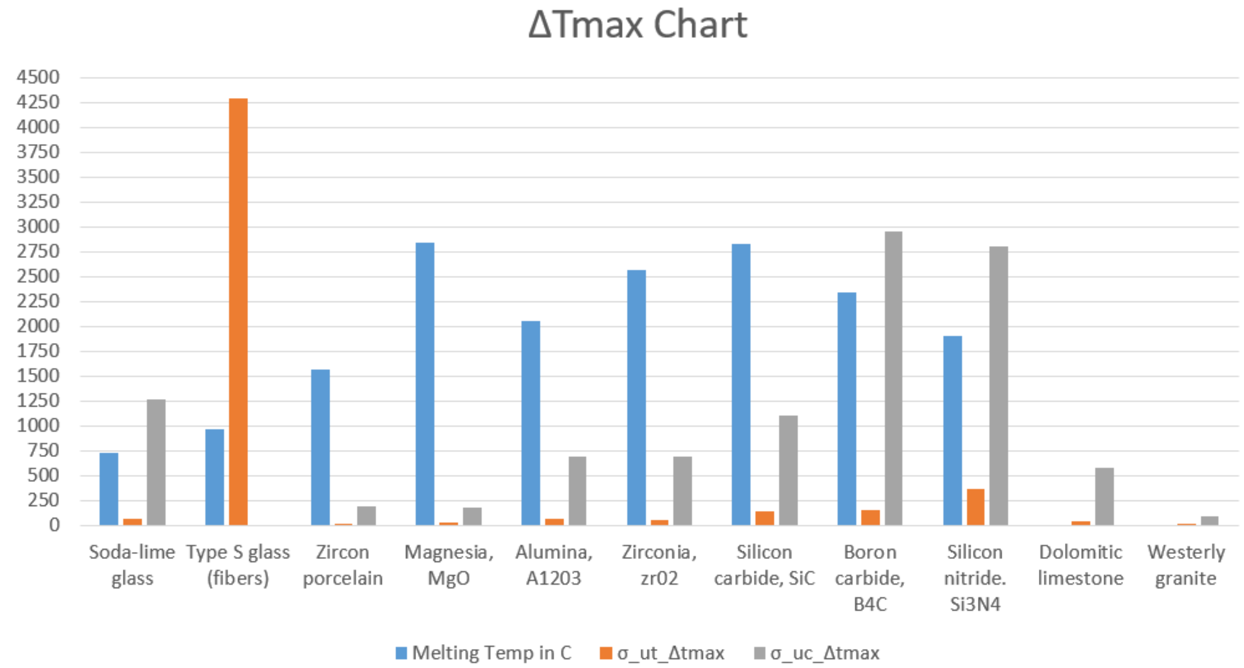
Given: - Thermal stress formula: $\sigma_x = \sigma_y = -\frac{E\alpha\Delta T}{1-\nu}$. - Ceramic properties (see Excel table).

Part (a): Calculate $\Delta T_{\max}$

Rearrange the formula to solve for ΔT:

$$\Delta T_{\max} = \frac{\sigma_u(1-\nu)}{E\alpha}$$

- For tension ($\sigma_u = \sigma_{ut}$): Downward shock (cooling). - For compression ($\sigma_u = \sigma_{uc}$): Upward shock (heating).



Part (b): Material Selection for Turbine Blades

Best Choice: Silicon Nitride (Si_3N_4) - High ΔT_{max} in tension (365.41°C) and compression (2801.45°C). - High melting temperature (1900°C) and strength.

5.35 A composite material is made with an aluminum alloy matrix and 25%, by volume, of unidirectional SiC fibers. Estimate the elastic constants E_X, E_Y, G_{XY}, ν_{XY}, and ν_{YX}.

Table 5.2 Elastic Constants for Various Materials at Ambient Temperature

Material	Elastic Modulus E, GPa (10^3 ksi)		Poisson's Ratio ν
(a) Metals			
Aluminum	70.3	(10.2)	0.345
Brass, 70Cu-30Zn	101	(14.6)	0.350
Copper	130	(18.8)	0.343
Iron; mild steel	212	(30.7)	0.293
Lead	16.1	(2.34)	0.44
Magnesium	44.7	(6.48)	0.291
Stainless steel, 2Ni-18Cr	215	(31.2)	0.283
Titanium	120	(17.4)	0.361
Tungsten	411	(59.6)	0.280
(c) Ceramics and glasses			
Alumina, Al_2O_3	400	(58.0)	0.22
Diamond	960	(139)	0.20
Magnesia, MgO	300	(43.5)	0.18
Silicon carbide, SiC	396	(57.4)	0.22
Fused silica glass	70	(10.2)	0.18
Soda-lime glass	69	(10.0)	0.20
Type E glass	72.4	(10.5)	0.22
Dolomitic limestone	69.0	(10.0)	0.281
Westerly granite	49.6	(7.20)	0.213

Given: - Composite material: 25% SiC fibers (by volume) in an aluminum alloy matrix. - Material properties:

- **SiC fibers (Table 5.2 (c))**: $E_f = 396\,\text{GPa}$, $\nu_f = 0.22$
- **Aluminum matrix (Table 5.2(a))**: $E_m = 70.3\,\text{GPa}$, $\nu_m = 0.345$

Objective: Estimate $E_x, E_y, G_{XY}, \nu_{XY}, \nu_{YX}$.

Step 1: Longitudinal Modulus (E_x)

Using the **rule of mixtures** for unidirectional composites:

$$E_x = E_f V_f + E_m V_m$$

Substitute $V_f = 0.25$, $V_m = 0.75$:

$$E_x = (396 \times 0.25) + (70.3 \times 0.75) = 99 + 52.725 = \boxed{151.7\,\text{GPa}}$$

Step 2: Transverse Modulus (E_y)

Using the **inverse rule of mixtures**:

$$\frac{1}{E_y} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\frac{1}{E_y} = \frac{0.25}{396} + \frac{0.75}{70.3} = 0.000631 + 0.010668 = 0.011299$$

$$E_y = \frac{1}{0.011299} = \boxed{88.5\,\text{GPa}}$$

Step 3: Shear Modulus (G_{XY})

First calculate shear moduli for fibers and matrix:

$$G_f = \frac{E_f}{2(1+\nu_f)} = \frac{396}{2 \times 1.22} = 162.3\,\text{GPa}, \quad G_m = \frac{E_m}{2(1+\nu_m)} = \frac{70.3}{2 \times 1.345} = 26.1\,\text{GPa}$$

Using the inverse rule of mixtures:

$$\frac{1}{G_{XY}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

$$\frac{1}{G_{XY}} = \frac{0.25}{162.3} + \frac{0.75}{26.1} = 0.00154 + 0.0287 = 0.03024$$

$$G_{XY} = \frac{1}{0.03024} = \boxed{33.1\,\text{GPa}}$$

Step 4: Poisson's Ratios

Using the **rule of mixtures** for ν_{XY}:

$$\nu_{XY} = \nu_f V_f + \nu_m V_m = (0.22 \times 0.25) + (0.345 \times 0.75) = \boxed{0.314}$$

Using the reciprocity relation for ν_{YX}:

$$\nu_{YX} = \nu_{XY} \times \frac{E_y}{E_x} = 0.314 \times \frac{88.5}{151.7} = \boxed{0.183}$$

Summary of Elastic Constants

$$E_x = 151.7\,\text{GPa}, \quad E_y = 88.5\,\text{GPa}, \quad G_{XY} = 33.1\,\text{GPa},$$
$$\nu_{XY} = 0.314, \quad \nu_{YX} = 0.183.$$

5.41 For unidirectional E-glass fibers used to reinforce epoxy, employ the reinforcement and matrix properties given in Table 5.3(a), and in the note below the table, to estimate E_X and E_Y for several volume fractions of reinforcement ranging from zero to 100%. Plot curves of E_X versus V_r, and E_Y versus V_r, on the same graph, and comment on the trends.

Table 5.3 Elastic Constants and Density for Fiber-Reinforced Epoxy with 60% Unidirectional Fibers by Volume

(a) Reinforcement			(b) Composite, $V_r = 0.60$				
Type	E_r GPa (10^3 ksi)	ν_r	E_X GPa (10^3 ksi)	E_Y GPa (10^3 ksi)	G_{XY} GPa (10^3 ksi)	ν_{XY}	ρ g/cm^3
E-glass	72.3 (10.5)	0.22	45 (6.5)	12 (1.7)	4.4 (0.64)	0.25	1.94
Kevlar 49	124 (18.0)	0.35	76 (11.0)	5.5 (0.8)	2.1 (0.3)	0.34	1.30
Graphite (T-300)	218 (31.6)	0.20	132 (19.2)	10.3 (1.5)	6.5 (0.95)	0.25	1.47
Graphite (GY-70)	531 (77.0)	0.20	320 (46.4)	5.5 (0.8)	4.1 (0.6)	0.25	1.61

Step-by-Step Solution

Given: - Unidirectional E-glass fibers in an epoxy matrix. - Volume fractions V_r ranging from 0 to 100%. - Material properties (Tables 5.2 and 5.3):

- **E-glass fibers**: $E_f = 72.3\,\text{GPa}$, $\nu_f = 0.22$
- **Epoxy matrix**: $E_m = 3.5\,\text{GPa}$, $\nu_m = 0.33$

Objective: Estimate E_x and E_y for varying V_r and plot the results.

Key Formulas

1. **Longitudinal Modulus (E_x):**

$$E_x = E_f V_r + E_m(1 - V_r)$$

2. **Transverse Modulus (E_y):**

$$\frac{1}{E_y} = \frac{V_r}{E_f} + \frac{1 - V_r}{E_m}$$

Sample Calculation for $V_r = 0.6$:

$$E_x = (72.3 \times 0.6) + (3.5 \times 0.4) = 43.38 + 1.4 = \boxed{44.78\,\text{GPa}},$$

$$\frac{1}{E_y} = \frac{0.6}{72.3} + \frac{0.4}{3.5} = 0.0083 + 0.1143 = 0.1226 \implies E_y = \boxed{8.16\,\text{GPa}}.$$

Results for All Volume Fractions

5.41 For unidirectional E-glass fibers used to reinforce epoxy, employ the reinforcement and matrix properties given in Table 5.3(a), and in the note below the table, to estimate E_X and E_Y for several volume fractions of reinforcement ranging from zero to 100%. Plot curves of E_X versus V_r, and E_Y versus V_r, on the same graph, and comment on the trends.

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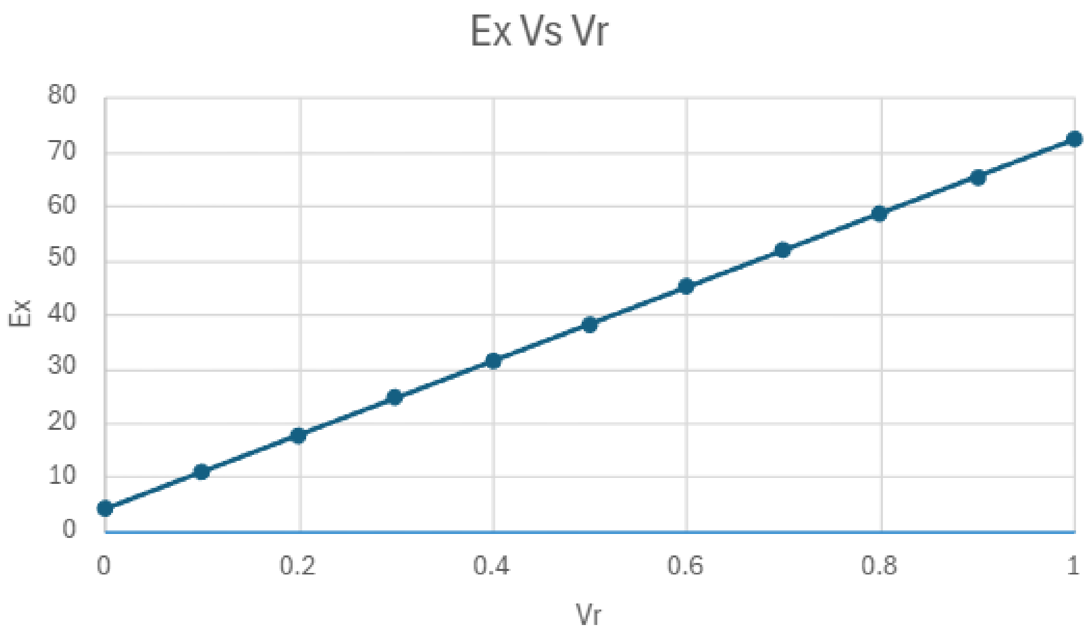
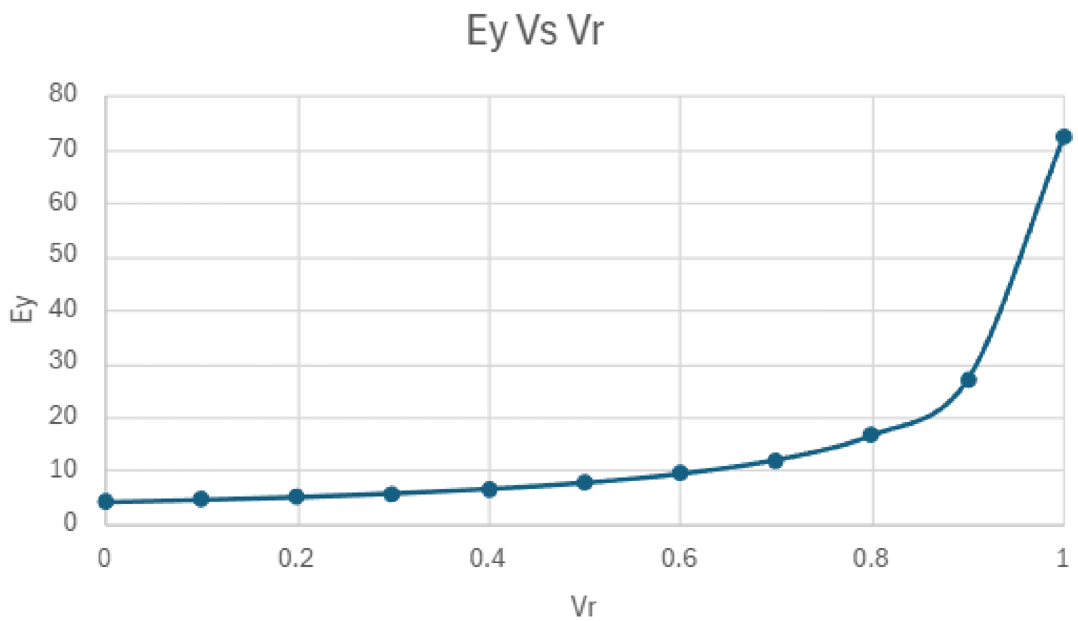
Plots of E_x and E_y vs V_r:





Trend Analysis

- **Longitudinal Modulus (E_x):** Increases linearly with V_r, dominated by the high E_f.
- **Transverse Modulus (E_y):** Increases non-linearly, heavily influenced by the low E_m.
- **Key Insight:** Fiber volume fraction significantly impacts E_x, while E_y is constrained by the matrix.