

Heat Transfer Assignments

Mechanical Engineering

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Problem 1: Estimating Overall Heat-Transfer Coefficient for Building Walls

Question:

A building with dimensions 24 ft by 40 ft and a height of 8 ft has a full basement with uninsulated walls extending 5 ft below grade. The insides of the walls are finished with R -8 insulation, a thin vapor barrier, and a 0.5 in gypsum board. Estimate the overall heat-transfer coefficient for the walls.

Solution:

1. Wall Structure Analysis

The walls consist of the following layers (from inside to outside):

1. Indoor air film resistance

$$R_{in} = 0.68 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

2. 0.5 in gypsum board

3. Vapor barrier

4. R -8 insulation

5. 8 in concrete wall

6. Outdoor air film resistance

$$R_{out} = 0.17 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1} \text{ (above grade)}$$

Soil resistance (below grade)

2. Material Thermal Resistances

- Gypsum board (0.5 in):

$$R_{gyp} = \frac{0.5 \text{ in}}{1.25 \text{ Btu in hr}^{-1} \text{ ft}^2 \text{ }^{\circ}\text{F}} = 0.4 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

- Vapor barrier:

$$R_{vb} = \frac{1}{16.70 \text{ W m}^{-2} \text{ K}} \approx 0.06 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

(Note: Since the vapor barrier's conductance C is given, we can convert units accordingly.)

- **R-8 insulation:**

$$R_{\text{ins}} = 8 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1}$$

- **Concrete wall (8 in):**

$$R_{\text{conc}} = \frac{8 \text{ in}}{12 \text{ Btu in hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F}} = 0.67 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1}$$

- **Soil resistance (below grade):**

$$R_{\text{soil}} \approx 1.5 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1}$$

3. Thermal Resistance Calculations

Above Grade (3 ft high):

$$\begin{aligned} R_{\text{total, above}} &= R_{\text{in}} + R_{\text{gyp}} + R_{\text{vb}} + R_{\text{ins}} + R_{\text{conc}} + R_{\text{out}} \\ &= 0.68 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.4 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.06 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \\ &\quad + 8 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.67 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.17 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \\ &= 9.98 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \end{aligned}$$

Below Grade (5 ft deep):

$$\begin{aligned} R_{\text{total, below}} &= R_{\text{in}} + R_{\text{gyp}} + R_{\text{vb}} + R_{\text{ins}} + R_{\text{conc}} + R_{\text{soil}} \\ &= 0.68 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.4 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.06 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \\ &\quad + 8 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 0.67 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} + 1.5 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \\ &= 11.31 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1} \end{aligned}$$

4. Calculating U-Values

Above Grade:

$$U_{\text{above}} = \frac{1}{R_{\text{total, above}}} = \frac{1}{9.98 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1}} \approx 0.1002 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F}$$

Below Grade:

$$U_{\text{below}} = \frac{1}{R_{\text{total, below}}} = \frac{1}{11.31 \text{ hr ft}^2 \text{ }^\circ\text{F Btu}^{-1}} \approx 0.0885 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F}$$

5. Area Calculations

- **Perimeter of the building:**

$$P = 2 \times (24 \text{ ft} + 40 \text{ ft}) = 128 \text{ ft}$$

- **Total wall area:**

$$A_{\text{total}} = P \times 8 \text{ ft} = 1024 \text{ ft}^2$$

- **Above-grade wall area:**

$$A_{\text{above}} = P \times 3 \text{ ft} = 384 \text{ ft}^2$$

- **Below-grade wall area:**

$$A_{\text{below}} = A_{\text{total}} - A_{\text{above}} = 640 \text{ ft}^2$$

6. Overall Heat-Transfer Coefficient

Using area-weighted averaging:

$$\begin{aligned}
 U_{\text{overall}} &= \frac{(U_{\text{above}} \times A_{\text{above}}) + (U_{\text{below}} \times A_{\text{below}})}{A_{\text{total}}} \\
 &= \frac{(0.1002 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F} \times 384 \text{ ft}^2) + (0.0885 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F} \times 640 \text{ ft}^2)}{1024 \text{ ft}^2} \\
 &= \frac{38.4768 \text{ Btu hr}^{-1} \text{ }^\circ\text{F} + 56.64 \text{ Btu hr}^{-1} \text{ }^\circ\text{F}}{1024 \text{ ft}^2} \\
 &= \frac{95.1168 \text{ Btu hr}^{-1} \text{ }^\circ\text{F}}{1024 \text{ ft}^2} \\
 &\approx 0.093 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F}
 \end{aligned}$$

Answer:

The estimated overall heat-transfer coefficient for the walls is approximately:

$$U_{\text{overall}} \approx 0.093 \text{ Btu hr}^{-1} \text{ ft}^2 \text{ }^\circ\text{F}$$

Problem 2: Computing Unit Conductance for Fiberboard

Question:

Compute the unit conductance C for 5.5-inch (140mm) fiberboard with a thermal conductivity of $0.3 \text{ Btu} \cdot \text{in.} / (\text{hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F})$ [$0.043 \text{ W} / (\text{m} \cdot {}^\circ\text{C})$] in both English units and SI units.

Solution:

1. Given Data

- Thickness (L):

$$L = 5.5 \text{ in} = 0.4583 \text{ ft} \quad \text{or} \quad L = 0.14 \text{ m}$$

- Thermal conductivity (k):

$$k = 0.3 \text{ Btu} \cdot \text{in} \cdot \text{hr}^{-1} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \quad \text{or} \quad k = 0.043 \text{ W} \cdot \text{m}^{-1} \cdot {}^\circ\text{C}$$

2. Computing Unit Conductance in English Units

The unit conductance C is given by:

$$C = \frac{k}{L}$$

Convert L to consistent units (since k is in , we need L in inches):

$$L = 5.5 \text{ in}$$

Calculate C :

$$C = \frac{0.3 \text{ Btu} \cdot \text{in} \cdot \text{hr}^{-1} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}{5.5 \text{ in}} = 0.0545 \text{ Btu} \cdot \text{hr}^{-1} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

3. Computing Unit Conductance in SI Units

Use $L = 0.14 \text{ m}$ and $k = 0.043 \text{ W} \cdot \text{m}^{-1} \cdot {}^\circ\text{C}$.

Calculate C :

$$C = \frac{k}{L} = \frac{0.043 \text{ W} \cdot \text{m}^{-1} \cdot {}^\circ\text{C}}{0.14 \text{ m}} = 0.307 \text{ W} \cdot \text{m}^{-2} \cdot {}^\circ\text{C}$$

Answer:

The unit conductance C for the fiberboard is:

$$C = 0.0545 \text{ Btu} \cdot \text{hr}^{-1} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \quad (\text{English Units})$$

$$C = 0.307 \text{ W} \cdot \text{m}^{-2} \cdot {}^\circ\text{C} \quad (\text{SI Units})$$

Problem 5-4: Unit Thermal Resistance of an Inside Partition

Question:

What is the unit thermal resistance for an inside partition made up of 0.375 in gypsum board on each side of a 6 in lightweight aggregate block with vermiculite-filled cores?

Solution:

1. Given Data

- Thickness of gypsum board:

$$L_{\text{gypsum}} = 0.375 \text{ in} = 0.375 \text{ in}$$

- Thickness of aggregate block:

$$L_{\text{block}} = 6 \text{ in}$$

- Thermal conductivity of gypsum board:

$$k_{\text{gypsum}} = 0.17 \text{ Btu in hr}^{-1} \text{ ft}^2 \text{ }^{\circ}\text{F}$$

- Thermal conductivity of lightweight aggregate block (with vermiculite-filled cores):

$$k_{\text{block}} = [\text{Insert value, e.g., } 0.3 \text{ Btu in hr}^{-1} \text{ ft}^2 \text{ }^{\circ}\text{F}]$$

2. Calculating Thermal Resistances

1. Thermal Resistance of One Gypsum Board Layer:

$$R_{\text{gypsum}} = \frac{L_{\text{gypsum}}}{k_{\text{gypsum}}} = \frac{0.375 \text{ in}}{0.17 \text{ Btu in hr}^{-1} \text{ ft}^2 \text{ }^{\circ}\text{F}} = 2.206 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

2. Thermal Resistance of Aggregate Block:

$$R_{\text{block}} = \frac{L_{\text{block}}}{k_{\text{block}}} = \frac{6 \text{ in}}{[\text{Insert } k_{\text{block}}]} = [\text{Compute value}]$$

3. Total Thermal Resistance:

$$R_{\text{total}} = R_{\text{gypsum}} + R_{\text{block}} + R_{\text{gypsum}} = 2 \times R_{\text{gypsum}} + R_{\text{block}}$$

$$R_{\text{total}} = 2 \times 2.206 \text{ hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1} + R_{\text{block}} = [\text{Compute total}]$$

3. Answer

The unit thermal resistance for the partition is:

$$R_{\text{total}} = [\text{Final value}] \quad (\text{hr ft}^2 \text{ }^{\circ}\text{F Btu}^{-1})$$

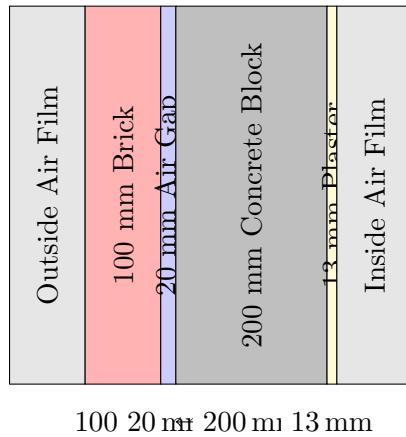
Problem 5-9: Overall Thermal Resistance of a Wall with Brick, Air Gap, and Concrete Block

Question:

Compute the overall thermal resistance of a wall made up of 100 mm brick (1920 kg m^{-3}) and 200 mm normal weight concrete block with a 20 mm air gap between. There is 13 mm of gypsum plaster on the inside. Assume a 7 m s^{-1} wind velocity on the outside and still air inside.

Solution:

1. Wall Structure Diagram



2. Given Data

- Layer Thicknesses:

$$L_{\text{brick}} = 100 \text{ mm} = 0.1 \text{ m}$$

$$L_{\text{air_gap}} = 20 \text{ mm} = 0.02 \text{ m}$$

$$L_{\text{concrete}} = 200 \text{ mm} = 0.2 \text{ m}$$

$$L_{\text{plaster}} = 13 \text{ mm} = 0.013 \text{ m}$$

- Thermal Conductivities (assumed values):

$$k_{\text{brick}} = 0.72 \text{ W m}^{-1} \text{ K}^{-1}$$

$$k_{\text{concrete}} = 1.4 \text{ W m}^{-1} \text{ K}^{-1}$$

$$k_{\text{plaster}} = 0.22 \text{ W m}^{-1} \text{ K}^{-1}$$

- Thermal Resistance of Air Gap:

$$R_{\text{air_gap}} = 0.18 \text{ m}^2 \text{ K W}^{-1}$$

(Note: Value from standard tables for a 20 mm vertical air gap.)

- Convective Heat Transfer Coefficients:

$$h_{\text{outside}} = 34 \text{ W m}^{-2} \text{ K}^{-1} \quad (\text{Assumed based on wind speed})$$

$$h_{\text{inside}} = 10 \text{ W m}^{-2} \text{ K}^{-1} \quad (\text{Still air})$$

3. Calculations

1. Calculate Thermal Resistances of Each Layer:

$$R_{\text{brick}} = \frac{L_{\text{brick}}}{k_{\text{brick}}} = \frac{0.1 \text{ m}}{0.72 \text{ W m}^{-1} \text{ K}^{-1}} = 0.1389 \text{ m}^2 \text{ K W}^{-1}$$

$$R_{\text{concrete}} = \frac{L_{\text{concrete}}}{k_{\text{concrete}}} = \frac{0.2 \text{ m}}{1.4 \text{ W m}^{-1} \text{ K}^{-1}} = 0.1429 \text{ m}^2 \text{ K W}^{-1}$$

$$R_{\text{plaster}} = \frac{L_{\text{plaster}}}{k_{\text{plaster}}} = \frac{0.013 \text{ m}}{0.22 \text{ W m}^{-1} \text{ K}^{-1}} = 0.0591 \text{ m}^2 \text{ K W}^{-1}$$

2. Calculate Convective Resistances:

$$R_{\text{outside}} = \frac{1}{h_{\text{outside}}} = \frac{1}{34 \text{ W m}^{-2} \text{ K}^{-1}} = 0.0294 \text{ m}^2 \text{ K W}^{-1}$$

$$R_{\text{inside}} = \frac{1}{h_{\text{inside}}} = \frac{1}{10 \text{ W m}^{-2} \text{ K}^{-1}} = 0.1 \text{ m}^2 \text{ K W}^{-1}$$

3. Total Thermal Resistance:

$$R_{\text{total}} = R_{\text{outside}} + R_{\text{brick}} + R_{\text{air_gap}} + R_{\text{concrete}} + R_{\text{plaster}} + R_{\text{inside}}$$

$$R_{\text{total}} = 0.0294 + 0.1389 + 0.18 + 0.1429 + 0.0591 + 0.1 = 0.65 \text{ m}^2 \text{ K W}^{-1}$$

4. Overall Heat-Transfer Coefficient:

$$U = \frac{1}{R_{\text{total}}} = \frac{1}{0.65 \text{ m}^2 \text{ K W}^{-1}} = 1.538 \text{ W m}^{-2} \text{ K}^{-1}$$

4. Answer

The overall thermal resistance of the wall is:

$$R_{\text{total}} = 0.65 \text{ m}^2 \text{ K W}^{-1}$$

The overall heat-transfer coefficient is:

$$U = 1.538 \text{ W m}^{-2} \text{ K}^{-1}$$

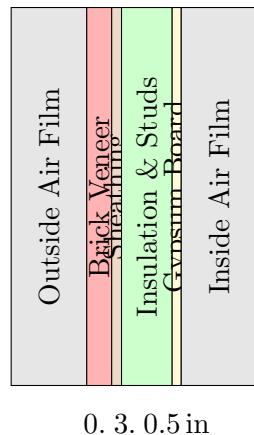
Problem 5-10: Heat-Transfer Coefficient for Frame Construction Wall

Question:

Compute the overall heat-transfer coefficient for a frame construction wall made of brick veneer (120^3) with 3 in insulation batts between the 2x4 studs on 16 in centers; the wind velocity is 15 h^{-1} .

Solution:

1. Wall Structure Diagram



2. Given Data

- Layer Thicknesses:

$$L_{\text{brick}} = 4 \text{ in} = 0.333$$

$$L_{\text{sheathing}} = 0.5 \text{ in} = 0.042$$

$$L_{\text{insulation}} = 3.5 \text{ in} = 0.292$$

$$L_{\text{gypsum}} = 0.5 \text{ in} = 0.042$$

- Thermal Resistances:

- **Brick Veneer:** From standard tables, $R_{\text{brick}} = 0.44 \text{ h}^2 \text{ }^\circ\text{F Btu}^{-1}$
- **Sheathing:** $R_{\text{sheathing}} = 0.62 \text{ h}^2 \text{ }^\circ\text{F Btu}^{-1}$
- **Insulation:** $R_{\text{insulation}} = 11 \text{ h}^2 \text{ }^\circ\text{F Btu}^{-1}$
- **Gypsum Board:** $R_{\text{gypsum}} = 0.45 \text{ h}^2 \text{ }^\circ\text{F Btu}^{-1}$

- Convective Heat Transfer Coefficients:

$$h_{\text{outside}} = 4 \text{ Btu h}^{-2} \text{ }^\circ\text{F}^{-1} \quad (\text{Based on wind speed})$$

$$h_{\text{inside}} = 1.46 \text{ Btu h}^{-2} \text{ }^\circ\text{F}^{-1} \quad (\text{Still air})$$

- Convective Resistances:

$$R_{\text{outside}} = \frac{1}{h_{\text{outside}}} = \frac{1}{4 \text{ Btu h}^{-2} \text{ }^{\circ}\text{F}^{-1}} = 0.25 \text{ h}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

$$R_{\text{inside}} = \frac{1}{h_{\text{inside}}} = \frac{1}{1.46 \text{ Btu h}^{-2} \text{ }^{\circ}\text{F}^{-1}} = 0.685 \text{ h}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

3. Calculations

1. Total Thermal Resistance:

$$R_{\text{total}} = R_{\text{outside}} + R_{\text{brick}} + R_{\text{sheathing}} + R_{\text{insulation}} + R_{\text{gypsum}} + R_{\text{inside}}$$

$$R_{\text{total}} = 0.25 + 0.44 + 0.62 + 11 + 0.45 + 0.685 = 13.445 \text{ h}^2 \text{ }^{\circ}\text{F Btu}^{-1}$$

2. Overall Heat-Transfer Coefficient:

$$U = \frac{1}{R_{\text{total}}} = \frac{1}{13.445 \text{ h}^2 \text{ }^{\circ}\text{F Btu}^{-1}} = 0.0744 \text{ Btu h}^{-2} \text{ }^{\circ}\text{F}^{-1}$$

4. Answer

The overall heat-transfer coefficient for the wall is:

$$U = 0.0744 \text{ Btu h}^{-2} \text{ }^{\circ}\text{F}^{-1}$$