HVAC Heating Load and Humidification Calculation

Given Data

- Total heating load of the building:

$$Q_{\text{total}} = 200{,}000 \text{ Btu/hr}$$

- Sensible heat factor (SHF):

$$\text{SHF} = 0.8$$

- Indoor (space) conditions to be maintained:

$$T_{\text{space}} = 72°F, \quad RH_{\text{space}} = 30\%$$

- Outdoor conditions (ventilation air):

$$T_{\text{out}} = 40°F, \quad RH_{\text{out}} = 20\%$$

- Required outdoor air flow:

$$V_{\text{out}} = 1000 \text{ cfm}$$

- Supply air temperature:

$$T_{\text{sup}} = 120°F$$

- Water vapor for humidification has an enthalpy of:

$$h_{\text{vapor}} = 1150 \text{ Btu/lb}_{\text{ma}}$$

- Assume sea-level pressure and standard air properties.

Step-by-Step Solution

Breaking Down the Total Load

The total heating load is given as:

$$Q_{\text{total}} = 200{,}000 \text{ Btu/hr}$$

Given the sensible heat factor (SHF = 0.8), we can separate the load into sensible and latent components:

$$Q_{\text{sens}} = \text{SHF} \times Q_{\text{total}} = 0.8 \times 200{,}000 = 160{,}000 \text{ Btu/hr}$$

The latent load is then:

$$Q_{\text{lat}} = Q_{\text{total}} - Q_{\text{sens}} = 200{,}000 - 160{,}000 = 40{,}000 \text{ Btu/hr}$$

Thus, the building load consists of:

$$Q_{\text{sens}} = 160{,}000 \text{ Btu/hr}, \quad Q_{\text{lat}} = 40{,}000 \text{ Btu/hr}.$$

(a) Conditions and Amount of Air Supplied

The sensible load in the space is handled by supplying air at a higher temperature than the room. The sensible load equation is:

$$Q_{\text{sens}} = \dot{m}_{\text{air}} c_p (T_{\text{sup}} - T_{\text{space}})$$

We know:

$$Q_{\text{sens}} = 160{,}000 \text{ Btu/hr}, \quad T_{\text{sup}} = 120^\circ F, \quad T_{\text{space}} = 72^\circ F.$$

The specific heat of air is approximately:

$$c_p \approx 0.24 \text{ Btu/lb}_{\text{da}}^\circ F.$$

To relate CFM to mass flow, we use the standard approximate density of air:

$$\rho_{\text{air}} \approx 0.075 \text{ lb}_{\text{da}}/\text{ft}^3.$$

Convert CFM to lb/hr of dry air:

$$1 \text{ cfm} = 1 \frac{\text{ft}^3}{\text{min}} \implies 1 \text{ cfm} \times 60 \frac{\text{min}}{\text{hr}} \times 0.075 \frac{\text{lb}_{\text{da}}}{\text{ft}^3} = 4.5 \text{ lb}_{\text{da}}/\text{hr}.$$

So, if V is the total supply airflow in CFM,

$$\dot{m}_{\text{air}} = 4.5V \text{ lb}_{\text{da}}/\text{hr}.$$

Plugging into the sensible heat equation:

$$160{,}000 = (4.5V)(0.24)(120 - 72)$$

Calculate the temperature difference:

$$120 - 72 = 48^\circ F.$$

Thus:

$$160{,}000 = 4.5 \times V \times 0.24 \times 48.$$

Compute the product $0.24 \times 48 = 11.52$, and then $4.5 \times 11.52 = 51.84$:

$$160{,}000 = V \times 51.84 \implies V = \frac{160{,}000}{51.84} \approx 3085 \text{ cfm}.$$

Hence, the total supply airflow required is about:

$$V_{\text{total}} \approx 3085 \text{ cfm}.$$

We are required to bring in:

$$V_{\text{out}} = 1000 \text{ cfm} \quad \text{from outdoors.}$$

The remainder must be recirculated:

$$V_{\text{recirc}} = V_{\text{total}} - V_{\text{out}} = 3085 - 1000 = 2085 \text{ cfm}.$$

Mixed Air Conditions

We have:

$$V_{\text{out}} = 1000 \text{ cfm}, \ T_{\text{out}} = 40^\circ F$$

$$V_{\text{recirc}} = 2085 \text{ cfm}, \ T_{\text{recirc}} = T_{\text{space}} = 72^\circ F$$

The mixed air temperature before heating:

$$T_{\text{mix}} = \frac{1000 \times 40 + 2085 \times 72}{3085}.$$

Compute numerator:

$$1000 \times 40 = 40{,}000, \quad 2085 \times 72 = 150{,}120.$$

Sum:

$$40{,}000 + 150{,}120 = 190{,}120.$$

Divide by 3085:

$$T_{\text{mix}} = \frac{190{,}120}{3085} \approx 61.6^\circ F.$$

Thus, the furnace sees mixed air at about $61.6^\circ F$ and heats it to $120^\circ F$.

(b) Temperature Rise Through the Furnace

The temperature rise in the furnace is:

$$\Delta T_{\text{furnace}} = T_{\text{sup}} - T_{\text{mix}} = 120 - 61.6 = 58.4^\circ F.$$

(c) Amount of Water Vapor Required

The latent load is 40,000 Btu/hr. To handle the latent load, we add water vapor. Each pound of water vapor added has an enthalpy of 1150 Btu/lb$_{\text{ma}}$.

The amount of water required per hour is:

$$\dot{m}_{\text{vapor}} = \frac{Q_{\text{lat}}}{h_{\text{vapor}}} = \frac{40{,}000 \text{ Btu/hr}}{1150 \text{ Btu/lb}} \approx 34.78 \text{ lb/hr}.$$

So approximately 34.8 lb/hr of water vapor must be added to maintain the required humidity level.

(d) Capacity of the Furnace

The total building load is 200,000 Btu/hr, consisting of:

$$Q_{\text{sens}} = 160{,}000 \text{ Btu/hr}, \quad Q_{\text{lat}} = 40{,}000 \text{ Btu/hr}.$$

The furnace (heating coil) is primarily responsible for supplying the sensible portion of the load. The humidification system supplies the latent portion. Therefore, the furnace capacity is the sensible portion:

$$\text{Furnace capacity} = 160{,}000 \text{ Btu/hr}.$$

Summary of Results

1. **Conditions and amount of air supplied:**

   $$V_{\text{total}} \approx 3085 \text{ cfm} \quad \text{with} \quad 1000 \text{ cfm outside air and } 2085 \text{ cfm return air}.$$

   Mixed air at about $61.6°F$ is heated to $120°F$ and delivered to the space at $72°F$ and 30% RH.

2. **Temperature rise through the furnace:**

   $$\Delta T_{\text{furnace}} = 58.4°F.$$

3. **Amount of water vapor required:**

   $$\dot{m}_{\text{vapor}} \approx 34.8 \text{ lb/hr}.$$

4. **Capacity of the furnace:**

   $$Q_{\text{furnace}} = 160{,}000 \text{ Btu/hr}.$$