Heat Transfer Assignments

Mechanical Engineering

February 22, 2025

Problem 1: Estimating Overall Heat-Transfer Coefficient for Building Walls

Question:

A building with dimensions 24 ft by 40 ft and a height of 8 ft has a full basement with uninsulated walls extending 5 ft below grade. The insides of the walls are finished with R-8 insulation, a thin vapor barrier, and a 0.5 in gypsum board. Estimate the overall heat-transfer coefficient for the walls.

Solution:

1. Wall Structure Analysis

The walls consist of the following layers (from inside to outside):

1. **Indoor air film resistance**
   $R_{\text{in}} = 0.68\,\text{hr}\,\text{ft}^2\,°\text{F}\,\text{Btu}^{-1}$
2. 0.5 in **gypsum board**
3. **Vapor barrier**
4. **R-8 insulation**
5. 8 in **concrete wall**
6. **Outdoor air film resistance**
   $R_{\text{out}} = 0.17\,\text{hr}\,\text{ft}^2\,°\text{F}\,\text{Btu}^{-1}$ *(above grade)*
   Soil resistance *(below grade)*

2. Material Thermal Resistances

- **Gypsum board (0.5 in):**

$$R_{\text{gyp}} = \frac{0.5\,\text{in}}{1.25\,\text{Btu}\,\text{in}\,\text{hr}^{-1}\,\text{ft}^2\,°\text{F}} = 0.4\,\text{hr}\,\text{ft}^2\,°\text{F}\,\text{Btu}^{-1}$$

- **Vapor barrier:**

$$R_{\text{vb}} = \frac{1}{16.70\,\text{W}\,\text{m}^{-2}\,\text{K}} \approx 0.06\,\text{hr}\,\text{ft}^2\,°\text{F}\,\text{Btu}^{-1}$$

  (Note: Since the vapor barrier's conductance C is given, we can convert units accordingly.)

- ***R*-8 insulation:**

$$R_{\text{ins}} = 8\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}$$

- **Concrete wall** $(8\,\text{in})$**:**

$$R_{\text{conc}} = \frac{8\,\text{in}}{12\,\text{Btu}\,\text{in}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}} = 0.67\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}$$

- **Soil resistance (below grade):**

$$R_{\text{soil}} \approx 1.5\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}$$

3. Thermal Resistance Calculations

Above Grade $(3\,\text{ft}$ **high):**

$$\begin{aligned}
R_{\text{total, above}} &= R_{\text{in}} + R_{\text{gyp}} + R_{\text{vb}} + R_{\text{ins}} + R_{\text{conc}} + R_{\text{out}} \\
&= 0.68\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.4\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.06\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} \\
&\quad + 8\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.67\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.17\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} \\
&= 9.98\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}
\end{aligned}$$

Below Grade $(5\,\text{ft}$ **deep):**

$$\begin{aligned}
R_{\text{total, below}} &= R_{\text{in}} + R_{\text{gyp}} + R_{\text{vb}} + R_{\text{ins}} + R_{\text{conc}} + R_{\text{soil}} \\
&= 0.68\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.4\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.06\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} \\
&\quad + 8\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 0.67\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + 1.5\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} \\
&= 11.31\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}
\end{aligned}$$

4. Calculating U-Values

Above Grade:

$$U_{\text{above}} = \frac{1}{R_{\text{total, above}}} = \frac{1}{9.98\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}} \approx 0.1002\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}$$

Below Grade:

$$U_{\text{below}} = \frac{1}{R_{\text{total, below}}} = \frac{1}{11.31\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}} \approx 0.0885\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}$$

5. Area Calculations

- **Perimeter of the building:**

$$P = 2 \times (24\,\text{ft} + 40\,\text{ft}) = 128\,\text{ft}$$

- **Total wall area:**

$$A_{\text{total}} = P \times 8\,\text{ft} = 1024\,\text{ft}^2$$

- **Above-grade wall area:**

$$A_{\text{above}} = P \times 3\,\text{ft} = 384\,\text{ft}^2$$

- **Below-grade wall area:**

$$A_{\text{below}} = A_{\text{total}} - A_{\text{above}} = 640\,\text{ft}^2$$

6. Overall Heat-Transfer Coefficient

Using area-weighted averaging:

$$\begin{aligned}
U_{\text{overall}} &= \frac{(U_{\text{above}} \times A_{\text{above}}) + (U_{\text{below}} \times A_{\text{below}})}{A_{\text{total}}} \\
&= \frac{(0.1002\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F} \times 384\,\text{ft}^2) + (0.0885\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F} \times 640\,\text{ft}^2)}{1024\,\text{ft}^2} \\
&= \frac{38.4768\,\text{Btu}\,\text{hr}^{-1}\,{}^\circ\text{F} + 56.64\,\text{Btu}\,\text{hr}^{-1}\,{}^\circ\text{F}}{1024\,\text{ft}^2} \\
&= \frac{95.1168\,\text{Btu}\,\text{hr}^{-1}\,{}^\circ\text{F}}{1024\,\text{ft}^2} \\
&\approx 0.093\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}
\end{aligned}$$

Answer:
The estimated overall heat-transfer coefficient for the walls is approximately:

$$U_{\text{overall}} \approx 0.093\,\text{Btu}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}$$

Problem 2: Computing Unit Conductance for Fiberboard

Question:

Compute the unit conductance C for 5.5-inch (140mm) fiberboard with a thermal conductivity of 0.3Btu·in./(hr·ft²·°F) [0.043 W/(m·°C)] in both English units and SI units.

Solution:

1. Given Data

- Thickness (L):

$$L = 5.5\,\text{in} = 0.4583\,\text{ft} \quad \text{or} \quad L = 0.14\,\text{m}$$

- Thermal conductivity (k):

$$k = 0.3\,\text{Btu in hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F} \quad \text{or} \quad k = 0.043\,\text{W m}^{-1}\,{}^\circ\text{C}$$

2. Computing Unit Conductance in English Units

The unit conductance C is given by:

$$C = \frac{k}{L}$$

Convert L to consistent units (since k is in , we need L in inches):

$$L = 5.5\,\text{in}$$

Calculate C:

$$C = \frac{0.3\,\text{Btu in hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}}{5.5\,\text{in}} = 0.0545\,\text{Btu hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}$$

3. Computing Unit Conductance in SI Units

Use $L = 0.14\,\text{m}$ and $k = 0.043\,\text{W m}^{-1}\,{}^\circ\text{C}$.

Calculate C:

$$C = \frac{k}{L} = \frac{0.043\,\text{W m}^{-1}\,{}^\circ\text{C}}{0.14\,\text{m}} = 0.307\,\text{W m}^{-2}\,{}^\circ\text{C}$$

Answer:

The unit conductance C for the fiberboard is:

$$C = 0.0545\,\text{Btu hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F} \quad \text{(English Units)}$$
$$C = 0.307\,\text{W m}^{-2}\,{}^\circ\text{C} \quad \text{(SI Units)}$$

Problem 5-4: Unit Thermal Resistance of an Inside Partition

Question:

What is the unit thermal resistance for an inside partition made up of 0.375 in gypsum board on each side of a 6 in lightweight aggregate block with vermiculite-filled cores?

Solution:

1. Given Data

- Thickness of gypsum board:

$$L_{\text{gypsum}} = 0.375\,\text{in} = 0.375\,\text{in}$$

- Thickness of aggregate block:

$$L_{\text{block}} = 6\,\text{in}$$

- Thermal conductivity of gypsum board:

$$k_{\text{gypsum}} = 0.17\,\text{Btu}\,\text{in}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}$$

- Thermal conductivity of lightweight aggregate block (with vermiculite-filled cores):

$$k_{\text{block}} = [\text{Insert value, e.g., } 0.3\,\text{Btu}\,\text{in}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}]$$

2. Calculating Thermal Resistances

1. **Thermal Resistance of One Gypsum Board Layer:**

$$R_{\text{gypsum}} = \frac{L_{\text{gypsum}}}{k_{\text{gypsum}}} = \frac{0.375\,\text{in}}{0.17\,\text{Btu}\,\text{in}\,\text{hr}^{-1}\,\text{ft}^2\,{}^\circ\text{F}} = 2.206\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}$$

2. **Thermal Resistance of Aggregate Block:**

$$R_{\text{block}} = \frac{L_{\text{block}}}{k_{\text{block}}} = \frac{6\,\text{in}}{[\text{Insert } k_{\text{block}}]} = [\text{Compute value}]$$

3. **Total Thermal Resistance:**

$$R_{\text{total}} = R_{\text{gypsum}} + R_{\text{block}} + R_{\text{gypsum}} = 2 \times R_{\text{gypsum}} + R_{\text{block}}$$

$$R_{\text{total}} = 2 \times 2.206\,\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1} + R_{\text{block}} = [\text{Compute total}]$$

3. Answer

The unit thermal resistance for the partition is:

$$R_{\text{total}} = [\text{Final value}] \quad \left(\text{hr}\,\text{ft}^2\,{}^\circ\text{F}\,\text{Btu}^{-1}\right)$$

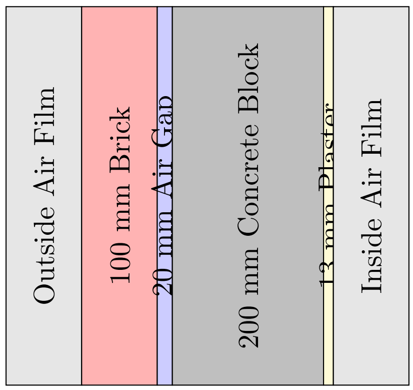
Problem 5-9: Overall Thermal Resistance of a Wall with Brick, Air Gap, and Concrete Block

Question:

Compute the overall thermal resistance of a wall made up of 100 mm brick ($1920\,\mathrm{kg\,m^{-3}}$) and 200 mm normal weight concrete block with a 20 mm air gap between. There is 13 mm of gypsum plaster on the inside. Assume a $7\,\mathrm{m\,s^{-1}}$ wind velocity on the outside and still air inside.

Solution:

1. Wall Structure Diagram



100 20 mm 200 mm 13 mm

2. Given Data

- **Layer Thicknesses:**

$$L_{\text{brick}} = 100\,\mathrm{mm} = 0.1\,\mathrm{m}$$
$$L_{\text{air\_gap}} = 20\,\mathrm{mm} = 0.02\,\mathrm{m}$$
$$L_{\text{concrete}} = 200\,\mathrm{mm} = 0.2\,\mathrm{m}$$
$$L_{\text{plaster}} = 13\,\mathrm{mm} = 0.013\,\mathrm{m}$$

- **Thermal Conductivities (assumed values):**

$$k_{\text{brick}} = 0.72\,\mathrm{W\,m^{-1}\,K^{-1}}$$
$$k_{\text{concrete}} = 1.4\,\mathrm{W\,m^{-1}\,K^{-1}}$$
$$k_{\text{plaster}} = 0.22\,\mathrm{W\,m^{-1}\,K^{-1}}$$

- **Thermal Resistance of Air Gap:**

$$R_{\text{air\_gap}} = 0.18\,\mathrm{m^2\,K\,W^{-1}}$$

(Note: Value from standard tables for a 20 mm vertical air gap.)

- **Convective Heat Transfer Coefficients:**

$$h_{\text{outside}} = 34\,\mathrm{W\,m^{-2}\,K^{-1}} \quad \text{(Assumed based on wind speed)}$$
$$h_{\text{inside}} = 10\,\mathrm{W\,m^{-2}\,K^{-1}} \quad \text{(Still air)}$$

3. Calculations

1. **Calculate Thermal Resistances of Each Layer:**

$$R_{\text{brick}} = \frac{L_{\text{brick}}}{k_{\text{brick}}} = \frac{0.1\,\text{m}}{0.72\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}} = 0.1389\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$
$$R_{\text{concrete}} = \frac{L_{\text{concrete}}}{k_{\text{concrete}}} = \frac{0.2\,\text{m}}{1.4\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}} = 0.1429\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$
$$R_{\text{plaster}} = \frac{L_{\text{plaster}}}{k_{\text{plaster}}} = \frac{0.013\,\text{m}}{0.22\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}} = 0.0591\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$

2. **Calculate Convective Resistances:**

$$R_{\text{outside}} = \frac{1}{h_{\text{outside}}} = \frac{1}{34\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}} = 0.0294\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$
$$R_{\text{inside}} = \frac{1}{h_{\text{inside}}} = \frac{1}{10\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}} = 0.1\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$

3. **Total Thermal Resistance:**

$$R_{\text{total}} = R_{\text{outside}} + R_{\text{brick}} + R_{\text{air\_gap}} + R_{\text{concrete}} + R_{\text{plaster}} + R_{\text{inside}}$$

$$R_{\text{total}} = 0.0294 + 0.1389 + 0.18 + 0.1429 + 0.0591 + 0.1 = 0.65\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$

4. **Overall Heat-Transfer Coefficient:**

$$U = \frac{1}{R_{\text{total}}} = \frac{1}{0.65\,\text{m}^2\,\text{K}\,\text{W}^{-1}} = 1.538\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

4. Answer

The overall thermal resistance of the wall is:

$$R_{\text{total}} = 0.65\,\text{m}^2\,\text{K}\,\text{W}^{-1}$$

The overall heat-transfer coefficient is:

$$U = 1.538\,\text{W}\,\text{m}^{-2}\,\text{K}^{-1}$$

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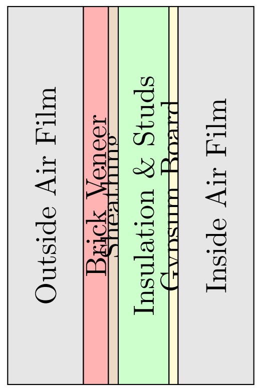
Problem 5-10: Heat-Transfer Coefficient for Frame Construction Wall

Question:

Compute the overall heat-transfer coefficient for a frame construction wall made of brick veneer ($120\,^3$) with 3 in insulation batts between the 2x4 studs on 16 in centers; the wind velocity is $15\,\mathrm{h}^{-1}$.

Solution:

1. Wall Structure Diagram



0. 3. 0.5 in

2. Given Data

- **Layer Thicknesses:**

$$
\begin{aligned}
L_{\text{brick}} &= 4\,\text{in} = 0.333 \\
L_{\text{sheathing}} &= 0.5\,\text{in} = 0.042 \\
L_{\text{insulation}} &= 3.5\,\text{in} = 0.292 \\
L_{\text{gypsum}} &= 0.5\,\text{in} = 0.042
\end{aligned}
$$

- **Thermal Resistances:**
  - **Brick Veneer:** From standard tables, $R_{\text{brick}} = 0.44\,\mathrm{h}^2\,{}^\circ\mathrm{F}\,\mathrm{Btu}^{-1}$
  - **Sheathing:** $R_{\text{sheathing}} = 0.62\,\mathrm{h}^2\,{}^\circ\mathrm{F}\,\mathrm{Btu}^{-1}$
  - **Insulation:** $R_{\text{insulation}} = 11\,\mathrm{h}^2\,{}^\circ\mathrm{F}\,\mathrm{Btu}^{-1}$
  - **Gypsum Board:** $R_{\text{gypsum}} = 0.45\,\mathrm{h}^2\,{}^\circ\mathrm{F}\,\mathrm{Btu}^{-1}$

- **Convective Heat Transfer Coefficients:**

$$
\begin{aligned}
h_{\text{outside}} &= 4\,\mathrm{Btu}\,\mathrm{h}^{-2}\,{}^\circ\mathrm{F}^{-1} \quad \text{(Based on wind speed)} \\
h_{\text{inside}} &= 1.46\,\mathrm{Btu}\,\mathrm{h}^{-2}\,{}^\circ\mathrm{F}^{-1} \quad \text{(Still air)}
\end{aligned}
$$

- **Convective Resistances:**

$$R_{\text{outside}} = \frac{1}{h_{\text{outside}}} = \frac{1}{4\,\text{Btu}\,\text{h}^{-2}\,{}^{\circ}\text{F}^{-1}} = 0.25\,\text{h}^{2}\,{}^{\circ}\text{F}\,\text{Btu}^{-1}$$

$$R_{\text{inside}} = \frac{1}{h_{\text{inside}}} = \frac{1}{1.46\,\text{Btu}\,\text{h}^{-2}\,{}^{\circ}\text{F}^{-1}} = 0.685\,\text{h}^{2}\,{}^{\circ}\text{F}\,\text{Btu}^{-1}$$

3. Calculations

1. **Total Thermal Resistance:**

$$R_{\text{total}} = R_{\text{outside}} + R_{\text{brick}} + R_{\text{sheathing}} + R_{\text{insulation}} + R_{\text{gypsum}} + R_{\text{inside}}$$

$$R_{\text{total}} = 0.25 + 0.44 + 0.62 + 11 + 0.45 + 0.685 = 13.445\,\text{h}^{2}\,{}^{\circ}\text{F}\,\text{Btu}^{-1}$$

2. **Overall Heat-Transfer Coefficient:**

$$U = \frac{1}{R_{\text{total}}} = \frac{1}{13.445\,\text{h}^{2}\,{}^{\circ}\text{F}\,\text{Btu}^{-1}} = 0.0744\,\text{Btu}\,\text{h}^{-2}\,{}^{\circ}\text{F}^{-1}$$

4. Answer

The overall heat-transfer coefficient for the wall is:

$$U = 0.0744\,\text{Btu}\,\text{h}^{-2}\,{}^{\circ}\text{F}^{-1}$$

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